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SIGNAL ANALYSIS OF A CORRELATION INTERFEROMETER

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The correlation interferometer is an example of a two-receiver correlation radiometer. Staelin (1974) has analyzed the performance of this radiometer. In this memorandum I will follow Staelin's analysis but will use a terminology more familiar to most radio astronomers.

The block diagram of a correlation interferometer is shown in Figure 1. The two receiver inputs $s_a(t)$ [or just $s(t)$] and $s_b(t)$ are identical except for a multiplicative constant:

$$s_b(t) = \sqrt{\frac{K_b}{K_a}}\, s_a(t),$$

where K is the sensitivity of the antenna in K Jy^{-1} ($K \sim 0.1$ K Jy^{-1} for a 25m antenna).

The autocorrelation function of the signal s at the input to the multiplier is

$$\phi_s(\tau) = \overline{s_1 s_2},$$

where $s_1 = s(t)$ and $s_2 = s(t + \tau)$. The DC input to the multiplier is

$$\phi_s(0) = kK_a SB,$$

where S is the flux density of the source.

The noises $n_a(t)$ [or just $n(t)$] and $n_b(t)$ are Gaussian with zero mean and are independent of each other and of the signals $s_a(t)$ and $s_b(t)$. Their autocorrelation functions at the inputs to the multiplier differ by a multiplicative constant equal to the ratio of the receiver temperatures, with

$$\phi_n(\tau) = \overline{n_1 n_2},$$

and

$$\phi_n(0) = kT_aB.$$

To get the output signal and noise powers, we must obtain first the autocorrelation function of the multiplier output:

$$\phi_m(\tau) = a^2\, \overline{v_a(t)\; v_b(t)\; v_a(t+\tau)\; v_b(t+\tau)}$$

$$= a^2G^2\, \overline{[(s_{a1} + n_{a1})(s_{b1} + n_{b1})(s_{a2} + n_{a2})(s_{a2} + n_{b2})]}.$$

Carrying out the indicated multiplications and averages, we obtain

$$\phi_m(\tau) = a^2G^2\, (\overline{s_{a1}\; s_{b1}\; s_{a2}\; s_{b2}} + \overline{s_{a1}\; s_{a2}} \cdot \overline{n_{b1}\; n_{b2}} + \overline{n_{a1}\; n_{a2}} \cdot \overline{s_{b1}\; s_{b2}} + \overline{n_{a1}\; n_{a2}} \cdot \overline{n_{b1}\; n_{b2}}),$$

or

$$\phi_m(\tau) = a^2\, G^2 \left[\frac{K_b}{K_a}\, \overline{s_1{}^2 s_2{}^2} + \left(\frac{T_b}{T_a} + \frac{K_b}{T_b}\right) \phi_s(\tau)\phi_n(\tau) + \frac{T_b}{T_a}\, \phi_n{}^2(\tau)\right].$$

Since

$$\overline{s_1{}^2 s_2{}^2} = \phi_s{}^2(0) + 2\phi_s{}^2(\tau),$$

we can expand the first term to obtain

$$\phi_m(\tau) = a^2G^2 \left[\frac{K_b}{K_a} \phi_s{}^2(0) + 2 \frac{K_b}{K_a} \phi_s{}^2(\tau) + \left(\frac{T_b}{T_a} + \frac{K_b}{K_a}\right)\phi_s(\tau)\phi_n(\tau) + \frac{T_b}{T_a} \phi_n{}^2(\tau)\right].$$

The associated power spectrum at the multiplier output is then

$$\Phi_m(f) = a^2G^2 \left[\frac{K_b}{K_a} \phi_s{}^2(0)\mu_o(f) + 2 \frac{K_b}{K_a} \int_{\infty}^{\infty} \Phi_s(\eta)\Phi_s(f-\eta)d\eta + \left(\frac{T_b}{T_a} + \frac{K_b}{K_a}\right) \int_{\infty}^{\infty}\Phi_s(\eta)\Phi_n(f-\eta)d\eta + \frac{T_b}{T_a} \int_{\infty}^{\infty}\Phi_n(\eta)\Phi_n(f-\eta)d\eta\right],$$

where $\mu_o(f)$ is the unit impulse at f=0.

