

VLA Test Memorandum 102

Site Coordinate Systems and Conversions

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Abstract

The conversions between geodetic coordinates, the New Mexico State Plane Coordinate System, and ground coordinates are described and illustrated by examples. The conversion between grid and geodetic azimuths is also treated. Appendices give numerical data for the VLA site.

I. Introduction

This memorandum was conceived originally to give alignment data for laying out and checking the observing stations. It quickly became evident, however, that the reference frames which are involved should be clarified first. This is the problem treated here. A later memorandum will discuss the observing stations.

Three different coordinate systems must be considered in setting up the VLA. Each is advantageous, depending on the context. Since coordinate conversions are somewhat hazardous because of the ease of making blunders, the present memorandum has been prepared as a "cookbook" in order to make the conversions as direct and foolproof as possible. Of course, anyone using it for the first time probably will deny that it is either direct or foolproof.

We are concerned with the following reference frames:

- A. Geodetic Coordinates (ϕ, λ): These are the geodetic latitude and longitude. They are customarily expressed in arc measure. They tie the VLA to the rest of the world. Astronomical azimuths measured in the field are identical with geodetic azimuths to the accuracy required in setting up the VLA.
- B. New Mexico State Plane Coordinates (x,y): This is a transverse Mercator projection of the spheroid onto a cylinder tangent to the spheroid (at mean sea level) along a standard meridian. Since the distortion of the projection increases rather rapidly away from the standard meridian, New Mexico actually has three plane coordinate systems covering different longitude

zones. The VLA site is in the West Zone, for which the standard meridian is $107^{\circ}50'00''$ W. The coordinates are expressed in feet away from an arbitrary origin. The x and y coordinates increase eastward and northward respectively.

- C. Ground Coordinates (X,Y): This frame is similar to the State Plane Coordinates, but differs in that it has been corrected for (a) the elevation of the site above mean sea level and (b) the local scale distortion of the transverse Mercator projection. The (X,Y) and (x,y) frames are oriented identically. The ground coordinates are the ones to use for all survey work on the site.

Conversion between the different systems are described in Sections II and III. The conversion between geodetic and grid azimuths is treated in Section IV. Appendix I gives numerical data for the VLA. Appendix II lists the rectangular coordinates of all of the permanent survey monuments on the site.

II. Conversion between State Plane Coordinates and Ground Coordinates

These systems differ by a constant scale factor which depends on elevation and location with respect to the standard meridian. For the VLA site, one has:

$$X = 1.0004134 x$$

$$Y = 1.0004134 y$$

$$x = 0.99958677 X$$

$$y = 0.99958677 Y$$

In both systems, the coordinates are expressed in feet from arbitrary (but fixed) origins.

III. Conversion between State Plane Coordinates and Geodetic Coordinates

These conversions necessarily are rather involved. They are discussed in general terms in Chapters 12 and 23 of Surveying for Civil Engineers by Philip Kissam (McGraw-Hill, 1956). Numerical tables for making the conversions for any place in New Mexico are given in Coast and Geodetic Survey Special Publication 324 (U.S. Government Printing Office, 1954). The tables are tough to use

because they are inconveniently arranged, the instructions are vague, and numerous interpolations are necessary. I have recast them in analytical form so that one can make the conversion with an HP-35 without touching the tables. With the tables, the conversions are difficult, whereas they are merely tedious with the HP-35. I have retained the notation of the tables with minor modifications. A caution: the equations given below embody a number of simplifying approximations which are correct near the VLA site center, but which start to break down at distances of 30 miles or more away from it.

The equations for making the conversion in each direction are given below, in parallel with an example of their use. Important: the calculations should always be carried to the number of decimal places used in the examples. For the sample calculation, we convert the State Plane Coordinates of the intersection of the wye arms into the corresponding geodetic latitude and longitude. Then the plane coordinates are rederived from these results.

