VLA SCIENTIFIC MEMORANDUM NO. 136

Robust Solution for Antenna Gains

Frederic R. Schwab

September, 1981

VLA visibility data bases frequently are contaminated by "wild" points, or, in statistical jargon, by <u>outliers</u>. The outliers may be due, for example, to transient problems in the receiver/i.f. chains, to sampler problems or correlator problems, or to faults in system error detection. Because that part of the standard calibration procedure which consists in solution for antenna gains is quite sensitive to discordant data points, the calibrator data must be edited carefully before calibration. I will outline a new method for the solution for antenna gains which is much less sensitive to outliers than is the standard method and which, moreover, in the presence of well-behaved random errors, performs nearly as well as the standard least-squares method. Therefore, much of the burden of data editing can be shifted to the post-calibration stage of data reduction. Because the usual map/CLEAN procedure is quite sensitive to wild points, editing is still required -- but it may be accomplished somewhat more easily once the data have been corrected for antenna gains. I shall pay no attention here to the problem of identifying outliers, but only to the task of describing a calibration scheme which is not too severely affected by them.

Both the standard calibration program (ANTSOL) and the self-calibration program - until its most recent implementation - have been based on the method of least-squares. A popular (at least in linear regression), but non-classical, approach to parameter estimation in the presence of outliers is, instead of solving for parameters which minimize the sum of squared residuals, to solve for parameters which minimize the sum of absolute values of the residuals. The latter approach is known as ℓ_1 minimization, and least-squares as ℓ_2, since in both cases one minimizes a discrete ℓ_p norm of the residuals, p = 1 or 2. To gain an intuitive feeling for the difference, consider that in the case of linear regression, if the constant term, or intercept, is included in the regression model, then the ℓ_2 solution has the property that the <u>mean</u> residual is equal to zero, whereas an ℓ_1 solution (the ℓ_1 solution to an overdetermined linear system is not necessarily unique) has the property that the <u>median</u> residual is zero. ℓ_1 solutions to linear problems can be computed by standard linear programming methods. Nonlinear ℓ_1 minimization, as is required for our problem, has not received much attention in the literature, but effective algorithms for this purpose recently have been published [1,2].

The basic idea of the approach to nonlinear ℓ_1 minimization that is outlined in [1] is this:

> Replace the problem
>
> find a vector x which minimizes $S(x) = \sum_i |f_i(x)|$
>
> by a family of problems
>
> find a vector x which minimizes $S_\varepsilon(x) = \sum_i [f_i^2(x)+\varepsilon]^{\frac{1}{2}}$
>
> and, for a sequence $\varepsilon_1 > \varepsilon_2 > \ldots$, tending to 0, apply an extrapolation procedure to the corresponding sequence of solution vectors, $x_1, x_2, \ldots$.

(The extrapolation procedure converts a convergent sequence to a sequence which converges no less rapidly -- in practice, for a variety of extrapolation methods in common use, the convergence usually is accelerated.) Such an indirect method is required because, for well-behaved f_i, S(x) may be nondifferentiable at points where some f_i is equal to 0. Extrapolation usually is required because the problems get more ill-conditioned as ε tends to 0. The extrapolation method recommended in [1] is a standard one due to Fiacco and McCormick. In the present implementation of the method in the self-calibration program I haven't employed extrapolation because, in a variety of tests, the solutions have tended to stabilize for small ε before numerical instability has set in. I do, however, solve a sequence of problems (in a typical case, say, with ε = 5, .5, and .05 mJy) since my minimization algorithms tend to require better initial guesses as ε decreases.

For general purpose ℓ_1 minimization the method outlined in [2] ought to be a more reliable method than that in [1]. For our specialized problem, I believe that the above approach is adequate.

For the case of complex antenna gains, we wish, given instantaneous observations $\tilde{V}_{ij}$ obtained on $n(n-1)/2$ baselines $i-j$ $(i<j)$, given corresponding model visibilities V_{ij}, and given weights w_{ij}, to solve for $g = (g_i)_{i=1}^{n}$ by minimizing the summation

$$S_\varepsilon(g) = \sum_{i<j} w_{ij} \left[|\tilde{V}_{ij} - g_i \bar{g}_j V_{ij}|^2 + \varepsilon \right]^{\frac{1}{2}} .$$

A necessary condition for S_ε to be minimized is that its gradient, $\nabla S_\varepsilon(g)$, be equal to 0. We can rewrite the equation $\nabla S_\varepsilon(g) = 0$ so that the k^{th} antenna gain, in terms of the others, is given by

$$g_k = \frac{\sum_{i<k} w_{ik} \left[|R_{ik}|^2 + \varepsilon\right]^{-\frac{1}{2}} g_i V_{ik} \overline{\tilde{V}}_{ik} + \sum_{k<j} w_{kj} \left[|R_{kj}|^2 + \varepsilon\right]^{-\frac{1}{2}} g_j \bar{V}_{kj} \tilde{V}_{kj}}{\sum_{i<k} w_{ik} \left[|R_{ik}|^2 + \varepsilon\right]^{-\frac{1}{2}} |g_i V_{ik}|^2 + \sum_{k<j} w_{kj} \left[|R_{kj}|^2 + \varepsilon\right]^{-\frac{1}{2}} |g_j V_{kj}|^2}$$

$\equiv f_k(g_1, g_2, \ldots, g_{k-1}, g_{k+1}, \ldots, g_n)$, where $R_{ij} \equiv \tilde{V}_{ij} - g_i \bar{g}_j V_{ij}$.

Choosing an initial guess, $g^{(0)}$, we can use an iterative relaxation method where, at the (m+1)st iteration, the k^{th} antenna gain (for fixed ε) is approximated by

$$g_k^{(m+1)} = g_k^{(m)} + \omega[f_k(g_1^{(m+1)}, g_2^{(m+1)}, \ldots, g_{k-1}^{(m+1)}, g_{k+1}^{(m)}, \ldots, g_n^{(m)}) - g_k^{(m)}].$$

In practice, I have found successive over-relaxation (with $\omega = 1.5$) to converge reliably and in about the same number of iterations as the corresponding method for ℓ_2 solutions.* Typically, for each ε, 5-10 iterations are required in order to achieve a relative error of 10^{-4} in the solutions, starting, for ε_1, with an initial guess $g^{(0)}=1$.

Phase errors generally have a more deleterious effect on maps than do amplitude errors, thus, especially in self-calibration, one is sometimes driven by signal-to-noise considerations to wish to reduce the number of parameters and so to solve only for antenna-based phase errors, $\psi = (\psi_i)_{i=1}^{n}$. We then have

$$S_\varepsilon(\psi) = \sum_{i<j} w_{ij} \left[|\tilde{V}_{ij} - e^{i(\psi_i - \psi_j)} V_{ij}|^2 + \varepsilon \right]^{\frac{1}{2}} .$$

S_ε is periodic of period 2π in each of the ψ_k, and we may use this fact to drive an iterative scheme to a local minimum of S_ε rather than to a

---

*for ℓ_2 solutions,

$$g_k = \frac{\sum_{i<k} w_{ik}\, g_i\, V_{ik}\, \overline{\tilde{V}}_{ik} + \sum_{k<j} w_{kj}\, g_j\, \overline{V}_{kj}\, \tilde{V}_{kj}}{\sum_{i<k} w_{ik} |g_i\, V_{ik}|^2 + \sum_{k<j} w_{kj} |g_j\, V_{kj}|^2} .$$

local maximum:

$$\psi_k^{(m+1)} = \psi_k^{(m)} - \omega \tan^{-1}\left(\frac{\partial S_\varepsilon/\partial\psi_k}{\partial^2 S_\varepsilon/\partial\psi_k^{\,2}}\right).$$

$$\frac{\partial S_\varepsilon}{\partial\psi_k} = -\sum_{i<k} w_{ik}\,[|R_{ik}|^2+\varepsilon]^{-\frac{1}{2}}\,\mathrm{Im}(L) + \sum_{k<j} w_{kj}\,[|R_{kj}|^2+\varepsilon]^{-\frac{1}{2}}\,\mathrm{Im}(M)$$

and

$$\frac{\partial^2 S_\varepsilon}{\partial\psi_k^{\,2}} = \sum_{i<k} w_{ik}\left\{[|R_{ik}|^2+\varepsilon]^{-\frac{1}{2}}\,\mathrm{Re}(L) - [|R_{ik}|^2+\varepsilon]^{-3/2}\,\mathrm{Im}^2(L)\right\}$$

$$+ \sum_{k<j} w_{kj}\left\{[|R_{kj}|^2+\varepsilon]^{-\frac{1}{2}}\,\mathrm{Re}(M) - [|R_{kj}|^2+\varepsilon]^{-3/2}\,\mathrm{Im}^2(M)\right\},$$

where $L \equiv e^{i(\psi_i-\psi_k)}$, $M \equiv e^{i(\psi_k-\psi_j)}$, and $R_{ij} \equiv \tilde{V}_{ij} - e^{i(\psi_i-\psi_j)}V_{ij}$. *

[See erratum sheet preceeding this memo]

In the circumstance in which the real parts of the visibility observations have an error distribution different from that of the imaginary parts, and perhaps in the circumstance in which the noise in the real part of an observation is independent of the noise in the imaginary part of that observation, it might be preferable to minimize the sum of the

---

*for ℓ_2 solutions,

$$2\,\frac{\partial S}{\partial\psi_k} = -\sum_{i<k} w_{ik}\mathrm{Im}(L) + \sum_{k<j} w_{kj}\mathrm{Im}(M) \quad \text{and}$$

$$2\,\frac{\partial^2 S}{\partial\psi_k^{\,2}} = \sum_{i<k} w_{ik}\mathrm{Re}(L) + \sum_{k<j} w_{kj}\mathrm{Re}(M).$$

absolute values of the real parts of the residuals plus the sum of the absolute values of the imaginary parts of the residuals, rather than the sum of the moduli of the complex residuals. However, I have tested this approach for the case of well-behaved random errors, with identical error distributions for real and imaginary, but independent errors in real part and imaginary part, and I found, in that case, very little difference in the computed solutions. In my implementation, this alternative solution method is much slower because more arithmetic is required. For brevity, the algebraic machinery for the alternative method has been omitted from this report.