Power spectral components of $\Phi_m(f)$ are sketched in Figure 2 for signal and noise with flat spectra passing through filters with rectangular passbands of width B.

The average output of the low-pass filter is simply the DC output of the multiplier, or

$$\overline{v_o} = \sqrt{P_{DC}} = aGkSB \sqrt{K_aK_b} .$$

To determine the fluctuations about the average it is necessary to specify the particular output filter; the simplest filter for our purpose is an ideal integrator with integration time τ. The power out of the ~~output~~ filter is

$$\overline{v_o{}^2(t)} = \int_{\infty}^{\infty}\Phi_o(f)\, df$$

$$= \int_{\infty}^{\infty}\Phi_m(f)|H(f)|^2df,$$

where $|H(f)|^2$ is the magnitude squared of the transfer function of the low-pass filter. For integration times of interest, H(f) will be negligible except near f=0, and for practical purposes $\Phi_m(f)$ may be replaced by its value at or near f=0. For an ideal integrator the impulse response is

$$h(t) = \begin{cases} \frac{1}{\tau}, & 0<t<\tau \\ 0, & \text{elsewhere}, \end{cases}$$

and the output noise power is

$$P_{noise} = \Phi_m(0)\int_{\infty}^{\infty}|H(f)|d^2f$$

$$= \Phi_m(0)\int_{\infty}^{\infty}h^2(t)dt$$

$$= \frac{a^2G^2k^2B}{\tau}\left[K_aK_bS^2 + \frac{K_aST_a}{2}\left(\frac{T_b}{T_a} + \frac{K_b}{K_a}\right) + \frac{T_a\ T_b}{2}\right].$$

The rms variation in the output is just $\sqrt{P_{noise}}$.

In terms of flux density at the input, we find

$$\Delta S_{rms} = \frac{\text{rms output}}{\text{sensitivity}}$$

$$= \frac{\sqrt{P_{noise}}}{\left|\frac{\partial\sqrt{P_{DC}}}{\partial S}\right|}.$$

Therefore,

$$\Delta S_{rms} = \frac{1}{\sqrt{B\tau}}\left[S^2 + \frac{S}{2}\left(\frac{T_a}{K_a} + \frac{T_b}{K_b}\right) + \frac{1}{2}\left(\frac{T_a}{K_a}\right)\left(\frac{T_b}{K_b}\right)\right]^{\frac{1}{2}}.$$

For identical antennas and receivers, we obtain

$$\Delta S_{rms} = \frac{1}{\sqrt{B\tau}}\left(S^2 + \frac{ST}{K} + \frac{1}{2}\frac{T^2}{K^2}\right)^{\frac{1}{2}}.$$

In the weak-source case,

$$\Delta S_{rms} = \frac{1}{\sqrt{2B\tau}} \frac{T}{K} ,$$

and in the strong-source case,

$$\Delta S_{rms} = \frac{S}{\sqrt{B\tau}} .$$

The corresponding expression for a total-power radiometer on an identical antenna is

$$\Delta S_{rms} = \frac{1}{\sqrt{B\tau}} \left(S + \frac{T}{K}\right).$$

Consequently, the correlation interferometer is more sensitive by a factor of $\sqrt{2}$ in the presence of independent sources of noise (e.g., receiver noise, ground pickup) at each antenna than the total-power radiometer but has the same sensitivity to sources of noise such as the source itself and the sky background that are common to both antennas.

Reference:

Staelin, D.H. 1974, The Detection and Measurement of Radio Astronomical Signals (Cambridge: The Massachusetts Institute of Technology).