Conversion of State Plane Coordinates to Geodetic Coordinates

Given: $x = 565285.633$

$y = 1120089.552$

Step	Formula	Example
1	$x' = x - 500000$	$x' = 65285.633$
2	$P = 1.43432 + 0.15516 (10^{-6}y) + 0.00523 (10^{-6}y)^2$	$P = 1.61467$
3	$y_0 = y - (10^{-4}x')^2 P - 0.01$	$y_0 = 1120020.721$
4	$y' = y_0 - 1091365.48$	$y' = 28655.241$
5	$\phi = 34^\circ + 2.748181175 \times 10^{-6}y' - 6.214237865 \times 10^{-16}(y')^2$	$\phi = 34.07874928$ $= 34^\circ 04' 43'' 497$
6	$H = 87.036675 - 2.501386 (10^{-6}y_0) - 0.096389 (10^{-6}y_0)^2$	$H = 84.11416$
7	$\Delta\lambda_0 = x'/H$ (nearest second only)	$\Delta\lambda_0 = 776''$
8	$a = 0.208 (10^{-6}y_0) - 1$	$a = -0.767$
9	$b = 0.3702 (10^{-2}\Delta\lambda_0) - 1.64 \times 10^{-4} (10^{-2}\Delta\lambda_0)^3$	$b = 2.796$
10	$\Delta\lambda = (x' - ab)/H$ (arc seconds)	$\Delta\lambda = 776''181$ $= 0^\circ 12' 56'' 181$
11	$\lambda = 107^\circ 50' 00'' - \Delta\lambda$	$\lambda = 107^\circ 37' 03'' 819$
	The geodetic coordinates are: $\phi = 34^\circ 04' 43'' 497$ N $\lambda = 107^\circ 37' 03'' 819$ W	

Conversion of Geodetic Coordinates to State Plane Coordinates

Given: $\phi = 34^{\circ}04'43''497$ N

$\lambda = 107^{\circ}37'03''819$ W

Step	Formula	Example
1	$\Delta\lambda = 107^{\circ}50'00'' - \lambda$ (arc seconds)	$\Delta\lambda = 776''181$
2	$\phi' = \phi - 34^{\circ}$ (decimal degrees)	$\phi' = 0^{\circ}0787492$
3	$H = 84.191934 - 0.986754 \phi' - 0.012844 (\phi')^2$	$H = 84.11415$
4	$V = 1.141439 + 0.016156 \phi' - 6.96 \times 10^{-4} (\phi')^2$	$V = 1.142707$
5	$a = 0.075 \phi' - 0.772$	$a = -0.766$
6	$b = 0.3702 (10^{-2}\Delta\lambda) - 1.64 \times 10^{-4} (10^{-2}\Delta\lambda)^3$	$b = 2.797$
7	$c = 3.333 \times 10^{-4} (10^{-2}\Delta\lambda) - 2.800 \times 10^{-4} (10^{-2}\Delta\lambda)^2 + 2.667 \times 10^{-6} (10^{-2}\Delta\lambda)^3$	$c = -0.013$
8	$x' = H\Delta\lambda + ab$	$x' = 65285.633$
9	$x = x' + 500000$	$x = 565285.633$
10	$y_o = 1091365.48 + 363877.03 \phi' + 29.94 (\phi')^2$	$y_o = 1120020.691$
11	$y = y_o + (10^{-2}\Delta\lambda)^2 V + c$	$y = 1120089.521$
	The State Plane Coordinates are: $x = 565285.633$ $y = 1120089.521$	

The result agrees with initial coordinates to within 0.031 ft, or 3/8 of an inch. The accuracy is entirely adequate for VLA purposes.

IV. Azimuth Conversions

The grid azimuth from point a to point b is given by*

$$A_{\text{grid}} = \arctan \frac{x_b - x_a}{y_b - y_a} = \arctan \frac{X_b - X_a}{Y_b - Y_a} .$$

*We use the astronomical convention that azimuth is reckoned clockwise from the north. In geodetic surveying, azimuth is frequently measured from the south-- a point to bear in mind when looking for the azimuth marks associated with a bench mark!

