NATIONAL RADIO ASTRONOMY OBSERVATORY
Post Office Box "O"
Socorro, New Mexico 87801

VLA SCIENTIFIC MEMORANDUM NO. 124

GRIDDING AND SIGNAL-TO-NOISE RATIOS

B. G. Clark

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VLA Scientific Memorandum No. 123 left the impression that, in the cases of practical importance for the VLA, the convolution-gridding process did not cause a variation in signal-to-noise ratio (SNR) over the map field. This is not true.

This is particularly obvious in one case of great importance: "pill box" convolution with resampling at the box width. Then the noise values at the various (u,v) points are independent random variables. The process of Fourier transforming is, in fact, adding these sample values together with weightings whose moduli do not vary over the output map. Therefore, the noise does not vary over the output map. However, it can be easily shown that if the actual u,v points are randomly and uniformly distributed with respect to the nearest resampling point, that the point spread function is multiplied by sinc x/2 sinc y/2, where (x,y) is the output plane location of the point spread function. This falls to $2/\pi$ at the edge of the field.

The property that the measured (u,v) samples are uniformly and randomly distributed relative to the nearest resampling point can be shown to hold for most cases of practical VLA importance. The correlation due to the Hermitian nature of the input also does not give rise to significant effects in the practical case.

The reason that Memo No. 123 is misleading is that it concerns itself with the case of resampling interval much shorter than the scale length of the convolution function, a case which might occur in some optical processor designs, but has no application to intentional convolution.

It is informative to calculate the magnitude of the effect for the numerical experiment of Memo No. 123, which can be easily done. The variance in the map plane is the Fourier transform of the autocorrelation function of the noise, which is the autocorrelation of the convolving function. For the numerical experiment, the "pillbox" function of 3.5 resampling intervals was used. The autocorrelation function is therefore 5/7, 3/7, and 1/7 for the points separated by 1, 2, and 3 intervals, respectively. Therefore, the variance along the x axis goes as

$$1+ 10/7 \cos x + 6/7 \cos 2x + 2/7 \cos 3x$$

whereas the amplitude of the point spread function goes as sinc 7x/4. The rms noise goes to 0.223 of the central value at the zero of the point spread function.

The ratio of the point spread function to the square root of the variance (i.e., the SNR) is very near one for most of the field displayed but appears in Figure 2b of Memo 123 to go about .9 at the edge on axis and to smaller values in the corners of the field where the point spread function is near zero. With the parameters given, the signal-to-noise ratio should fall to .855 at the edge of the field (a reduction of one contour interval). Therefore the numerical experiment is in accord with the analysis.

The effect is clearly present only in the corners of Figure 2b, but would have been easily visible had a display been employed that was more appropriate for the data in hand.