The results of a series of tests comparing the ℓ_1 gain solution method with the ℓ_2 method are presented below. The data used in each test run simulate the observations of a one Jansky point source (located at the visibility phase reference position) by a 27 element interferometer. The visibility model exactly represents the source; i.e., $V_{ij} \equiv 1$. The observations $\tilde{V}_{ij}$ $(1 \leq i < j \leq 27)$ for each run are these model visibilities, modified by 27 multiplicative complex gains $g = (g_i)_{i=1}^{27}$, possibly with random noise added in. Weights $w_{ij} \equiv 1$ were used in all cases. In order to simulate wild observations, in some cases a number of the data points have been chosen at random and have no relation to the visibility model or to g.

Any solution to one of the minimization problems described above is unique only up to a multiplicative constant of unit modulus; that is, if $\hat{g}$ is one local minimizer, then $c\hat{g}$ is another, whenever $|c|=1$. The solution algorithms are such that, with no special attention to the fact that the problems are underdetermined, they always converge upon some solution* (barring unusual circumstances). In order most fairly to compare a com-

---

*Interestingly, if the argument of some gain is held fixed, then the rate of convergence is diminished.

puted solution, $\hat{g}$, with the true solution, g, each computed solution has been multiplied by that factor, c, of unit modulus, appropriate to minimize $\Sigma|g_i - c\hat{g}_i|^2$. In terms of the true solution, c is given by $\Sigma g_i \bar{\hat{g}}_i \,/\, |\Sigma g_i \bar{\hat{g}}_i|$.

<u>Behavior when the data are contaminated only by well-behaved random errors</u>

The ℓ_1 method clearly would be unattractive if, given data contaminated only by well-behaved random errors, it were to behave much worse than the least-squares method. Two series of tests were performed in which the model observations were contaminated by adding varying levels of zero mean (approximately-) Gaussian distributed random noise (independently) to the real parts and to the imaginary parts of the visibility observations; i.e., $\tilde{V}_{ij} = g_i\bar{g}_jV_{ij}$ + noise ($=g_i\bar{g}_j$ + noise, for our 1 Jy point model). To test the two phase solution methods, the antenna gains were chosen to be of unit modulus, with phases drawn at random from the uniform distribution on [0°, 360°]. To test the complex gain solution methods, the phases of the antenna gains were chosen in the same manner as before, and the moduli were drawn from the uniform distribution on [.5, 1.5]. Ten trials were performed for each of eleven choices of standard deviation of the random noise. A different choice of gains and a different choice of noise were made for each trial (except that the same data were used for both the ℓ_1 and ℓ_2 solution tests; hence there will be some correlation in the mean solution errors, between ℓ_1 and ℓ_2). The rms error in each gain solution was computed as $\sqrt{n^{-1}\Sigma|g_i-\hat{g}_i|^2}$. The mean of these errors, for each set of ten trials, is shown in Table 1 and in the plot of Figure 1. For S/N $\gtrsim$ 3 (resp., $\gtrsim$ 2) the rms gain solution error remains $\lesssim$ .1 for the phase solutions (resp., for the gain solutions). Excluding the points at 2.0 Jy random noise, the ℓ_1 phase solution error (resp., gain solution error) is typically about 13% (resp., 14%) greater than the ℓ_2 error. The results of two representative trials (which were not included in the test sample) are shown in Figure 2.

Another test which might be of interest is the case $\tilde{V}_{ij}=g_i\bar{g}_j(V_{ij}+\text{noise})$. Trials in which the observations were contaminated by noise of this form gave results which were very similar to those described above.

<u>Sensitivity of the solution methods to observations chosen at random to be outliers</u>

To test the sensitivity of the solution methods to randomly chosen outliers, some probability was assigned according to which an observation was chosen to be made a wild point. The wild points were assigned phases drawn at random from the uniform distribution on [0°, 360°] and amplitudes drawn at random from the uniform distribution on [0,2] Jy. The "tame" observations were contaminated by 0.2 Jy Gaussian noise, as in the random noise tests, above. The results for complex gain solutions and for phase solutions are shown in detail in Figures 3 and 4, respectively. For data made wild with probabilities .05, .10, .20, and .50, the mean observed rms gain solution errors in ten trials were 6.3, 6.6, 8.1, and 23 ($\times 10^{-2}$) (9.7, 12, 19, and 40 ($\times 10^{-2}$)) for the ℓ_1 (resp., for the ℓ_2) solution method. In the corresponding test of the phase solution methods, assigning the same probabilities, the mean observed rms errors were 4.1, 4.8, 4.9, and 7.7 ($\times 10^{-2}$) (5.3, 7.3, 10.0, and 27 ($\times 10^{-2}$)) for the ℓ_1 (resp., for the ℓ_2) solution method.

To test the sensitivity to the presence of extreme outliers, two trials were run in which each observation, with probability .10, was assigned a random phase and a random amplitude in the range $[0,10^{10}]$ Jy. The results are shown in Figures 5a and 5b. The rms ℓ_1 (resp., ℓ_2) gain solution errors observed were 7.2 and 4.5 ($\times 10^{-2}$) (resp., 4.2×10^{7} and 1.3×10^{0}) for the complex gain solution and for the phase solution, respectively. This test, with ten trials in each case, was repeated for the case of wild amplitudes in the range [0,.01] Jy. The results are shown in Figures 5c and 5d. The mean observed rms gain solution errors for ℓ_1 (resp., for ℓ_2) were 7.2 and 4.6 ($\times 10^{-2}$) (10.9 and 4.1 ($\times 10^{-2}$)) in the complex gain solutions and in the phase solutions, respectively. In the presence of extreme outliers, the ℓ_1 method appears in all cases, except that of phase solutions with very small amplitude outliers, to outperform the ℓ_2 method.

Sensitivity of the solution methods to bad i.f.'s

An occasional problem with VLA data is that the data associated with a single i.f. may be bad; sometimes the system error detection facility fails to recognize the problem. In these tests, 0.2 Jy Gaussian distributed random noise was added to the observations, as above, except that all the visibilities associated with antenna 5 were assigned a given amplitude and a random phase uniformly distributed on [0°, 360°]. Representative results, along with the rms gain solution errors that were observed in ten trials with identical error distributions, are displayed in Figures 6 and 7. For amplitudes associated with antenna 5 of 0.1, 1.0, 2.0, and 5.0 Jy, mean rms gain solution errors (excluding antenna 5) of 5.8, 5.9, 6.6, and 8.2 ($\times 10^{-2}$) (5.2, 5.3, 6.6, and 80 ($\times 10^{-2}$)) were observed for the ℓ_1 (resp., the ℓ_2) solutions when solving for full complex gains. In ten trials with 26 antennas and 0.2 Jy random noise, with no outliers, the mean observed rms gain solution error was 5.9±.5 and 5.3±.4 ($x10^{-2}$) for the ℓ_1 and ℓ_2 solutions, respectively.

When solving only for phase errors, for visibility amplitudes associated with antenna 5 of 0.5, 1.0, 2.0, and 100.0 Jy, mean rms gain solution errors (excluding antenna 5) of 5.1, 4.6, 5.3, and 5.4 ($x10^{-2}$) (4.8, 5.2, 7.3, and 126 ($x10^{-2}$)) were observed with the ℓ_1 (resp., the ℓ_2) solution method. In ten trials with 26 antennas, 0.2 Jy random noise, and no outliers, the mean observed rms gain solution error was 4.6±.6 and 4.1±.5 ($x10^{-2}$) for the ℓ_1 and ℓ_2 solutions, respectively.

Clearly it is desirable to detect the presence of a bad i.f. before solving for antenna gains, no matter which solution method is used. In the case of full complex gain solutions, if there is a bad i.f., and if the associated amplitudes are very large, then the ℓ_1 method is preferable. For phase solutions, the ℓ_1 method is extremely insensitive to a bad i.f. In both cases, the ℓ_1 method is strikingly superior to the least-squares method only when the errors associated with the bad i.f. are very large.

A few additional tests were run: If the observations associated with antenna 5 are assigned not a random phase, but each is assigned, instead, a phase of 45 degrees, then the conclusions above remain unaltered. In the case of not a single bad i.f., but of five bad i.f.'s, it is still preferable to detect the presence of the bad i.f.'s before solving for the gains (the solutions are quite inferior to those for 22 good i.f.'s), and the ℓ_2 solutions still are not much inferior to the ℓ_1 solutions except when the amplitudes of the bad points are very large.

Conclusions

The ℓ_1 method for gain solutions appears to be an attractive alternative to the least-squares method, both in self-calibration and, perhaps more so, in standard calibration. In the case of standard calibration in the pipeline data reduction system, the ℓ_1 method has appeal because in the pipeline it might not be possible to correct data editing errors before calibration. The appeal in self-calibration lies in the fact that, in combination with a scheme for the identification of outliers, one would be able first to solve for the gains and then to identify and flag the outliers, resting assured that the gain solutions probably were not unduly influenced by the outliers (otherwise it would be necessary to recompute the gains after flagging data). Since the ℓ_1 method performs nearly as well as ℓ_2 in the presence of well-behaved errors, any decision not to use ℓ_1 in preference to ℓ_2 probably would be based either upon faith that there were not many harmful outliers or upon considerations of computational speed. In a careful implementation, especially if the gains are computed in an array processor, the penalty due to the added computational complexity ought not to be too great.

Cornwell's recent error analysis of calibration [3] is a useful complement to the test results reported here. I plan to do a related empirical study of the sensitivity of simultaneous solution for antenna gains and for constant or slowly time-varying additive or multiplicative correlator based errors, to errors in the observations and to perturbations in the source model.

The problem of identifying outliers merits careful attention. [4] ought to be a useful reference on this subject.

TABLE 1.