This differs from the geodetic azimuth by an amount

$$\Delta\alpha = A_{\text{geodetic}} - A_{\text{grid}} .$$

In seconds of arc, we have

$$\Delta\alpha = Mx'$$

where

$$M = 0.0065794 + 6.89 \times 10^{-10} (y - 10^6)$$

for the VLA site, and

$$x' = x - 500000 .$$

Thus for the intersection of the arms of the wye, where $x = 565285.633$ and $y = 1120089.552$, we have $\Delta\alpha = 434''9 = 0^{\circ}07'14''9$.

Along a given arm of the VLA, the grid azimuth of the arm remains constant, whereas the corresponding geodetic azimuth changes with distance from the center because of the convergence of the meridians. The variation is considerable; the extreme range of $\Delta\alpha$ over the VLA is more than $13'$ of arc. $\Delta\alpha$ changes most rapidly along the southeast arm, where it increases by nearly $20''$ of arc per kilometer (see Appendix I).

Appendix ICoordinate and Azimuth Conversion Data for the VLA

Values at the Wye Center:

$$\left. \begin{array}{l} \phi = 34^{\circ}04'43''.497 \text{ N} \\ \lambda = 107^{\circ}37'03''.819 \text{ W} \end{array} \right\} \text{geodetic coordinates}$$

$$\left. \begin{array}{l} x = 565285.633 \\ y = 1120089.552 \end{array} \right\} \text{state plane coordinates}$$

$$\left. \begin{array}{l} X = 565519.322 \\ Y = 1120552.598 \end{array} \right\} \text{ground coordinates}$$

Arm azimuths (outward from the center);

Arm	Geodetic Azimuth		Nominal* Grid azimuth
	At wye center	At end of arm	
N	355°00'00"	354°59'25"	354°52'45"
SE	115°00'00"	115°06'54"	114°52'45"
SW	236°00'00"	235°53'40"	235°52'45"

*Actual survey values are different by a few arc seconds; this is of no importance.

Changes per kilometer of arm length (outward from the center):

Arm	Ground Coordinates		N.M. Plane Coordinates		$\Delta\alpha$
	ΔX	ΔY	Δx	Δy	
N	-292.836	+3267.745	-292.715	+3266.395	-1"842
SE	+2976.368	-1380.269	+2975.138	-1379.699	+19"714
SW	-2716.064	-1840.354	-2714.942	-1839.594	-18"095

These data can be used to compute (X,Y) , (x,y) , and $\Delta\alpha$ for points along the arms of the wye, if the distance from the center is specified. Example: At a point on the SE arm, 15 km from the wye center, one has

$$\begin{aligned} X &= 610164.8 \\ Y &= 1099848.6 \\ x &= 609912.7 \\ y &= 1099394.1 \\ \Delta\alpha &= 12'11'' \end{aligned}$$

The geodetic azimuth of the arm at this point is

$$A_{\text{grid}} + \Delta\alpha = 115^{\circ}04'56'' .$$

Example: What is the distance and geodetic azimuth from the point in the previous example to a point 10 km from the center on the N arm? For the latter point,

$$\begin{aligned} X_N &= 562591.0 \\ Y_N &= 1153230.0 \end{aligned}$$

The grid azimuth to it from the first point is

$$A_{\text{grid}} = \arctan \frac{562591.0 - 610164.8}{1153230.0 - 1099848.6} = \begin{cases} 318^{\circ}29'23.97'' \\ 318^{\circ}17'33'' \end{cases} .$$

The geodetic azimuth is

$$A_{\text{grid}} + 12'11'' = 318^{\circ}29'44'' .$$

The distance is

$$D = [(X_N - X)^2 + (Y_N - Y)^2]^{1/2} = 71504.1 \text{ ft.}$$

Appendix IILocations of Permanent Survey Monuments
on the VLA Site

The following three pages are copies of the report of Limbaugh Engineers, Inc. on the positions of benchmarks they set on the VLA site (letter to J. H. Lancaster, dated 25 July 1973).

