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ORDERS OF MAGNITUDE OF SOME INSTRUMENTAL EFFECTS

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Self calibration, marvelous though it is, cannot correct quite a large number of instrumental effects because they do not correspond to antenna gain effects. The purpose of this memo is to estimate order of magnitudes for some of these. The alternate title of this memo is therefore, "What Self-Cal Won't Do For You".

1. Antsol's "Closure Errors"

Consider the case of a "perfectly calibrated" point source at field center. The output on each baseline, due to the defects in closure, will be, not S, the source flux, but $(1+\alpha_{mn})S$, where α_{mn} is the closure defect on the m-n baseline as printed by antsol. The error pattern (E(x,y)) this induces in a map is, obviously, the Fourier transform of $\alpha_{mn}\ S\cdot W$, where W is the weighting function (ignoring the effects of convolution). By Parsevel's Theorem

$$<E^2(x,y)> = <W^2 \alpha_{mn}{}^2> \; S^2$$

if W and α are independent (if I didn't have this parenthesis here, I would have sneaked a very iffy hypothesis past you without your noticing, wouldn't I have?), then

$$\text{rms dynamic range} = \frac{\sqrt{<W^2>}}{<W>} \cdot \sqrt{<\alpha_{mn}{}^2>}$$

The lefthand factor is (for large maps anyway) essentially identical to that for the rms value of dirty beam sidelobes. Since dirty beam sidelobes tend to be 1% to 3% rms, and the α_{mn} tend to have an rms (dominated by the imaginary part) of 1% to 2%, the predicted limitation of this effect on dynamic range is of order 0.1% to 0.01% (highest for snapshots which have bad dirty beams). The success of trying to calibrate the effect is not known.

2. Bandwidth

I have shown (VLA Scientific Memo #118) that under the premise of identical bandpasses, the effect of bandwidth smearing is representable as an integral convolution. The implication is, that since this is a linear operation, self-calibration should work normally, producing a map which can later be corrected by the procedure I outlined.

However, Dick Thompson has shown (VLA Electronics Memo #192) that the presence of large delay errors can cause small bandpass mismatches to contribute much more to closure errors than they do in the case of well matched delays. It would not surprise me if the "effective closure errors" (as measured by self-cal's ability to make high dynamic range maps) were of order 10%, rather than the 1-2% we measure on calibrators, for a source near the first null of the bandwidth pattern on the longest baseline. Moral - if the field is dominated by a strong point, put it at the phase tracking center if you possibly can.

For fun I give yet another expression for the effects of bandwidth smearing. For a circular gaussian beam of width θ (full width, half maximum) and a gaussian passband of width W at frequency F ($W \ll F$), the effect is approximately, in the vicinity of (x_o, y_o), to convolve the beam

$$e^{-c\frac{(x^2+y^2)}{2\theta^2}}$$

(where c = ln 256) with a one dimensional "bandwidth beam"

$$e^{-c\,\frac{(xx_o+yy_o)^2F^2}{2W^2(x_o{}^2+y_o{}^2)^2}}$$

for the cross-section $y = y_o = 0$ this convolution gives

$$\left(1 + \frac{W^2}{F^2}\frac{x_o^2}{\theta^2}\right)^{-\frac{1}{2}} e^{-c\,\frac{x^2}{2\left(\theta^2 + \frac{W^2}{F^2}x_o^2\right)}}$$

Note that with other parameters constant, the synthesized beam θ is inversely proportional to F so that the amplitude reduction (the first factor) is independent of F.

3. Beam and Pointing

It has long been known, that, for identical, perfectly pointed, elements, the effects of the element beam can be removed by a division in the map plane. A reasonable (Arnold Rots has more reasonable) estimate of the beam is

$$P(r) \approx 1 - 1.2\times10^{-3}\,(rF)^2 + 3.5\times10^{-7}\,(rF)^4$$

where r is in minutes of arc and F is in Gigahertz.