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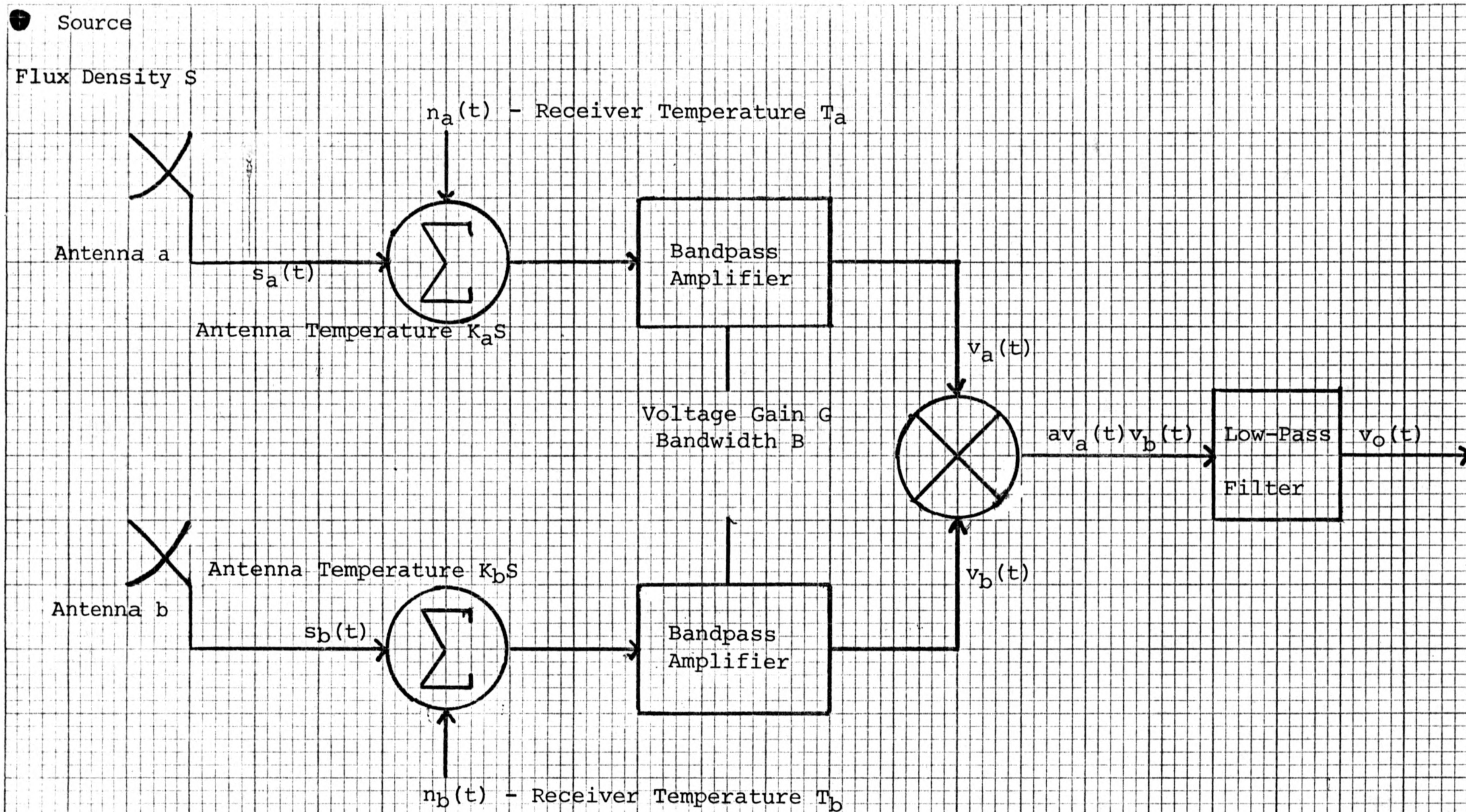