VLA - PERMANENT SURVEY MONUMENTS

JOB #3778 - NORTH WYE

New Mexico State Plane Coord. System - West Zone
 Delta Alpha = +00°07'15"
 Mean Ground Datum = 7000'
 Ground to Grid Conv. Fact. = .999586770

MONUMENT		STATE PLANE COORDINATES		GROUND COORDINATES		ELEV.
Number	Station	X	Y	X	Y	
VLA						
Centerpoint	0+00.000	565,285.633	1,120,089.552	565,519.322	1,120,552.598	6,964.52
N-1-1	51+20.885	564,828.582	1,125,187.875	565,062.083	1,125,653.029	6,962.64
N-1	81+36.037	564,559.418	1,128,189.739	564,792.807	1,128,656.133	6,971.85
N-2	92+46.618	564,460.281	1,129,295.425	564,693.629	1,129,762.277	6,966.28
N-3	130+06.821	564,124.592	1,133,039.054	564,357.801	1,133,507.453	6,969.00
N-4	163+03.149	563,830.296	1,136,320.850	564,063.384	1,136,790.606	6,970.68
N-4-1	176+39.546	563,710.990	1,137,651.356	563,944.029	1,138,121.662	6,971.42
N-5	226+31.864	563,265.212	1,142,621.661	563,498.066	1,143,094.022	
N-6	277+34.881	562,809.527	1,147,702.175	563,042.193	1,148,176.636	6,985.84
N-7	327+30.249	562,363.415	1,152,675.510	562,595.896	1,153,152.027	
N-8	378+71.809	561,904.205	1,157,794.389	562,136.497	1,158,273.022	6,990.31
N-8-1	402+53.675	561,691.476	1,160,165.747	561,923.680	1,160,645.361	6,989.97
N-9	436+36.516	561,389.288	1,163,533.661	561,621.367	1,164,014.667	6,994.10
N-9-1	457+78.395	561,197.947	1,165,666.087	561,429.947	1,166,147.975	6,996.90
N-10	489+57.124	560,914.002	1,168,830.790	561,145.884	1,169,313.986	7,000.10
N-11	538+70.007	560,475.097	1,173,721.991	560,706.798	1,174,207.209	7,004.86
N-11-1	587+07.733	560,042.848	1,178,538.360	560,274.370	1,179,025.569	7,025.89
N-12	609+45.976	559,842.849	1,180,766.721	560,074.289	1,181,254.851	7,035.55
N-13	621+69.912	559,733.474	1,181,985.252	559,964.868	1,182,473.886	7,027.98
N-14	624+66.688	559,706.953	1,182,280.719	559,938.336	1,182,769.474	7,017.23

NOTE: ALL MONUMENTS ARE BRASS CAPS SET IN CONCRETE

VAL - PERMANENT SURVEY MONUMENTS

Job #3778 - SOUTHEAST WYE

New Mexico State Plane Coord. System - West Zone
 Delta Alpha = +00°07'15"
 Mean Ground Datum = 7000'
 Ground to Grid Conv. Fact. = .999586770

MONUMENT		STATE PLANE COORDINATES		GROUND COORDINATES		ELEV.
Number	Station	X	Y	X	Y	
VLA						
Centerpoint	0+00.000	565,285.633	1,120,089.552	565,519.323	1,120,552.598	6,964.52
SE-1-1	38+89.100	568,812.435	1,118,454.234	569,047.583	1,118,916.604	6,967.98
SW-1	80+42.190	572,578.616	1,116,707.869	572,815.321	1,117,169.517	6,998.72
SE-2	109+01.421	575,171.484	1,115,505.584	575,409.260	1,115,966.735	7,018.55
SE-3	126+80.276	576,784.616	1,114,757.577	577,023.059	1,115,218.419	7,031.37
SE-4	164+77.763	580,228.335	1,113,160.772	580,468.202	1,113,620.953	7,036.35
SE-5	207+34.768	584,088.786	1,111,370.788	584,330.248	1,111,830.229	7,059.53
SE-6	227+42.301	585,909.317	1,110,526.676	586,151.532	1,110,985.769	7,074.72
SE-7	243+53.287	587,370.246	1,109,849.315	587,613.065	1,110,308.127	7,076.04
SE-8	255+33.735	588,440.742	1,109,352.984	588,684.004	1,109,811.592	7,076.56
SE-9	306+57.204	593,086.993	1,107,198.779	593,332.175	1,107,656.496	7,064.50
SE-10	350+35.042	597,057.071	1,105,358.097	597,303.894	1,105,815.053	7,057.69
SE-11	383+11.735	600,028.533	1,103,980.434	600,276.635	1,104,436.820	7,060.21
SE-12	429+26.090	604,213.198	1,102,040.414	604,462.980	1,102,495.998	7,058.22
SE-13	464+59.704	607,417.730	1,100,554.787	607,668.837	1,101,009.757	7,064.76
SE-14	501+65.371	610,778.287	1,098,996.815	611,030.783	1,099,451.141	7,071.85
SE-15	553+71.000	615,499.116	1,096,808.220	615,753.564	1,097,261.641	7,124.57
SE-16	578+80.681	617,775.076	1,095,753.090	618,030.465	1,096,206.075	7,117.62
SE-17	621+67.945	621,663.079	1,093,950.624	621,920.075	1,094,402.864	7,052.56
SW-17-1	651+77.999	624,392.822	1,092,685.143	624,650.947	1,093,136.860	
SE-18	695+63.591	627,463.131	1,091,261.780	627,722.525	1,091,712.909	6,898.67
SE-18-1	687+66.712	627,647.336	1,091,176.385	627,906.806	1,091,627.478	6,898.34
SE-18-2	690+29.052	627,885.245	1,091,066.093	628,144.814	1,091,517.140	6,883.64