Of more concern to us here is the variation in pointing, either as a function of time or of antenna. If radiation comes from all over the beam, self-cal cannot remove this effect because different pieces of the source vary independently. The rms amplitude fluctuation on a source at radius r is approximately the slope of the beam times our 0.2 minute rms one dimensional nighttime pointing error.

$$\frac{\Delta A}{A} = 4.8\times10^{-4}\,rF^2 - 2.8\times10^{-7}\,r^3F^4$$

We may here repeat the moral, that if you are dominated by a strong point, put it at the phase tracking center if you possibly can.

4. Non-Coplanar Baselines

This isn't really a dynamic range or proper instrumental problem - it can be corrected analytically and exactly. However, the fact is that we have no software to do so.

The VLA is, to an excellent approximation, instantaneously coplanar. Jerry Hudson showed in his thesis that for an instantaneously coplanar array, the instantaneous beam at a given point on the map wanders with time on a path with length of order r^2 (where r is the radius from the center of the field in radians). The coefficient depends very much on the circumstances of the observation, ranging from near zero for snapshots to about the cosecant of limiting elevation for full synthesis. For an order of magnitude estimate, let us take uniform motion along a path of length r^2. Then, with a Gaussian beam of size θ, the reduction in intensity is given by

$$\frac{\int_{-\frac{r^2}{2}}^{\frac{r^2}{2}} e^{-c\frac{x^2}{2\theta^2}}\,dx}{r^2}$$

For small r, this deviates from unity by a term going as r^4, so there is a rather sharp cutoff of the useful field. For correction factors near unity, the above expression reduces to

$$1 - \frac{c}{24}\frac{r^4}{\theta^2} \text{ (r and } \theta \text{ in radians).}$$

5. Finite Integration Time

This bears an interesting similarity to bandwidth smearing. In fact, for an object at the north pole, it reduces to a distorted azimuthal convolution in the same way that bandwidth smearing reduces to a distorted radial convolution. Unfortunately at other declinations the situation is not so simple. For an object at x,y (relative to phase tracking center) the instantaneous phase is

$$2\pi F\ (ux+vy)$$

The phase rate is

$$2\pi F\ (\frac{du}{dt}x + \frac{dv}{dt}y)$$

and the loss in amplitude by integrating for a time Δt is given by

$$R \approx \sin\ (\pi F\ (\frac{du}{dt}x + \frac{dv}{dt}y)\ \Delta t)/(\pi F\ (\frac{du}{dt}x + \frac{dv}{dt}y))$$

$$\approx 1 - \frac{1}{6}\ [\pi F\ (\frac{du}{dt}x + \frac{dv}{dt}y)\ \Delta t]^2$$

To see how this affects things we can make reasonable assumptions about the symmetry of the array:

$$< \frac{du}{dt} \frac{dv}{dt} > = 0$$

$$< \frac{du^2}{dt} > = <u^2>$$

$$< \frac{dv^2}{dt} > = <u^2> \sin^2\delta$$

Then $$R \approx 1 - \frac{1}{24} (x^2+\sin^2\delta \; y^2) <(2\pi Fu)^2> \Delta t^2$$

If we have a Gaussian beam of width θ

$$e^{-c \frac{x^2}{2\theta^2}}$$

The u,v distribution must approximate to its transform

$$e^{- \frac{(2\pi Fu\theta)^2}{2c}}$$

whence

$$<u^2> = \frac{c}{(2\pi F\theta)^2}$$

and the amplitude loss is approximately

$$R \simeq 1 - \frac{1}{24} (x^2+\sin^2\delta \; y^2) \frac{c}{\theta^2} \Delta t^2$$

For order of magnitude work we can say

$$R \approx 1 - (\frac{n}{3000} \frac{r}{\theta})^2$$

where the integration time is n times 10 seconds. Maps are severely affected by integration time at a few times closer than $\frac{3000}{n}$ beamwidths from the center. Once again though, I can pronounce my moral: If your field is dominated by a strong point source, put it at the phase tracking center if you possibly can.