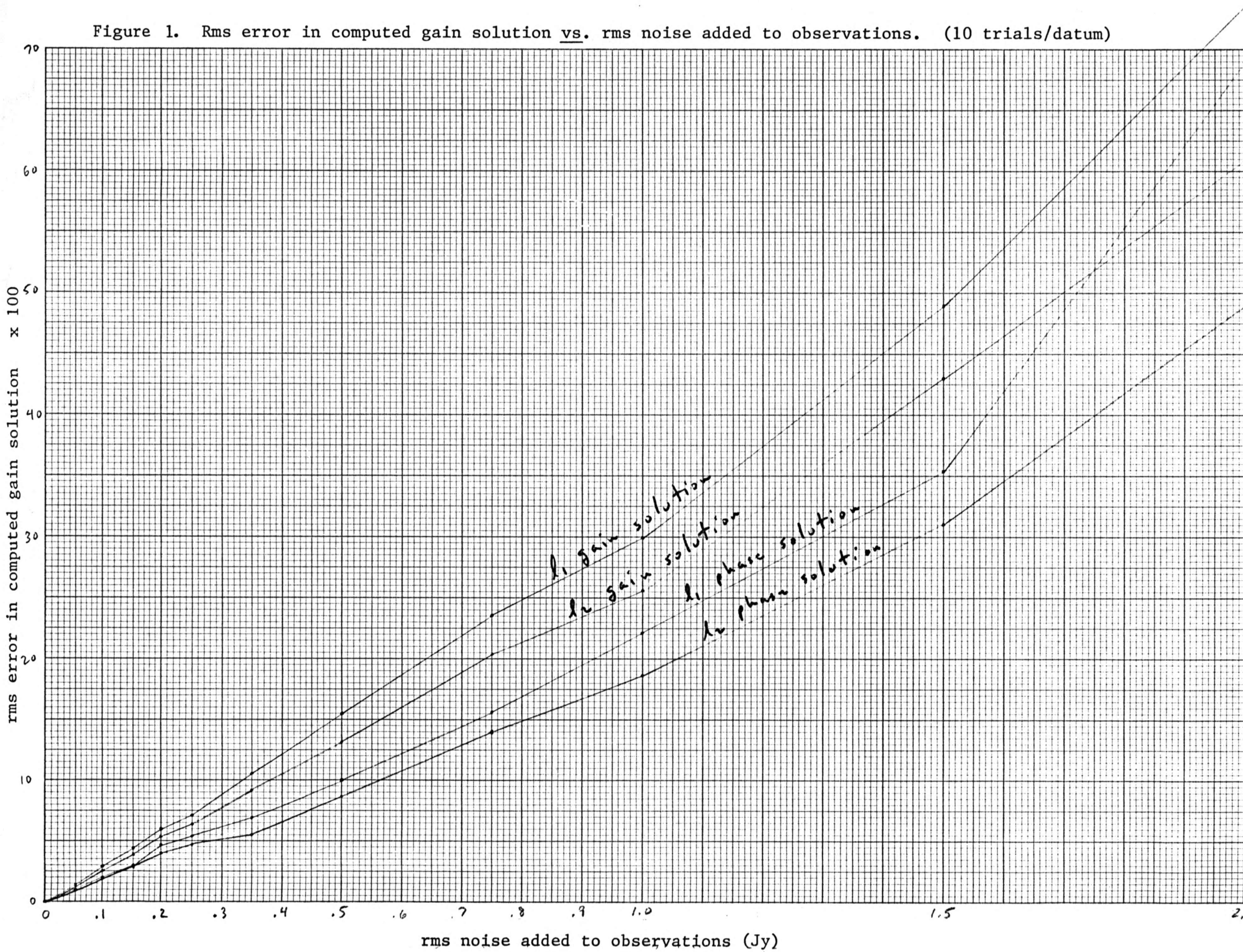
rms error in computed gain solution x100

rms noise (Jy)	phase solutions ℓ_1	phase solutions ℓ_2	complex gain solutions ℓ_1	complex gain solutions ℓ_2
.05	1.0 ± .1	1.0 ± .1	1.4 ± .2	1.3 ± .2
.10	2.0 ± .3	1.9 ± .3	2.9 ± .4	2.6 ± .3
.15	3.0 ± .3	2.9 ± .3	4.4 ± .5	3.8 ± .4
.20	4.7 ± .6	4.0 ± .5	6.0 ± .6	5.3 ± .5
.25	5.4 ± .6	4.7 ± .7	7.1 ± .8	6.4 ± .7
.35	6.9 ± .8	6.0 ± .4	11 ± 1	9.2 ± 1
.50	10 ± 1	8.7 ± 1	16 ± 2	13 ± 1
.75	16 ± 3	14 ± 2	24 ± 3	20 ± 2
1.00	22 ± 3	19 ± 3	30 ± 4	26 ± 4
1.50	35 ± 6	31 ± 4	49 ± 7	43 ± 5
2.00	69 ± 23	49 ± 12	74 ± 17	61 ± 10

10 trials/datum

simulated observations of 1 Jy point source

Figure 1. Rms error in computed gain solution vs. rms noise added to observations. (10 trials/datum)



REFERENCES

1. El-Attar, R. A., M. Vidyasagar, and S. R. K. Dutta, "An algorithm for ℓ_1-norm minimization with application to nonlinear ℓ_1-approximation", SIAM J. Num. Anal., 16 (1979), 70-86.

2. Murray, W. and M. L. Overton, "A projected Lagrangian algorithm for nonlinear ℓ_1 approximation", SIAM J. Sci. Stat. Comp., 2 (1981), 207-224.

3. Cornwell, T., "An error analysis of calibration", VLA Scientific Memo. No. 135, September 1981.

4. Barnett, V., and T. Lewis, Outliers in Statistical Data, Wiley, Chichester, 1978.

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	0.74019	18.96	0.68422	16.14	0.69212	16.55
2	0.76571	-142.61	0.76475	-147.41	0.76112	-146.70
3	1.46513	135.12	1.43573	134.16	1.42796	134.83
4	0.69369	139.70	0.78972	140.11	0.78809	137.89
5	1.20463	-112.25	1.19658	-112.99	1.17391	-113.42
6	0.99270	114.12	0.98797	114.17	0.99179	112.62
7	1.25279	73.59	1.29354	71.34	1.29637	72.74
8	1.10175	-132.58	1.14848	-134.57	1.12624	-133.90
9	0.68656	-166.74	0.68576	-161.29	0.68157	-161.84
10	1.30140	64.92	1.29272	65.87	1.29708	66.12
11	0.59614	86.63	0.56424	84.91	0.57076	85.48
12	1.31672	161.50	1.34419	160.76	1.35684	159.88
13	1.21291	33.59	1.14781	33.99	1.13274	33.16
14	1.21615	115.15	1.26026	116.42	1.27399	117.71
15	0.66908	-77.36	0.73036	-79.32	0.69313	-78.14
16	0.71566	40.06	0.70078	42.26	0.68445	43.15
17	0.87360	155.69	0.85458	155.59	0.85356	156.47
18	0.74752	66.06	0.82046	64.84	0.78710	65.27
19	0.73871	41.38	0.72071	44.40	0.73614	43.78
20	0.90312	61.31	0.92570	58.27	0.94986	58.03
21	0.74135	111.30	0.73756	113.17	0.74264	111.79
22	1.10917	-134.93	1.02903	-137.61	1.03807	-137.41
23	0.55400	-21.38	0.45843	-25.68	0.48013	-24.61
24	0.64669	-132.47	0.53754	-129.21	0.62701	-129.51
25	1.22916	160.26	1.23900	167.84	1.23783	166.14
26	1.37211	-116.55	1.40111	-115.99	1.40680	-116.18
27	1.17198	8.24	1.20896	7.63	1.21970	7.94
RMS SOLUTION ERROR:			6.45E-02		5.76E-02	

complex gain solution
0.2 Jy random noise

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 0.000E+00 JY

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	21.48	1.00000	19.56	1.00000	20.72
2	1.00000	-150.77	1.00000	-154.83	1.00000	-150.31
3	1.00000	119.24	1.00000	119.42	1.00000	118.35
4	1.00000	86.25	1.00000	91.90	1.00000	87.84
5	1.00000	129.27	1.00000	136.07	1.00000	135.03
6	1.00000	156.29	1.00000	161.00	1.00000	159.96
7	1.00000	-58.31	1.00000	-64.59	1.00000	-65.39
8	1.00000	121.55	1.00000	113.52	1.00000	117.62
9	1.00000	97.42	1.00000	102.99	1.00000	98.68
10	1.00000	167.88	1.00000	165.80	1.00000	168.37
11	1.00000	104.85	1.00000	109.47	1.00000	103.14
12	1.00000	-93.02	1.00000	-91.30	1.00000	-93.20
13	1.00000	81.85	1.00000	75.48	1.00000	76.08
14	1.00000	-78.73	1.00000	-77.62	1.00000	-77.98
15	1.00000	-154.90	1.00000	-150.37	1.00000	-151.80
16	1.00000	-11.80	1.00000	-16.54	1.00000	-15.28
17	1.00000	-81.36	1.00000	-81.81	1.00000	-76.75
18	1.00000	-149.09	1.00000	-152.95	1.00000	-151.72
19	1.00000	-156.57	1.00000	-153.85	1.00000	-155.18
20	1.00000	32.21	1.00000	32.30	1.00000	29.34
21	1.00000	-120.13	1.00000	-125.01	1.00000	-124.30
22	1.00000	98.61	1.00000	96.11	1.00000	95.71
23	1.00000	-134.91	1.00000	-131.66	1.00000	-129.81
24	1.00000	-28.66	1.00000	-27.06	1.00000	-25.22
25	1.00000	56.97	1.00000	50.29	1.00000	56.09
26	1.00000	71.30	1.00000	82.90	1.00000	77.83
27	1.00000	66.68	1.00000	64.44	1.00000	65.79
RMS SOLUTION ERROR:			8.35E-02		6.03E-02	

phase solution
0.35 Jy random noise

BADIF=F
NOISEAMP= 0.35 JY
WILDPCT= 0.00
WILDAMP= 0.000E+00 JY

Figure 2. Two representative trials illustrating behavior of the solution methods in the presence of random noise.

