VLA Test Memorandum 103

Calculation of Baseline Parameters from Survey Data

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Abstract: The procedure for calculating the components of baseline vectors from the geodetic positions and elevations of the observing stations is developed, with allowance for the ellipsoidal figure of the earth.

I. Introduction

When the tests in interferometer mode begin, we will need reasonably accurate initial estimates of the baseline constants. The ground coordinates (see VLA Test Memorandum 102) and elevation above mean sea level will be known for each station, by survey. The present memorandum shows how to convert these data into baseline constants. The VLA is large enough that the ellipsoidal shape of the earth must be taken into account.

Section II gives the definitions and relationships necessary for working with the ellipsoid. Section III states the numerical parameters of the adopted reference ellipsoid. Section IV gives the procedure to be used in computing baseline constants.

All lengths are expressed in meters.

II. The Ellipsoid

The material in this section is taken from Chapter II of Spherical and Practical Astronomy as Applied to Geodesy by I. I. Mueller (Frederick Ungar Publishing Co., 1969).

The reference ellipsoid is a figure of revolution which closely approximates the shape of a hypothetical surface which is everywhere at mean sea level. It is defined by its equatorial radius a and its polar radius b. The flattening of the ellipsoid is defined as

$$f = \frac{a - b}{a} .$$

The intersection of the ellipsoid with a plane which includes the polar axis is an ellipse whose eccentricity is

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{2f - f^2} .$$

The geodetic latitude ϕ of a point on the ellipsoid is the angle between the local vertical* and the equatorial plane. The geocentric latitude ϕ' is the angle between the equatorial plane and the line joining the point to the center of the ellipsoid. The geocentric and geodetic latitudes are related by

$$\tan \phi' = (1-f)^2 \tan \phi = (1-e^2) \tan \phi , \qquad (1)$$

The distance from the point to the center of the ellipsoid is the geocentric radius

$$\rho = a \left(\frac{1-e^2}{1-e^2 \cos^2 \phi'} \right)^{1/2} \qquad (2)$$

The radius of curvature of an arc on the surface of the ellipsoid depends on its direction and the latitude. In a north-south direction, one has the meridional radius of curvature

$$M = \frac{a (1-e^2)}{(1-e^2 \sin^2 \phi)^{3/2}} . \qquad (3)$$

In an east-west direction, one has the transverse radius of curvature

$$N = \frac{a}{(1-e^2 \sin^2 \phi)^{1/2}} . \qquad (4)$$

*This is the normal to the ellipsoid, strictly speaking. The vertical as actually measured with a plumb line will generally differ by a small amount due to mass inhomogeneities in the vicinity; latitude measured with respect to this vertical is the astronomical latitude.

At azimuth $\underline{\alpha}$, the radius of curvature is

$$R_\alpha = \left[\frac{\cos^2\alpha}{M} + \frac{\sin^2\alpha}{N}\right]^{-1} . \quad (5)$$

Now consider a cartesian reference frame with its origin at the center of the ellipsoid. Let the u-axis be positive toward the intersection of the meridian plane at longitude λ_o with the equatorial plane, and let the w-axis be positive northward along the axis of rotation of the ellipsoid. Finally, let the v-axis be positive westward. Then the (u,v,w) frame is parallel to the (x,y,z) frame used at NRAO to express the components of a baseline vector of an interferometer. The coordinates of a point on the surface of the ellipsoid are

$$\begin{aligned} u &= N \cos\phi \cos(\lambda - \lambda_o) \\ v &= N \cos\phi \sin(\lambda - \lambda_o) \\ w &= N (1-e^2) \sin\phi \end{aligned}$$

where the geodetic longitude λ is reckoned positive toward the west.

If a point lies at some distance h above mean sea level, its coordinates in the above frame are

$$\left.\begin{aligned} u &= (N+h) \cos\phi \cos(\lambda - \lambda_o) \\ v &= (N+h) \cos\phi \sin(\lambda - \lambda_o) \\ w &= [N(1-e^2)+h] \sin\phi \end{aligned}\right\} \quad (6)$$

This set of equations will be used for calculating the baseline constants (Section IV).

III. The Parameters of the Ellipsoid

We adopt the reference ellipsoid recommended by the IAU (see the American Ephemeris and Nautical Almanac for 1968, p. ix). The defining parameters are

$$\begin{aligned} a &= 6378160 \text{ meters} \\ 1/f &= 298.25 . \end{aligned}$$

We shall need the following derived parameters:

$$e^2 = 0.0066945419,$$
$$1-e^2 = 0.9933054581.$$

IV. Calculation of the Baseline Constants

The position of an observing station will be expressed by a vector $\underset{\sim}{L}$ which runs from the intersection of the wye arms to the station. Its components are

$$\left.\begin{aligned} L_x &= u - u_o \\ L_y &= v - v_o \\ L_z &= w - w_o \end{aligned}\right\} \qquad (7)$$

where (u_o, v_o, w_o) refers to the wye intersection. We have for this point

$$\phi_o = 34° \; 04' \; 43''.497$$
$$\lambda_o = 107° \; 37' \; 03''.819$$
$$h_o = 2122.786 \text{ m}$$

These correspond to

$$u_o = 5\;290\;146.195 \text{ m}$$
$$v_o = 0$$
$$w_o = 3\;554\;886.722 \text{ m} .$$

Therefore the vector position of an observing station at position (ϕ,λ,h) is given by

$$\left.\begin{aligned} L_x &= (N+h) \cos\phi \cos\Delta\lambda - 5\;290\;146.195 \\ L_y &= (N+h) \cos\phi \sin\Delta\lambda \\ L_z &= [N(1-e^2)+h] \sin\phi - 3\;554\;886.722 \end{aligned}\right\} \qquad (8)$$

where

$$\Delta\lambda = \lambda - 107° \; 37' \; 03''.819 .$$

It is important to note that N must be evaluated separately for each observing station--it varies much too rapidly with position to be treated as a site constant.

The baseline vector between two observing stations is simply

$$\underset{\sim}{B} = (\underset{\sim}{L}_2 - \underset{\sim}{L}_1)\,\lambda^{-1}$$

where λ is now the observing wavelength (and not some longitude!). The corresponding baseline constants are

$$\left.\begin{aligned} B_x &= (L_{x,2} - L_{x,1})\,\lambda^{-1} \\ B_y &= (L_{y,2} - L_{y,1})\,\lambda^{-1} \\ B_z &= (L_{z,2} - L_{z,1})\,\lambda^{-1} \end{aligned}\right\} \qquad (9)$$