NOTE: ALL MONUMENTS ARE BRASS CAPS SET IN CONCRETE.

VLA - PERMANENT SURVEY MONUMENTS

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VLA						
Centerpoint	0+00.000	565,285.634	1,120,089.552	565,519.323	1,120,552.598	6,964.52
SW-1	36+90.056	562,232.123	1,118,020.414	562,464.550	1,118,482.604	6,973.46
SW-2	80+52.444	558,622.245	1,115,574.297	558,853.180	1,116,035.476	6,979.80
SW-2-1	104+00.647	556,679.098	1,114,257.604	556,909.238	1,114,718.239	6,996.35
SW-3	142+78.433	553,470.215	1,112,083.237	553,699.020	1,112,542.973	7,041.65
SW-4	158+94.004	552,133.318	1,111,177.358	552,361.570	1,111,636.719	7,030.83
SW-5	204+46.780	548,365.845	1,108,624.557	548,592.540	1,109,082.863	6,980.19
SW-6	255+63.989	544,131.281	1,105,755.296	544,356.225	1,106,212.416	6,960.70
SW-7	291+70.157	541,147.115	1,103,733.303	541,370.826	1,104,189.587	6,972.92
SW-8	333+45.571	537,691.866	1,101,392.164	537,914.148	1,101,847.480	6,970.62
SW-9	367+23.566	534,896.481	1,099,498.161	535,117.603	1,099,952.694	6,967.76
SW-9-1	377+78.376	534,023.592	1,098,906.747	534,244.358	1,099,361.036	6,962.33
SW-10	423+98.658	530,200.164	1,096,316.238	530,419.346	1,096,769.456	6,953.38
SW-10-1	461+37.045	527,106.526	1,094,220.191	527,324.432	1,094,672.543	6,949.17
SW-11	502+52.351	523,700.982	1,091,912.807	523,917.480	1,092,364.205	6,945.44
SW-11-1	513+80.815	522,767.139	1,091,280.102	522,983.251	1,091,731.238	6,943.35
SW-12	563+58.752	518,647.725	1,088,489.079	518,862.134	1,088,939.061	6,936.70
SW-13	590+72.268	516,402.192	1,086,967.676	516,615.673	1,087,417.029	6,931.46
SW-13-1	602+27.724	515,446.000	1,086,319.854	515,659.086	1,086,768.940	6,929.46
SW-14	651+14.428	511,402.032	1,083,580.062	511,613.446	1,084,028.015	6,916.65
SW-15	687+29.204	508,410.630	1,081,553.409	508,620.807	1,082,000.524	6,906.28
SW-16	690+29.328	508,162.263	1,081,385.142	508,372.338	1,081,832.188	6,904.76
3694-810		565,214.247	1,117,414.298	565,447.906	1,117,876.238	7,000.50

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