a)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.09987	175.55	1.10575	176.47	1.08752	174.58
2	1.29303	-159.41	1.27749	-156.39	1.27422	-154.79
3	0.61480	-165.89	0.57944	-166.36	0.59109	-164.01
4	0.57045	-41.25	0.59579	-44.39	0.58571	-42.62
5	1.38634	-30.68	1.38917	-30.73	1.42108	-30.14
6	1.16762	-29.93	1.16249	-30.43	1.10786	-29.28
7	0.86445	-127.50	0.87108	-127.60	0.88695	-128.03
8	0.92914	-172.03	0.94576	-171.35	0.93915	-169.73
9	1.39668	130.43	1.39778	130.97	1.35248	130.36
10	1.49578	-152.29	1.51442	-152.21	1.51556	-153.93
11	0.73202	72.65	0.74676	71.39	0.75172	72.99
12	1.11831	17.02	1.09703	16.85	1.10938	16.16
13	0.79723	120.42	0.72391	120.86	0.65234	116.01
14	1.14240	-115.59	1.17060	-114.35	1.10677	-115.58
15	1.48000	-84.94	1.46717	-85.24	1.43612	-86.42
16	1.27359	-58.11	1.18470	-56.07	1.10403	-57.98
17	0.90325	-55.64	0.93710	-56.92	0.85860	-61.70
18	1.48209	-70.85	1.47415	-72.69	1.38243	-70.86
19	1.42598	-117.88	1.39516	-116.03	1.40999	-116.87
20	1.15649	-164.13	1.08549	-163.01	1.14431	-163.68
21	1.34873	96.78	1.42663	94.54	1.36209	96.12
22	0.86594	-70.74	0.89129	-75.02	0.78422	-72.71
23	1.07673	89.69	1.00044	89.27	0.85259	93.17
24	0.84003	3.49	0.85852	3.11	0.86306	3.02
25	1.19390	135.01	1.25216	134.58	1.21547	135.62
26	1.02587	118.11	1.00143	117.55	1.01138	119.31
27	0.71339	63.24	0.70277	64.03	0.68903	62.31
RMS SOLUTION ERROR:			4.87E-02		7.90E-02	

P = .05

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT= 4.27
WILDAMP= 2.000E+00 JY

Error (x100), 10 trials:

ℓ_1	ℓ_2
4.9	7.9
6.0	8.3
7.7	11.5
6.9	12.3
7.1	8.3
6.1	8.5
6.1	9.0
5.3	11.2
6.5	11.1
6.4	8.6
6.3±.8	9.7±1.5

b)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.03603	-167.97	0.98723	-166.38	0.95980	-169.82
2	1.04794	120.58	0.96317	127.30	0.85818	130.04
3	1.23007	146.39	1.12397	148.53	0.90677	151.03
4	1.36942	-171.53	1.36984	-173.71	1.34115	-175.32
5	0.95301	84.06	0.92536	88.28	0.99482	86.72
6	1.31045	126.89	1.27214	126.89	1.34328	126.66
7	0.74670	77.62	0.70197	80.23	0.71677	83.37
8	1.26754	51.60	1.32080	53.83	1.33723	53.61
9	1.11677	49.27	1.09359	49.50	1.12345	50.77
10	0.57825	-75.60	0.64781	-74.69	0.65505	-73.73
11	1.13125	-106.90	1.12292	-105.42	1.11717	-104.97
12	0.93848	173.66	1.01378	170.18	0.95913	167.64
13	0.65374	132.69	0.60178	132.61	0.67130	131.44
14	0.74572	-39.50	0.75758	-44.47	0.80087	-46.92
15	0.96338	116.11	1.00164	121.65	0.92168	120.59
16	0.64555	132.99	0.62377	127.30	0.62282	128.67
17	1.24006	20.08	1.27101	17.81	1.15968	17.94
18	1.08485	177.85	0.99267	-179.10	0.91631	179.34
19	1.15035	-114.84	1.17818	-115.62	1.02868	-115.50
20	1.48891	-15.10	1.46719	-14.00	1.44712	-12.91
21	0.51830	-26.13	0.44965	-34.64	0.49257	-27.46
22	1.42208	-128.21	1.45165	-130.58	1.36105	-131.09
23	0.81563	173.64	0.74464	174.37	0.71679	176.22
24	0.84111	145.62	0.84432	149.30	0.81621	143.74
25	0.95564	36.44	1.02483	34.33	0.85131	36.85
26	1.30061	-51.49	1.29051	-54.53	1.31544	-54.22
27	0.62514	-123.07	0.66384	-117.08	0.55885	-114.82
RMS SOLUTION ERROR:			7.08E-02		1.11E-01	

P = .10

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT= 7.69
WILDAMP= 2.000E+00 JY

Error (x100), 10 trials:

ℓ_1	ℓ_2
7.1	11.1
5.7	12.2
6.2	12.0
7.0	15.0
6.0	10.3
6.6	14.3
7.5	11.7
6.0	11.5
6.6	11.0
7.2	12.1
6.6±.6	12.1±1.4

c)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	0.95525	35.54	0.95540	33.00	0.92261	33.54
2	1.32810	-164.86	1.32439	-166.20	1.28921	-165.66
3	1.15311	-69.50	0.94256	-70.70	0.64490	-85.96
4	0.87469	-141.64	0.77347	-138.59	0.69590	-134.31
5	1.15979	-142.17	1.15037	-143.70	1.10333	-144.23
6	1.13292	12.43	1.03599	12.43	1.03740	15.39
7	0.77551	172.47	0.64340	171.51	0.63290	-176.91
8	0.59030	85.87	0.59526	85.44	0.50245	77.58
9	1.23228	-23.64	1.20271	-26.64	1.21977	-31.83
10	0.93168	-38.53	0.92028	-39.98	0.82875	-46.22
11	1.11962	5.28	1.12212	4.91	1.04042	0.39
12	1.15024	-178.61	1.10251	-174.54	1.15473	-174.01
13	1.33274	-113.77	1.24590	-112.64	0.95622	-121.70
14	1.07911	119.71	1.12003	121.37	0.95473	119.61
15	0.77755	87.09	0.71653	75.66	0.71486	69.52
16	1.45331	79.65	1.40563	80.09	1.41202	79.58
17	1.31692	70.46	1.19946	71.54	0.94190	60.43
18	1.36141	-137.95	1.34840	-138.37	1.34782	-140.55
19	1.19244	-86.47	1.04827	-86.52	0.79421	-78.01
20	0.69609	-57.94	0.57087	-55.56	0.59242	-50.69
21	0.90624	-34.08	0.87487	-32.03	0.65114	-27.56
22	0.96320	141.10	0.97310	143.51	0.94393	150.00
23	1.29700	-132.72	1.38871	-131.66	1.24063	-128.16
24	0.55852	-49.38	0.59908	-41.41	0.62667	-39.99
25	1.29520	-43.37	1.24077	-44.79	1.20497	-46.46
26	1.37174	-105.10	1.38121	-104.11	1.42295	-102.04
27	0.59017	95.18	0.60469	94.56	0.57631	103.30
RMS SOLUTION ERROR:			9.00E-02		2.10E-01	

P = .20

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT=23.65
WILDAMP= 2.000E+00 JY

Error (x100), 10 trials:

ℓ_1	ℓ_2
9.0	21.8
8.1	16.6
7.6	19.7
7.7	21.0
7.7	21.9
7.0	16.7
7.8	20.5
10.0	21.8
8.2	13.7
8.1	17.3
8.1±.8	19.1±2.7

d)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.45545	-15.37	1.18775	-21.31	1.26605	-24.82
2	0.65221	93.25	0.76069	92.51	0.55517	104.63
3	1.10179	-29.40	1.17332	-27.03	1.03583	-36.25
4	1.18879	21.59	0.50927	30.31	0.46101	30.39
5	0.75231	-36.02	0.38514	-39.09	0.17413	-26.71
6	1.00271	-64.64	0.87724	-67.23	0.83766	-67.47
7	1.40320	-161.06	1.29659	-162.43	1.26399	-162.30
8	1.41176	-15.65	1.37163	-15.30	1.29466	-13.78
9	0.71407	84.14	0.60204	92.45	0.38167	95.42
10	0.55770	-34.67	0.30543	-20.32	0.40567	-20.37
11	1.15709	51.78	1.19098	52.25	0.79930	59.02
12	1.47797	176.70	1.20073	179.24	1.23994	-173.75
13	1.16361	-111.79	0.97005	-111.12	0.53385	-96.74
14	0.97334	-164.16	0.80209	-160.95	0.87004	-157.15
15	1.20262	-81.10	1.14625	-84.10	0.91577	-84.33
16	1.38072	95.95	1.14901	97.44	0.70267	99.66
17	0.95544	140.73	0.90068	143.26	0.55276	139.93
18	1.11079	-110.79	1.07004	-113.96	0.74870	-116.47
19	1.26092	-55.90	1.21799	-55.95	1.16655	-48.54
20	0.55951	93.24	0.50000	90.23	0.59379	75.22
21	0.89571	27.55	0.36021	20.16	0.38610	-20.13
22	0.75306	107.63	0.60746	110.60	0.49540	112.49
23	0.92375	-171.60	0.74162	-160.90	0.60494	-145.69
24	0.83562	-2.48	0.57799	-15.90	0.59846	-24.51
25	1.09958	58.38	0.91446	60.80	0.97715	47.60
26	1.25923	30.95	1.15901	29.39	0.77003	16.73
27	0.55964	137.15	0.62153	146.77	0.47444	120.30
RMS SOLUTION ERROR:			2.38E-01		4.04E-01	

P = .50

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT=49.86
WILDAMP= 2.000E+00 JY

Error (x100), 10 trials:

ℓ_1	ℓ_2
23.8	40.4
18.1	37.0
19.7	59.1
24.8	39.0
14.5	29.2
25.3	41.3
25.5	47.3
26.1	42.6
29.7	46.4
18.5	38.0
22.6±4.4	40.0±4.8

Figure 3. Sensitivity of complex gain solutions to outliers. The "tame" observations had random noise of 0.2 Jy added. Each observation, with a given probability, P, was made a wild point by assigning it a random phase and an amplitude in the range 0-2 Jy.