6. Truncation of u,v

Currently the databases contain a fixed point u and v, with the least significant bit of 4 ns. This is, of course a pillbox convolution - resampling of the data, whose deleterious effects are well known on a larger scale. To estimate its importance, we can fall back on the usual analysis of the pillbox convolution (assuming it is unaffected by the later convolution-resampling in mapmaking, as it definitely is not in the case of the direct Fourier transform). The reduction in amplitude at (x,y) is

$$\sin(\pi x F\Delta)\ \sin(\pi y F\Delta)/(\pi x F\Delta \pi y F\Delta)$$

where Δ is the 4 ns truncation interval. This is approximately

$$1 - \frac{1}{24}(2\pi F\Delta r)^2 \quad .$$

Again, the reiterated moral is invoked.

G. Summary: What is Important

It is useful to know which of the above effects limits your work, so that you can take special care. To see that, we can put all the above effects in a comparable form, by noting the leading term of their reduction in amplitude on a source at half power beamwidth, and assuming that this is a measure of their importance in more subtle aspects of mapping. The only error which does not lend itself readily to this procedure is the pointing error. I put it in as equivalent to a multiplicative error of $1 - 2.4\times10^{-4}\ rF^2$ error which would be the case if, halfway through the observation, all antennas leaned away from the source, and stayed there. The actual situation is much more complicated, and this may result in a slight over emphasis of pointing errors relative to the others. In the process of summarizing, I shall convert from beamwidths, θ, which I have been using, to arm lengths, L. For L in kilometers, θ in seconds I take $\theta \approx \frac{50}{FL}$, F in Gigahertz.

Bandwidth:

Correctable: $1 - 290 \frac{W^2}{F^2} L^2$

Uncorrectable: Unknown, but perhaps only a factor of 10 less important, say $1 - 30 \frac{W^2 L^2}{F^2}$

Beam:

Correctable: ½

Uncorrectable: Equivalent to $1 - 4.6 \times 10^{-3} F$

Non-Coplanar Baseline:

Correctable: $1 - 4.5 \times 10^{-3} \frac{L^2}{F^2}$

Integration Time:

Uncorrectable: $1 - 6.4 \times 10^{-5} n^2 L^2$

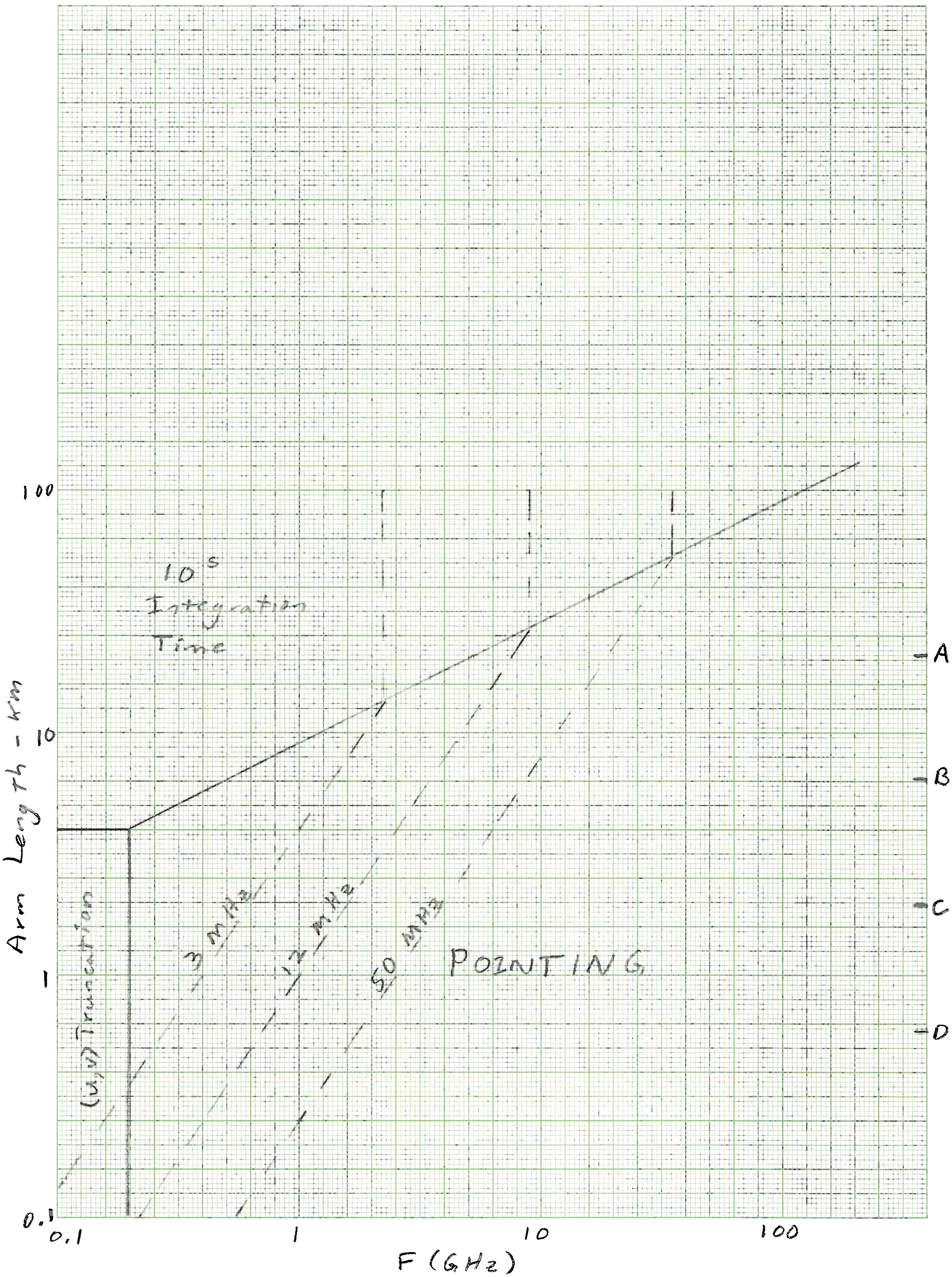
Truncation of u,v:

Uncorrectable?: $1 - 8.9 \times 10^{-4}$

It is interesting to note that the non-coplanar baseline effect at half power on the beam has the same functional form as a bandwidth correction, and is about equal to the uncorrectable effects of a 12 MHz bandwidth, and is less important than the full bandwidth effect at 6 MHz. This arises because the reciprocal of 12 MHz is 80 ns, about equal to the dish diameter. However, because the effect rises as the fourth power of radius, it becomes much more serious if you map to the the beam null.

The attached figure shows which uncorrectable errors dominate in various parts of the armlength-frequency plane. Pointing effects dominate except in A configuration at low frequencies where the errors are dominated by the effects of the 20 second integration (the boundary between these regions is the solid line). The area to the left of the dashed lines (labeled by bandwidth) is dominated by the uncorrectable bandwidth error. The u,v truncation error is a dominant error only in C or D array, and then only at frequencies near 100 MHz, where pointing error is vastly reduced by the very large beam.

I would like to close with a caveat that the approximations used here are such that a factor of two difference from an alternate calculation of the same quantity in the summary expressions in this section should be unsurprising, and in some particular circumstances a full order of magnitude may have been lost.



100
10
1
0.1
Arm Length - km
0.1
1
10
100
F (GHz)
10 s Integration Time
(u,v) Truncation
3 MHz
12 MHz
50 MHz
POINTING
A
B
C
D