Figure 1. Correlation Interferometer

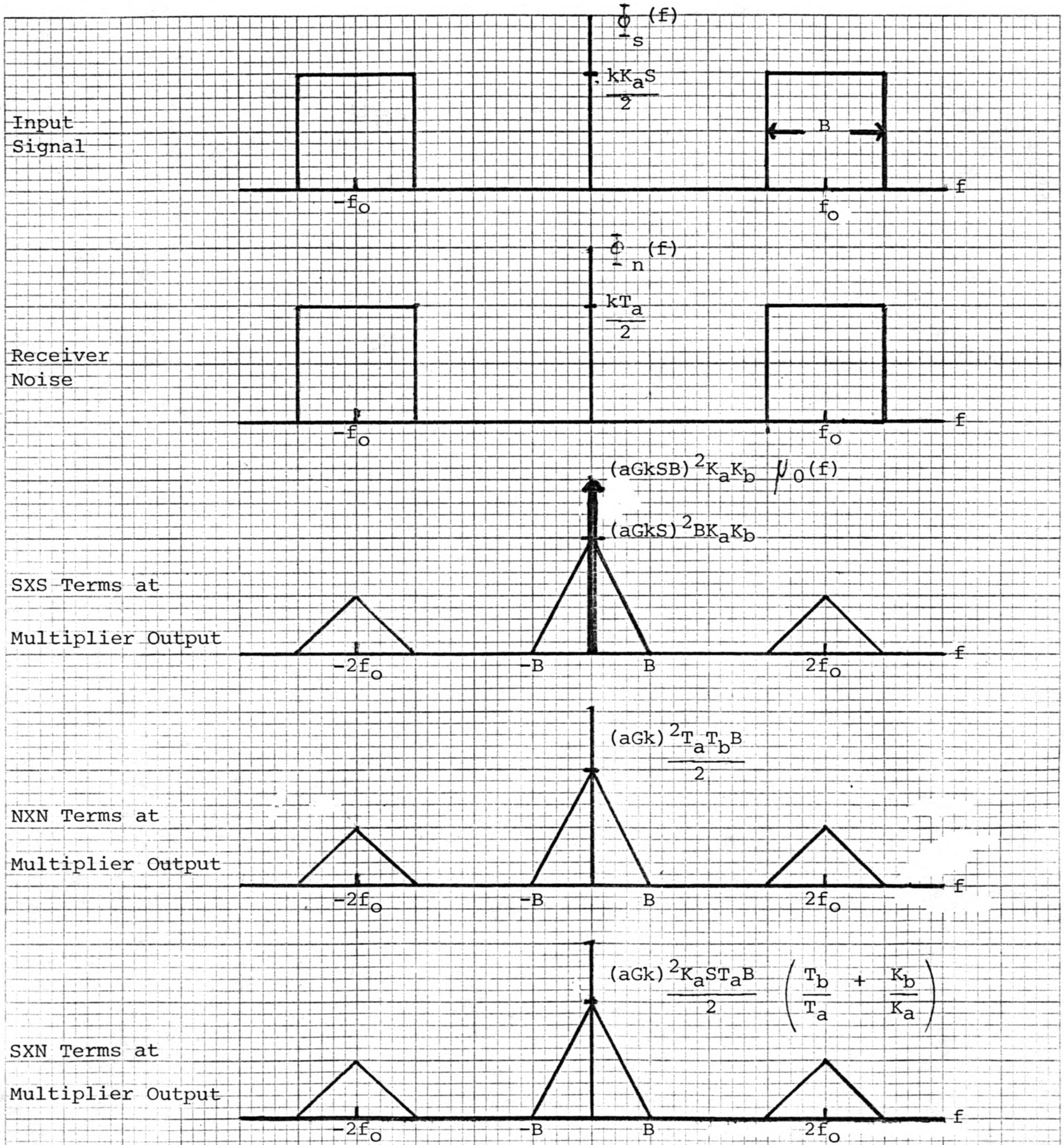

Figure 2. Multiplier Input and Output Power Spectral Densities