a)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	69.31	1.00000	70.29	1.00000	66.15
2	1.00000	123.02	1.00000	122.27	1.00000	123.31
3	1.00000	-34.80	1.00000	-35.62	1.00000	-30.54
4	1.00000	-37.59	1.00000	-37.78	1.00000	-38.76
5	1.00000	88.77	1.00000	88.21	1.00000	85.15
6	1.00000	-83.87	1.00000	-83.90	1.00000	-79.60
7	1.00000	46.27	1.00000	48.72	1.00000	48.84
8	1.00000	-106.97	1.00000	-105.61	1.00000	-109.64
9	1.00000	-61.13	1.00000	-59.49	1.00000	-60.27
10	1.00000	-163.46	1.00000	-163.66	1.00000	-162.93
11	1.00000	127.66	1.00000	128.00	1.00000	127.90
12	1.00000	-36.63	1.00000	-38.23	1.00000	-38.39
13	1.00000	-119.26	1.00000	-120.21	1.00000	-120.04
14	1.00000	-130.06	1.00000	-130.99	1.00000	-130.17
15	1.00000	163.58	1.00000	163.87	1.00000	166.31
16	1.00000	-56.73	1.00000	-54.62	1.00000	-50.08
17	1.00000	-62.78	1.00000	-63.34	1.00000	-63.08
18	1.00000	143.22	1.00000	145.71	1.00000	147.23
19	1.00000	61.93	1.00000	63.63	1.00000	64.15
20	1.00000	132.99	1.00000	137.37	1.00000	130.75
21	1.00000	18.73	1.00000	13.81	1.00000	11.36
22	1.00000	-35.26	1.00000	-38.81	1.00000	-37.83
23	1.00000	-69.17	1.00000	-68.32	1.00000	-67.70
24	1.00000	77.18	1.00000	80.03	1.00000	79.74
25	1.00000	116.60	1.00000	119.12	1.00000	119.78
26	1.00000	-7.96	1.00000	-11.58	1.00000	-10.72
27	1.00000	36.29	1.00000	31.44	1.00000	31.46
RMS SOLUTION ERROR:			3.93E-02		5.51E-02	

P = .05

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT= 4.27
WILDAMP= 2.000E+00 JY

Error (x100), 10 trials:

l_1	l_2
3.9	5.5
4.9	7.2
3.9	4.0
3.8	4.0
4.6	5.0
3.9	5.5
4.1	3.7
3.7	4.8
4.0	7.0
4.5	6.6
4.1±.4	5.3±1.2

b)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	155.79	1.00000	158.83	1.00000	157.07
2	1.00000	53.73	1.00000	57.19	1.00000	63.19
3	1.00000	-130.79	1.00000	-134.91	1.00000	-137.61
4	1.00000	-117.67	1.00000	-120.10	1.00000	-120.39
5	1.00000	-100.74	1.00000	-100.37	1.00000	-103.49
6	1.00000	51.77	1.00000	53.76	1.00000	52.55
7	1.00000	102.00	1.00000	102.70	1.00000	102.24
8	1.00000	-12.29	1.00000	-13.53	1.00000	-9.50
9	1.00000	-65.26	1.00000	-64.53	1.00000	-59.81
10	1.00000	36.27	1.00000	36.54	1.00000	36.37
11	1.00000	161.70	1.00000	163.04	1.00000	163.83
12	1.00000	37.82	1.00000	35.65	1.00000	40.23
13	1.00000	-169.90	1.00000	-168.42	1.00000	-170.26
14	1.00000	8.38	1.00000	7.37	1.00000	3.37
15	1.00000	-81.39	1.00000	-78.19	1.00000	-74.82
16	1.00000	-114.51	1.00000	-115.68	1.00000	-115.46
17	1.00000	129.89	1.00000	127.76	1.00000	128.99
18	1.00000	-114.53	1.00000	-116.15	1.00000	-117.49
19	1.00000	49.26	1.00000	53.37	1.00000	52.74
20	1.00000	48.93	1.00000	47.89	1.00000	40.43
21	1.00000	-38.75	1.00000	-35.89	1.00000	-35.37
22	1.00000	-18.64	1.00000	-22.49	1.00000	-21.85
23	1.00000	125.98	1.00000	123.12	1.00000	123.61
24	1.00000	65.74	1.00000	58.17	1.00000	46.44
25	1.00000	107.22	1.00000	108.17	1.00000	108.68
26	1.00000	-65.95	1.00000	-61.07	1.00000	-62.25
27	1.00000	14.01	1.00000	15.85	1.00000	18.33
RMS SOLUTION ERROR:			4.91E-02		9.03E-02	

P = .10

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT= 9.97
WILDAMP= 2.000E+00 JY

Error (x100) 10 trials:

l_1	l_2
4.4	9.0
5.9	7.5
3.4	6.0
4.4	7.0
5.1	6.8
5.7	6.8
4.6	7.1
4.2	6.5
5.4	8.7
4.6	7.5
4.8±.7	7.3±.9

c)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	-96.10	1.00000	-94.04	1.00000	-93.11
2	1.00000	-122.88	1.00000	-123.54	1.00000	-126.53
3	1.00000	109.31	1.00000	107.25	1.00000	104.09
4	1.00000	-37.78	1.00000	-40.19	1.00000	-35.20
5	1.00000	112.37	1.00000	113.23	1.00000	114.04
6	1.00000	131.33	1.00000	132.79	1.00000	144.44
7	1.00000	-99.09	1.00000	-99.66	1.00000	-95.95
8	1.00000	106.06	1.00000	106.20	1.00000	106.48
9	1.00000	19.59	1.00000	20.46	1.00000	22.49
10	1.00000	105.94	1.00000	110.52	1.00000	117.67
11	1.00000	114.25	1.00000	117.63	1.00000	123.54
12	1.00000	130.90	1.00000	132.94	1.00000	135.52
13	1.00000	126.01	1.00000	124.19	1.00000	123.22
14	1.00000	91.44	1.00000	92.04	1.00000	95.70
15	1.00000	62.40	1.00000	60.50	1.00000	58.86
16	1.00000	-96.74	1.00000	-96.53	1.00000	-96.16
17	1.00000	-79.27	1.00000	-75.63	1.00000	-73.74
18	1.00000	13.28	1.00000	12.50	1.00000	11.22
19	1.00000	-146.57	1.00000	-146.84	1.00000	-152.45
20	1.00000	-146.16	1.00000	-151.79	1.00000	-170.58
21	1.00000	159.40	1.00000	164.35	1.00000	168.90
22	1.00000	-87.88	1.00000	-91.85	1.00000	-88.97
23	1.00000	-80.08	1.00000	-79.96	1.00000	-82.47
24	1.00000	51.45	1.00000	49.16	1.00000	44.67
25	1.00000	-174.50	1.00000	-174.54	1.00000	-173.79
26	1.00000	-48.44	1.00000	-45.48	1.00000	-40.83
27	1.00000	-103.85	1.00000	-108.53	1.00000	-119.33
RMS SOLUTION ERROR:			4.61E-02		1.33E-01	

P = .20

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT=19.66
WILDAMP= 2.000E+00 JY

Error (x100), 10 trials:

l_1	l_2
4.6	13.3
5.5	8.4
4.6	7.8
5.1	12.7
5.5	11.3
4.6	8.4
4.5	8.7
5.6	12.9
4.6	8.7
4.2	7.6
4.9±.5	10.0±2.2

d)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	53.98	1.00000	51.33	1.00000	36.20
2	1.00000	-150.05	1.00000	-148.39	1.00000	-154.83
3	1.00000	-50.60	1.00000	-46.55	1.00000	-43.37
4	1.00000	-151.20	1.00000	-154.52	1.00000	-149.58
5	1.00000	109.96	1.00000	106.44	1.00000	94.13
6	1.00000	-102.37	1.00000	-107.98	1.00000	-115.28
7	1.00000	153.34	1.00000	153.56	1.00000	138.63
8	1.00000	144.69	1.00000	143.28	1.00000	147.37
9	1.00000	150.29	1.00000	149.62	1.00000	146.79
10	1.00000	43.99	1.00000	50.78	1.00000	83.42
11	1.00000	-105.30	1.00000	-107.73	1.00000	-113.88
12	1.00000	28.46	1.00000	37.61	1.00000	49.01
13	1.00000	-165.94	1.00000	-163.85	1.00000	-156.28
14	1.00000	160.65	1.00000	157.21	1.00000	157.66
15	1.00000	-57.77	1.00000	-60.29	1.00000	-62.50
16	1.00000	-81.93	1.00000	-81.91	1.00000	-70.81
17	1.00000	-73.57	1.00000	-74.78	1.00000	-76.09
18	1.00000	-60.21	1.00000	-60.99	1.00000	-59.09
19	1.00000	110.24	1.00000	108.07	1.00000	134.15
20	1.00000	-8.73	1.00000	-10.09	1.00000	-10.94
21	1.00000	84.52	1.00000	82.84	1.00000	76.45
22	1.00000	-85.57	1.00000	-87.35	1.00000	-109.06
23	1.00000	48.21	1.00000	49.91	1.00000	41.71
24	1.00000	159.54	1.00000	156.11	1.00000	163.80
25	1.00000	-49.27	1.00000	-42.34	1.00000	-36.06
26	1.00000	-153.85	1.00000	-152.32	1.00000	-153.57
27	1.00000	97.94	1.00000	101.88	1.00000	128.00
RMS SOLUTION ERROR:			6.17E-02		2.34E-01	

P = .50

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT=50.14
WILDAMP= 2.000E+00 JY

Error (x100), 10 trials:

l_1	l_2
6.2	23
7.4	24
6.0	25
6.1	18
8.2	27
8.3	26
6.7	22
8.6	32
9.6	29
10.1	45
7.7±1.4	27±7

Figure 4. Sensitivity of phase solutions to outliers. The "tame" observations had random noise of 0.2 Jy added. Each observation, with probability P, was made a wild point by assigning it a random phase and an amplitude in the range 0-2 Jy.

a)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.06404	-176.62	1.09113	-177.39	2.17052E+08	-176.62
2	1.42777	-16.99	1.46043	-17.57	7.67304E-09	-19.00
3	0.56165	-0.97	0.58336	-3.09	4.31308E-09	-7.41
4	1.11222	-22.82	1.17535	-18.44	5.14358E-09	-19.52
5	0.57124	-128.79	0.57232	-129.55	4.20181E-09	-120.27
6	0.66483	92.82	0.67166	94.46	9.60900	5.31
7	1.15405	-71.38	1.17541	-76.62	5.78927E-09	-84.55
8	1.07025	-100.66	1.11000	-104.89	5.90402E-09	-109.42
9	0.83387	-41.34	0.86388	-38.05	2.37950E-06	35.56
10	1.10392	141.36	0.94621	137.81	4.82642E-09	152.10
11	1.26570	-59.67	1.25826	-63.78	2.83481E-07	143.41
12	0.62248	107.11	0.60065	101.31	2.56449E-06	-95.98
13	1.05118	122.48	1.03443	120.05	3.91315E-09	124.09
14	1.15020	139.54	1.16112	139.33	5.23420E-09	144.85
15	1.18574	169.46	1.17918	169.29	6.79405E-09	-170.36
16	0.87057	46.91	0.96176	44.27	2.62833E-06	52.42
17	0.87204	45.82	0.92053	50.50	5.13266E-09	51.37
18	0.88262	-168.33	0.89531	-165.50	4.49944E-09	-164.00
19	0.90399	-177.80	0.95606	-179.94	5.65765E-09	179.97
20	1.33120	-69.26	1.29318	-67.25	5.91572E-09	-49.74
21	0.81568	174.06	0.85079	173.01	2.92069E-09	165.75
22	0.77071	165.71	0.74168	173.50	1.94785E-06	162.85
23	0.80968	-144.75	0.87252	-149.86	2.20337E-07	69.30
24	0.87156	120.19	0.87079	120.48	1.56973E-06	-89.30
25	1.37900	18.23	1.35452	19.16	7.78925	59.50
26	1.32882	-29.03	1.29914	-27.56	6.70176E-09	-30.39
27	1.16755	-37.10	1.06350	-34.81	15.02539	22.92
RMS SOLUTION ERROR:			7.24E-02		4.19E+07	

Wild amplitudes 0-10[10] Jy

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT=12.54
WILDAMP= 1.000E+10 JY

b)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	155.35	1.00000	151.24	1.00000	93.95
2	1.00000	129.72	1.00000	133.79	1.00000	-102.08
3	1.00000	7.20	1.00000	4.64	1.00000	149.90
4	1.00000	93.67	1.00000	94.07	1.00000	9.31
5	1.00000	-40.05	1.00000	-42.44	1.00000	-161.50
6	1.00000	160.74	1.00000	161.77	1.00000	107.74
7	1.00000	172.12	1.00000	170.09	1.00000	-66.89
8	1.00000	-59.43	1.00000	-56.23	1.00000	-76.07
9	1.00000	-72.51	1.00000	-71.81	1.00000	-129.07
10	1.00000	-113.37	1.00000	-113.25	1.00000	0.38
11	1.00000	-85.55	1.00000	-85.85	1.00000	-31.01
12	1.00000	-6.22	1.00000	-4.49	1.00000	-11.30
13	1.00000	-44.86	1.00000	-47.29	1.00000	-15.26
14	1.00000	63.35	1.00000	59.58	1.00000	89.53
15	1.00000	-8.49	1.00000	-4.27	1.00000	-106.46
16	1.00000	-67.17	1.00000	-67.57	1.00000	-68.49
17	1.00000	143.73	1.00000	146.61	1.00000	-131.20
18	1.00000	136.02	1.00000	137.51	1.00000	-164.59
19	1.00000	-69.45	1.00000	-68.29	1.00000	-133.90
20	1.00000	136.35	1.00000	133.63	1.00000	42.66
21	1.00000	-164.39	1.00000	-163.24	1.00000	-111.04
22	1.00000	163.17	1.00000	165.30	1.00000	-166.02
23	1.00000	78.39	1.00000	75.76	1.00000	-54.47
24	1.00000	36.13	1.00000	34.78	1.00000	-26.12
25	1.00000	-24.01	1.00000	-20.08	1.00000	47.02
26	1.00000	96.45	1.00000	98.63	1.00000	-82.50
27	1.00000	110.52	1.00000	122.43	1.00000	-78.91
RMS SOLUTION ERROR:			4.46E-02		1.29E+00	

Wild amplitudes 0-10[10] Jy

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT= 9.69
WILDAMP= 1.000E+10 JY

c)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.28363	-64.74	1.35151	-63.72	1.33158	-63.01
2	1.29126	154.16	1.25781	152.27	1.26648	151.96
3	0.79231	-144.69	0.78768	-140.56	0.77700	-141.56
4	0.98288	156.90	1.03237	153.87	0.98083	155.53
5	1.24375	-93.39	1.30073	-94.19	1.21198	-94.49
6	1.04492	55.62	1.08765	57.36	1.10118	56.09
7	0.65671	69.60	0.60228	73.64	0.62876	72.48
8	1.47164	30.89	1.50149	31.16	1.27188	31.97
9	1.46661	79.75	1.46351	79.88	1.45340	80.23
10	0.58417	-174.54	0.60570	-173.01	0.60649	-174.48
11	0.52152	-79.79	0.46013	-81.65	0.46355	-83.15
12	1.02416	178.41	0.98975	179.76	0.96246	179.25
13	1.22574	-48.41	1.26836	-45.89	1.23516	-47.24
14	1.29320	-68.21	1.33028	-68.09	1.37174	-67.66
15	0.56503	-162.30	0.55845	-167.25	0.57374	-169.05
16	0.94767	166.90	0.86902	168.16	0.86039	160.27
17	1.49023	21.08	1.44564	18.25	1.20640	19.39
18	1.30890	-3.24	1.27352	-2.05	1.18448	-3.28
19	0.94936	49.45	0.94435	46.62	0.89933	46.61
20	1.29253	-127.99	1.23066	-128.34	1.24195	-127.46
21	1.38467	-154.94	1.33400	-155.52	1.16237	-155.28
22	0.73357	86.78	0.71109	86.67	0.65576	83.14
23	0.70913	-161.10	0.70013	-156.98	0.66634	-158.22
24	1.28775	81.61	1.27882	82.50	1.10192	81.23
25	0.96651	121.57	0.82576	120.68	0.81129	122.10
26	0.79743	90.25	0.83073	88.49	0.85834	89.74
27	1.07763	112.70	1.03538	113.00	1.10978	112.62
RMS SOLUTION ERROR:			5.97E-02		1.08E-01	

Wild amplitudes 0-.01 Jy

Error (x100), 10 trials:

l_1	l_2
6.0	10.8
7.2	15.1
7.3	10.1
7.5	10.3
6.6	8.5
6.4	10.3
7.1	11.5
8.2	9.9
9.0	11.7
6.4	10.6
7.2±.9	10.9±1.6

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT= 9.97
WILDAMP= 1.000E-02 JY

d)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	170.57	1.00000	168.84	1.00000	169.13
2	1.00000	61.78	1.00000	59.65	1.00000	58.38
3	1.00000	-32.58	1.00000	-29.14	1.00000	-30.22
4	1.00000	170.12	1.00000	174.31	1.00000	172.84
5	1.00000	133.93	1.00000	132.03	1.00000	129.75
6	1.00000	116.48	1.00000	119.90	1.00000	117.56
7	1.00000	-96.81	1.00000	-94.28	1.00000	-94.50
8	1.00000	37.13	1.00000	34.98	1.00000	34.62
9	1.00000	74.25	1.00000	73.19	1.00000	73.60
10	1.00000	91.77	1.00000	92.13	1.00000	93.22
11	1.00000	85.60	1.00000	84.38	1.00000	84.06
12	1.00000	-82.32	1.00000	-87.92	1.00000	-87.61
13	1.00000	124.07	1.00000	121.32	1.00000	121.32
14	1.00000	-123.62	1.00000	-123.91	1.00000	-122.01
15	1.00000	-128.92	1.00000	-127.35	1.00000	-127.25
16	1.00000	-30.14	1.00000	-28.64	1.00000	-28.17
17	1.00000	157.98	1.00000	159.40	1.00000	157.62
18	1.00000	133.09	1.00000	130.72	1.00000	130.81
19	1.00000	32.56	1.00000	34.02	1.00000	34.92
20	1.00000	119.92	1.00000	120.76	1.00000	122.46
21	1.00000	8.41	1.00000	10.67	1.00000	11.45
22	1.00000	-11.07	1.00000	-13.70	1.00000	-13.29
23	1.00000	-140.90	1.00000	-141.68	1.00000	-140.95
24	1.00000	-53.64	1.00000	-49.39	1.00000	-50.27
25	1.00000	109.06	1.00000	107.45	1.00000	109.57
26	1.00000	12.93	1.00000	13.40	1.00000	13.05
27	1.00000	82.99	1.00000	81.51	1.00000	81.96
RMS SOLUTION ERROR:			4.19E-02		4.10E-02	

Wild amplitudes 0-.01 Jy

Error (x100), 10 trials:

l_1	l_2
4.2	4.1
4.9	4.3
4.9	4.6
4.2	3.3
4.0	4.1
4.3	3.3
4.7	4.2
5.1	3.8
5.5	4.7
4.7	4.6
4.6±.5	4.2±.4

BADIF=F
NOISEAMP= 0.20 JY
WILDPCT= 9.40
WILDAMP= 1.000E-02 JY

Figure 5. Sensitivity to the presence of extreme outliers. "Tame" observations have 0.2 Jy random noise added. Observations were made wild with probability .10 . a) and b) illustrate the sensitivity to extremely high amplitude outliers, c) and d) to low amplitude.

a)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.49261	125.34	1.50945	126.49	1.51124	125.75
2	1.00722	138.45	0.98822	136.70	0.90021	136.43
3	0.80554	124.41	0.80954	125.69	0.80093	125.07
4	0.80325	-170.35	0.87014	-173.21	0.39112	-172.84
5	0.63127	-167.89	0.01095	-77.83	6.86291E-03	-31.45
6	1.42669	153.38	1.46252	152.27	1.45846	153.08
7	0.91442	-120.91	0.85093	-120.73	0.84603	-121.60
8	0.82201	-151.92	0.85772	-157.93	0.82466	-152.43
9	1.01742	-53.24	1.02659	-53.17	1.01900	-54.69
10	0.76099	-81.55	0.73510	-85.48	0.71312	-85.03
11	1.15247	20.47	1.10232	22.47	1.12220	23.09
12	0.70161	161.69	0.69304	156.75	0.71526	150.96
13	0.81597	-131.06	0.80268	-131.90	0.79351	-132.26
14	1.06494	-38.55	1.11247	-38.31	1.09066	-30.67
15	1.39981	51.14	1.44730	51.50	1.42780	50.66
16	1.44739	-53.02	1.37220	-55.99	1.40359	-54.84
17	0.54301	114.35	0.56008	120.43	0.55305	118.50
18	0.73123	-61.11	0.71999	-62.51	0.73427	-62.51
19	1.46302	32.37	1.46091	33.59	1.45961	33.88
20	0.69367	55.17	0.72132	53.50	0.71290	54.85
21	1.23073	-121.49	1.21043	-121.51	1.20554	-121.47
22	1.45715	-163.76	1.46028	-163.04	1.50032	-132.02
23	0.79703	113.76	0.74991	112.90	0.77065	112.75
24	0.95245	-65.05	0.95765	-66.04	0.94353	-66.14
25	1.07108	127.22	1.01985	129.02	1.01869	120.86
26	1.14891	-24.35	1.10014	-24.57	1.20247	-24.56
27	0.57726	130.32	0.67140	134.22	0.64750	134.05
RMS SOLUTION ERROR: (EXCLUDING ANTENNA 5)			5.79E-02		4.94E-02	

0.2 Jy random noise
0.1 Jy wild points on if 5

BADIF=T
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 1.000E-01 JY

Error (x100), 10 trials:

l_1	l_2
4.2	4.2
5.2	4.3
6.2	5.1
5.8	4.9
5.4	4.8
6.1	5.3
5.5	4.9
6.5	6.2
6.2	5.4
6.4	5.9
5.8±.7	5.2±.6

b)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.43904	177.39	1.44957	178.87	1.43397	179.93
2	0.82349	-104.74	0.86908	-104.54	0.87093	-104.30
3	1.29562	20.87	1.27982	17.72	1.28337	18.69
4	1.14319	142.52	1.13478	141.94	1.17417	141.14
5	1.47205	-137.30	0.07471	-39.16	0.03964	-38.29
6	1.11549	8.78	1.08379	7.90	1.09856	6.65
7	1.43107	165.00	1.45600	166.64	1.45442	165.51
8	0.99948	64.07	1.00286	61.20	0.98301	63.73
9	1.02169	-35.75	0.95123	-36.43	0.97877	-36.22
10	0.83403	-161.35	0.78289	-157.76	0.80982	-158.58
11	1.29460	-70.73	1.26004	-76.77	1.25725	-77.33
12	1.15390	28.10	1.14111	28.71	1.15260	28.71
13	0.59211	-168.24	0.51075	-161.06	0.54194	-163.40
14	1.19746	-107.88	1.20421	-108.26	1.20349	-106.17
15	0.64206	-160.97	0.70475	-158.05	0.70304	-159.30
16	1.02005	95.79	1.09638	93.03	1.10245	93.91
17	1.14913	101.02	1.20674	102.66	1.19619	101.11
18	1.24647	-43.00	1.14430	-44.52	1.15867	-44.08
19	1.09917	53.23	1.10398	55.72	1.12065	56.32
20	0.94912	-88.70	1.00303	-86.32	1.01943	-87.61
21	0.70953	119.76	0.74442	115.06	0.77451	117.19
22	1.31067	121.93	1.27969	119.00	1.25030	121.72
23	1.29213	-6.02	1.20635	-7.55	1.28015	-7.62
24	0.07247	111.75	0.07475	112.92	0.85396	111.54
25	0.84908	121.01	0.86383	122.04	0.86279	121.70
26	1.41296	-115.08	1.44376	-120.33	1.44160	-119.96
27	1.43104	-36.31	1.42295	-34.81	1.40458	-35.72
RMS SOLUTION ERROR: (EXCLUDING ANTENNA 5)			6.38E-02		5.41E-02	

0.2 Jy random noise
1.0 Jy wild points on if 5

BADIF=T
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 1.000E+00 JY

Error (x100), 10 trials:

l_1	l_2
5.5	5.4
6.0	5.2
6.4	5.4
5.9	5.5
5.4	4.5
5.2	4.4
6.9	6.2
6.3	5.8
6.1	5.4
5.1	5.0
5.9±.6	5.3±.5

c)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	0.57740	-172.65	0.64083	-171.15	0.59960	-173.97
2	0.91130	-110.37	0.92431	-116.49	0.92566	-115.10
3	1.14140	-98.15	1.10825	-92.53	1.14996	-97.69
4	1.40376	-108.46	1.45420	-107.44	1.46079	-109.37
5	0.90561	-56.03	0.79830	130.68	0.49556	131.04
6	0.52075	114.67	0.54725	118.85	0.51031	117.90
7	1.20371	-165.07	1.26775	-164.94	1.24291	-163.46
8	0.64330	-10.40	0.61008	-9.17	0.63141	-3.88
9	0.61061	-60.94	0.50005	-65.66	0.60311	-60.52
10	1.42377	-55.94	1.40135	-60.94	1.41346	-62.65
11	0.76361	-67.65	0.72082	-67.91	0.71337	-60.37
12	0.73373	79.02	0.71429	77.25	0.72550	74.32
13	1.43086	146.28	1.41129	145.31	1.34127	144.98
14	1.03923	-59.62	1.03681	-60.92	1.05920	-59.02
15	1.30673	-90.80	1.36773	-89.93	1.37119	-89.50
16	1.47816	-99.80	1.55363	-97.29	1.57086	-97.04
17	0.60331	-123.72	0.70009	-122.29	0.72304	-123.10
18	1.44479	-30.76	1.40914	-29.36	1.45821	-29.46
19	0.51755	131.77	0.47304	121.90	0.50536	124.42
20	1.29812	-3.22	1.30008	-0.75	1.30641	-0.53
21	0.60330	152.61	0.69305	151.54	0.66383	145.05
22	0.55301	111.53	0.64655	99.09	0.63156	101.29
23	1.34382	157.96	1.26501	159.02	1.32430	157.88
24	0.66070	159.56	0.64506	164.51	0.64929	162.56
25	0.52232	35.05	0.59254	39.42	0.50290	37.66
26	1.27764	46.11	1.27766	42.87	1.28903	42.62
27	1.12370	-136.45	1.11135	-138.79	1.10761	-137.03
RMS SOLUTION ERROR: (EXCLUDING ANTENNA 5)			7.05E-02		6.63E-02	

0.2 Jy random noise
2.0 Jy wild points on if 5

BADIF=T
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 2.000E+00 JY

Error (x100), 10 trials:

l_1	l_2
7.4	6.4
7.0	6.7
7.7	8.6
5.3	4.9
6.1	6.3
7.1	7.5
6.1	7.6
6.2	5.6
5.8	5.4
7.7	7.3
6.6±.8	6.6±1.1

d)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	0.80032	-155.13	0.86604	-158.54	0.26257	-25.51
2	1.38293	-156.47	1.35344	-158.32	0.26750	176.10
3	0.83320	95.19	0.79736	93.42	0.26554	-178.09
4	0.53048	-159.79	0.53567	-155.47	0.26353	66.27
5	1.40239	29.02	2.12296	36.75	10.83932	79.79
6	0.53176	-12.94	0.40467	-12.55	0.26526	22.09
7	1.17409	-170.85	1.13246	-169.99	0.26592	-123.65
8	0.57950	-43.20	0.66541	-39.70	0.26494	-69.24
9	0.85937	-13.64	0.85242	-13.61	0.26014	154.53
10	1.06276	-3.67	1.02825	-4.91	0.26069	-130.05
11	1.40069	-1.91	1.52077	-3.66	0.26625	12.47
12	0.05641	86.30	0.86589	90.07	0.26226	-127.07
13	0.89684	-114.40	0.86893	-115.17	0.26340	-1.58
14	0.57959	127.03	0.61343	132.24	0.26290	-124.97
15	1.05260	100.10	1.00378	107.07	0.26615	92.07
16	0.87641	-90.96	0.96979	-92.78	0.26649	-60.25
17	0.89491	20.40	0.96922	22.76	0.26750	12.63
18	0.50626	56.04	0.55545	53.74	0.26301	-173.26
19	1.26269	-23.51	1.31733	-25.30	0.26713	-30.73
20	1.41324	-5.03	1.35590	-10.70	0.25826	149.65
21	0.73000	-148.34	0.60173	-149.02	0.26209	92.76
22	1.45339	81.73	1.44917	79.09	0.26603	56.87
23	1.19100	-10.65	1.10432	-13.64	0.26263	-93.10
24	0.69273	150.79	0.71002	155.29	0.26426	34.55
25	0.50269	-0.79	0.61351	0.91	0.26399	1.04
26	1.32501	51.64	1.30016	49.72	0.26124	-57.51
27	0.70249	-61.20	0.62082	-63.03	0.26391	44.33
RMS SOLUTION ERROR: (EXCLUDING ANTENNA 5)			6.62E-02		9.74E-01	

0.2 Jy random noise
5.0 Jy wild points on if 5

BADIF=T
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 5.000E+00 JY

Error (x100), 10 trials:

l_1	l_2
6.2	116
6.6	97.4
11.2	102
8.0	111
7.0	95.2
9.7	116
7.5	65.1
6.2	23.1
13.9	96.5
5.4	14.2
8.2±2.5	84.2±38

Figure 6. Sensitivity of complex gain solutions to a bad i.f.

a)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	-83.33	1.00000	-89.16	1.00000	-87.66
2	1.00000	175.96	1.00000	177.17	1.00000	176.92
3	1.00000	104.16	1.00000	102.91	1.00000	101.39
4	1.00000	41.27	1.00000	38.62	1.00000	38.12
5	1.00000	-179.03	1.00000	-63.08	1.00000	-50.63
6	1.00000	159.48	1.00000	152.71	1.00000	155.18
7	1.00000	-23.14	1.00000	-23.61	1.00000	-24.45
8	1.00000	-3.20	1.00000	-1.74	1.00000	-1.62
9	1.00000	66.98	1.00000	68.90	1.00000	68.63
10	1.00000	-39.49	1.00000	-44.08	1.00000	-44.46
11	1.00000	-153.38	1.00000	-153.79	1.00000	-153.14
12	1.00000	144.23	1.00000	140.08	1.00000	140.32
13	1.00000	101.36	1.00000	103.17	1.00000	103.52
14	1.00000	-34.06	1.00000	-35.26	1.00000	-34.84
15	1.00000	-125.43	1.00000	-127.82	1.00000	-128.40
16	1.00000	-24.60	1.00000	-26.79	1.00000	-25.50
17	1.00000	24.03	1.00000	23.21	1.00000	21.49
18	1.00000	177.96	1.00000	175.81	1.00000	175.16
19	1.00000	-150.36	1.00000	-156.14	1.00000	-155.69
20	1.00000	-68.37	1.00000	-72.34	1.00000	-70.97
21	1.00000	84.04	1.00000	81.91	1.00000	82.08
22	1.00000	64.26	1.00000	58.44	1.00000	58.33
23	1.00000	-111.91	1.00000	-111.35	1.00000	-112.68
24	1.00000	-152.31	1.00000	-152.30	1.00000	-154.14
25	1.00000	-117.07	1.00000	-121.21	1.00000	-120.15
26	1.00000	95.03	1.00000	96.06	1.00000	95.22
27	1.00000	120.70	1.00000	119.35	1.00000	120.56

RMS SOLUTION ERROR: (EXCLUDING ANTENNA 5) 5.61E-02 5.07E-02

Error (x100), 10 trials:

ℓ_1	ℓ_2
4.6	4.5
4.9	2.7
5.6	5.1
4.9	4.8
5.7	5.6
5.9	5.3
6.2	5.5
4.9	4.4
4.4	5.3
4.2	4.2
5.1±.7	4.8±.6

0.2 Jy random noise
0.5 Jy wild points on if 5

BADIF=T
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 5.000E-01 JY

b)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	8.00	1.00000	8.13	1.00000	9.14
2	1.00000	109.35	1.00000	114.64	1.00000	114.93
3	1.00000	-79.02	1.00000	-79.96	1.00000	-80.24
4	1.00000	177.12	1.00000	179.22	1.00000	-178.47
5	1.00000	-50.31	1.00000	-61.49	1.00000	-84.73
6	1.00000	-43.30	1.00000	-50.18	1.00000	-45.00
7	1.00000	-34.28	1.00000	-35.25	1.00000	-35.02
8	1.00000	-132.05	1.00000	-135.92	1.00000	-133.26
9	1.00000	-130.64	1.00000	-127.50	1.00000	-124.60
10	1.00000	58.74	1.00000	59.00	1.00000	60.89
11	1.00000	-144.71	1.00000	-139.01	1.00000	-140.47
12	1.00000	58.64	1.00000	58.77	1.00000	61.86
13	1.00000	-91.28	1.00000	-93.07	1.00000	-91.72
14	1.00000	-179.62	1.00000	179.61	1.00000	178.69
15	1.00000	-146.24	1.00000	-144.08	1.00000	-144.42
16	1.00000	-174.26	1.00000	-175.12	1.00000	-174.93
17	1.00000	106.07	1.00000	107.58	1.00000	107.44
18	1.00000	2.75	1.00000	5.76	1.00000	8.87
19	1.00000	-48.60	1.00000	-46.61	1.00000	-44.67
20	1.00000	-8.77	1.00000	-9.62	1.00000	-10.38
21	1.00000	162.28	1.00000	163.19	1.00000	163.56
22	1.00000	-36.60	1.00000	-36.88	1.00000	-37.51
23	1.00000	114.29	1.00000	111.72	1.00000	111.90
24	1.00000	36.47	1.00000	37.56	1.00000	38.88
25	1.00000	-5.84	1.00000	-4.49	1.00000	-4.88
26	1.00000	-175.89	1.00000	-172.84	1.00000	-172.99
27	1.00000	-118.59	1.00000	-119.42	1.00000	-120.26

RMS SOLUTION ERROR: (EXCLUDING ANTENNA 5) 4.65E-02 5.07E-02

Error (x100), 10 trials:

ℓ_1	ℓ_2
5.0	5.5
5.1	5.9
5.4	6.3
4.7	5.1
3.7	4.9
4.2	5.1
4.0	5.5
5.3	5.6
3.4	3.9
4.1	4.6
4.6±.6	5.2±.6

0.2 Jy random noise
1.0 Jy wild points on if 5

BADIF=T
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 1.000E+00 JY

c)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	125.00	1.00000	120.62	1.00000	123.38
2	1.00000	-95.03	1.00000	-95.53	1.00000	-96.61
3	1.00000	105.51	1.00000	104.19	1.00000	102.43
4	1.00000	-110.07	1.00000	-109.18	1.00000	-105.00
5	1.00000	100.14	1.00000	-140.33	1.00000	-145.20
6	1.00000	3.07	1.00000	2.13	1.00000	-2.42
7	1.00000	-157.02	1.00000	-159.51	1.00000	-160.37
8	1.00000	155.76	1.00000	152.91	1.00000	153.54
9	1.00000	158.76	1.00000	160.91	1.00000	161.05
10	1.00000	-51.26	1.00000	-49.65	1.00000	-50.65
11	1.00000	-120.38	1.00000	-134.04	1.00000	-132.56
12	1.00000	-19.11	1.00000	-23.61	1.00000	-20.40
13	1.00000	-165.65	1.00000	-171.54	1.00000	-169.41
14	1.00000	-55.11	1.00000	-61.26	1.00000	-64.07
15	1.00000	87.94	1.00000	84.52	1.00000	80.96
16	1.00000	-157.98	1.00000	-161.99	1.00000	-164.55
17	1.00000	-32.77	1.00000	-32.41	1.00000	-29.13
18	1.00000	97.54	1.00000	98.15	1.00000	97.72
19	1.00000	142.77	1.00000	139.64	1.00000	130.16
20	1.00000	-125.69	1.00000	-126.20	1.00000	-129.63
21	1.00000	90.41	1.00000	93.05	1.00000	94.38
22	1.00000	-8.45	1.00000	-8.05	1.00000	-8.11
23	1.00000	-129.13	1.00000	-128.65	1.00000	-132.59
24	1.00000	-154.51	1.00000	-150.74	1.00000	-150.41
25	1.00000	-109.07	1.00000	-113.30	1.00000	-109.94
26	1.00000	-40.92	1.00000	-46.49	1.00000	-40.92
27	1.00000	140.07	1.00000	145.42	1.00000	145.08

RMS SOLUTION ERROR: (EXCLUDING ANTENNA 5) 5.92E-02 7.37E-02

Error (x100), 10 trials:

ℓ_1	ℓ_2
5.4	7.4
6.0	7.8
5.5	7.6
4.3	7.2
4.6	7.4
5.5	7.8
5.8	6.8
3.4	6.0
5.5	7.4
5.8	8.1
5.3±.7	7.1±.6

0.2 Jy random noise
2.0 Jy wild points on if 5

BADIF=T
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 2.000E+00 JY

d)

ANT	TRUE SOLUTION AMP	TRUE SOLUTION PHASE	L1 SOLUTION AMP	L1 SOLUTION PHASE	L2 SOLUTION AMP	L2 SOLUTION PHASE
1	1.00000	-172.88	1.00000	-177.77	1.00000	126.89
2	1.00000	-11.62	1.00000	-11.91	1.00000	18.77
3	1.00000	-61.55	1.00000	-62.58	1.00000	-26.81
•	1.00000	-11.01	1.00000	-13.86	1.00000	99.36
5	1.00000	24.82	1.00000	98.70	1.00000	94.13
6	1.00000	27.88	1.00000	26.38	1.00000	39.64
7	1.00000	-30.71	1.00000	-40.14	1.00000	-77.89
8	1.00000	-50.94	1.00000	-50.93	1.00000	-67.67
9	1.00000	157.08	1.00000	156.39	1.00000	171.08
10	1.00000	-79.23	1.00000	-83.15	1.00000	-53.50
11	1.00000	-154.69	1.00000	-159.74	1.00000	152.64
12	1.00000	-90.77	1.00000	-91.80	1.00000	-109.42
13	1.00000	-8.97	1.00000	-8.93	1.00000	-105.39
14	1.00000	-179.92	1.00000	-179.13	1.00000	37.31
15	1.00000	-72.30	1.00000	-60.81	1.00000	101.03
16	1.00000	144.20	1.00000	137.32	1.00000	46.58
17	1.00000	85.70	1.00000	84.17	1.00000	45.05
18	1.00000	-24.95	1.00000	-23.70	1.00000	-20.30
19	1.00000	91.85	1.00000	90.80	1.00000	83.56
20	1.00000	-25.93	1.00000	-29.42	1.00000	56.45
21	1.00000	125.91	1.00000	127.63	1.00000	-176.54
22	1.00000	131.38	1.00000	131.37	1.00000	-127.35
23	1.00000	16.98	1.00000	14.24	1.00000	3.64
24	1.00000	78.31	1.00000	72.87	1.00000	150.44
25	1.00000	-168.44	1.00000	-168.08	1.00000	-7.37
26	1.00000	-101.06	1.00000	-106.14	1.00000	-143.96
27	1.00000	-144.82	1.00000	-145.13	1.00000	158.06

RMS SOLUTION ERROR: (EXCLUDING ANTENNA 5) 5.78E-02 1.08E+00

Error (x100), 10 trials:

ℓ_1	ℓ_2
5.8	108
6.1	132
6.1	125
5.6	116
5.0	134
5.8	123
4.3	127
3.9	136
6.1	140
5.4	119
5.4±.7	126±.4

0.2 Jy random noise
100.0 Jy wild points on if 5

BADIF=T
NOISEAMP= 0.20 JY
WILDPCT= 0.00
WILDAMP= 1.000E+02 JY

Figure 7. Sensitivity of phase solutions to a bad i.f.