VLA Test Memorandum 104

Tolerances for the Intersection of the Azimuth, Elevation, and Collimation Axes of the VLA Antennas

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1 April 1974

Abstract: The effects of imperfect coincidence of the azimuth, elevation, and collimation axes on fringe phase are evaluated. It is shown that the azimuth and elevation axes should be made to intersect as closely as practicable, while the placement of the collimation axis is not critical.

I. Phase Error Due to Imperfect Axis Intersection

Figure 1 shows the basis of the problem. The azimuth, elevation, and collimation* axes of an antenna ideally should intersect at a common point P. In practice, however, the intersection is unlikely to be perfect, owing to the finite errors of fabrication and assembly. As a result, when two nominally similar antennas are used together as an interferometer, the length and orientation of the baseline will vary somewhat with the azimuth and elevation toward which they are pointed. This in turn introduces a position-dependent shift in fringe phase.

The baseline vector $\vec{B}$ between two antennas 1 and 2 is nominally the line joining the points P_1 and P_2. The actual baseline vector $\vec{B}'$, however, is the line joining P_1' and P_2' (see Fig. 1), which varies in length and orientation with changes in the pointing elevation h and the pointing azimuth z. We have

$$\vec{B}' = \vec{B} + (\vec{D}_2 - \vec{D}_1) \qquad \ldots(1)$$

where $\vec{D}$ is the vectorial separation of P and P'. If the interferometer is pointed in the direction $\vec{S}$, and receives radiation from a point source which

*The collimation axis is the axis of symmetry of the primary reflector. It points toward the object being observed.

is displaced from this direction by $\Delta\vec{S}$, the fringe phase component due to $\vec{D}_1$ and $\vec{D}_2$ is

$$\Delta\Phi = (\vec{D}_2 - \vec{D}_1) \cdot (\vec{S} + \Delta\vec{S}) \text{ turns} , \qquad \ldots(2)$$

How does $\vec{D}$ depend on the errors of axis intersection? It can be seen from Fig. 1 that the displacement of P' from P can be represented by three constants, which we assume to be expressed in units of the observing wavelength:

a = lateral offset between the azimuth and elevation axes, reckoned positive toward the pointing azimuth Z;

b = collimation axis offset component perpendicular to the elevation axis, reckoned positive if it is in the direction Z ± 180° when h = 90°; and

c = collimation axis offset component parallel to the declination axis, reckoned positive in the direction Z - 90°,

a and b are both perpendicular to c, but they are perpendicular to each other only when h = 0. The north, east, and zenith components are denoted respectively by D_N, D_E, and D_V.

It then follows from Fig. 1 that

$$\vec{D} = \begin{bmatrix} D_N \\ D_E \\ D_V \end{bmatrix} = \begin{bmatrix} (a - b \sin h) \cos Z + c \sin Z \\ -c \cos Z + (a - b \sin h) \sin Z \\ b \cos h \end{bmatrix} \qquad \ldots(3)$$

In the same alt-az frame,

$$\vec{S} + \Delta\vec{S} = \begin{bmatrix} \cos h \cos Z - \sin h \cos Z \,\Delta h - \cos h \sin Z \,\Delta Z \\ \cos h \sin Z - \sin h \sin Z \,\Delta h + \cos h \cos Z \,\Delta Z \\ \sin h + \cos h \,\Delta h \end{bmatrix} \qquad \ldots(4)$$

where the displacement of the source from $\vec{S}$ is given by the elevation offset Δh and the azimuth offset ΔZ (both expressed in radians). Now define

$$\alpha = a_2 - a_1 ,$$
$$\beta = b_2 - b_1 ,$$
$$\gamma = c_2 = c_1 .$$

Then

$$\vec{D}_2 - \vec{D}_1 = \begin{bmatrix} (\alpha-\beta \sin h) \cos Z + \gamma \sin Z \\ -\gamma \cos Z + (\alpha-\beta \sin h) \sin Z \\ \beta \cos h \end{bmatrix} . \qquad \ldots(5)$$

The scalar product of (4) and (5) reduces to

$$\boxed{\Delta\Phi = \alpha \cos h - \Delta h (\alpha \sin h - \beta) - \gamma\Delta Z \cdot \cos h} . \qquad \ldots(6)$$

It can seen from this result that the first term, which depends on the minimum distances between the azimuth and elevation axes of the two antennas, is strongly dominant. The collimation axis offset parameters β and γ appear only in products with Δh and ΔZ, which are small quantities since are concerned with a source within the primary beam.

II. Offset between Azimuth and Elevation Axes

The phase effect due the offset between the azimuth and elevation axes alone is

$$\Delta\Phi' = \alpha (\cos h - \Delta h \sin h) .$$

Let the linear equivalent of α be $A = \lambda\alpha$. Δh cannot much exceed the HPBW of the primary beam, which is approximately λd^{-1} (where d is the diameter of the antennas). Therefore

$$|\Delta\Phi' - \alpha \cos h| \lesssim A\, d^{-1} \sin h .$$

Even if A were as large as a centimeter, the right-hand side of this inequality could not exceed 4×10^{-4} turns, or $0\overset{\circ}{.}14$, for $d = 25$ meters. Hence it is entirely sufficient to take

$$\Delta\Phi' = \alpha \cos h \text{ .}$$

$|\Delta\Phi'|$ should be kept below 0.25 (i.e., 90°) if it is not to introduce a potentially messy calibration problem. This requires $\alpha < \lambda/4$, or $a < \lambda/8$ for the individual antennas. Since the shortest VLA wavelength is 1.3 cm, this requires that the azimuth and elevation axes intersect within

$$0.163 \text{ cm} = 0.064 \text{ inch .}$$

E-Systems has agreed to a tolerance of ±0.030 inch in setting up the VLA antennas. Thus in practice we can expect that

$$|\Delta\Phi'| \le 0.117 \cos h \text{ (turns)} = 42° \cos h \text{ .}$$

We can live with this, but the measurement of α clearly will have to be a regular part of the VLA calibration procedure, and the on-line reduction programs will have to correct for it.

It obviously is not practical to try to make $\Delta\Phi'$ negligible by adjusting the antennas. To ensure that $|\Delta\Phi'| < 1°$ at λ1.3 cm, for example, the axes would have to intersect to within 0.0007 inch--an accuracy which would be very difficult to achieve and preserve.

III. Collimation Axis Offset

The phase effect due solely to the offset of the collimation axis is

$$\Delta\Phi'' = -\beta\, \Delta h \sin h - \gamma \Delta Z \cos h \text{ .}$$

E-Systems will keep within a tolerance of ±0.250 inch in setting the linear equivalents of <u>b</u> and <u>c</u>. Hence

$$\left.\begin{array}{c} |\lambda\beta| \\ |\lambda\gamma| \end{array}\right\} \le 0.5 \text{ inch} = 1.27 \text{ cm}$$

or, nearly enough,

$$\left.\begin{array}{c} |\beta| \\ |\gamma| \end{array}\right\} \le 1 \quad \text{for } \lambda = 1.3 \text{ cm .}$$

The greatest possible values of $|\Delta h|$ and $|\Delta Z \cos h|$ are approximately the HPBW of the primary beam, or 5×10^{-4} radians at $\lambda = 1.3$ cm. Therefore

$$|\Delta\Phi''| \leq 5 \times 10^{-4} \; (1 + \sin h) \quad \text{turns} \; .$$

The worst case occurs near the zenith, where

$$|\Delta\Phi''| \leq 10^{-3} \text{ turns} = 0.^{\circ}4 \; .$$

Thus the phase effect of the collimation offset should be negligible at all times.

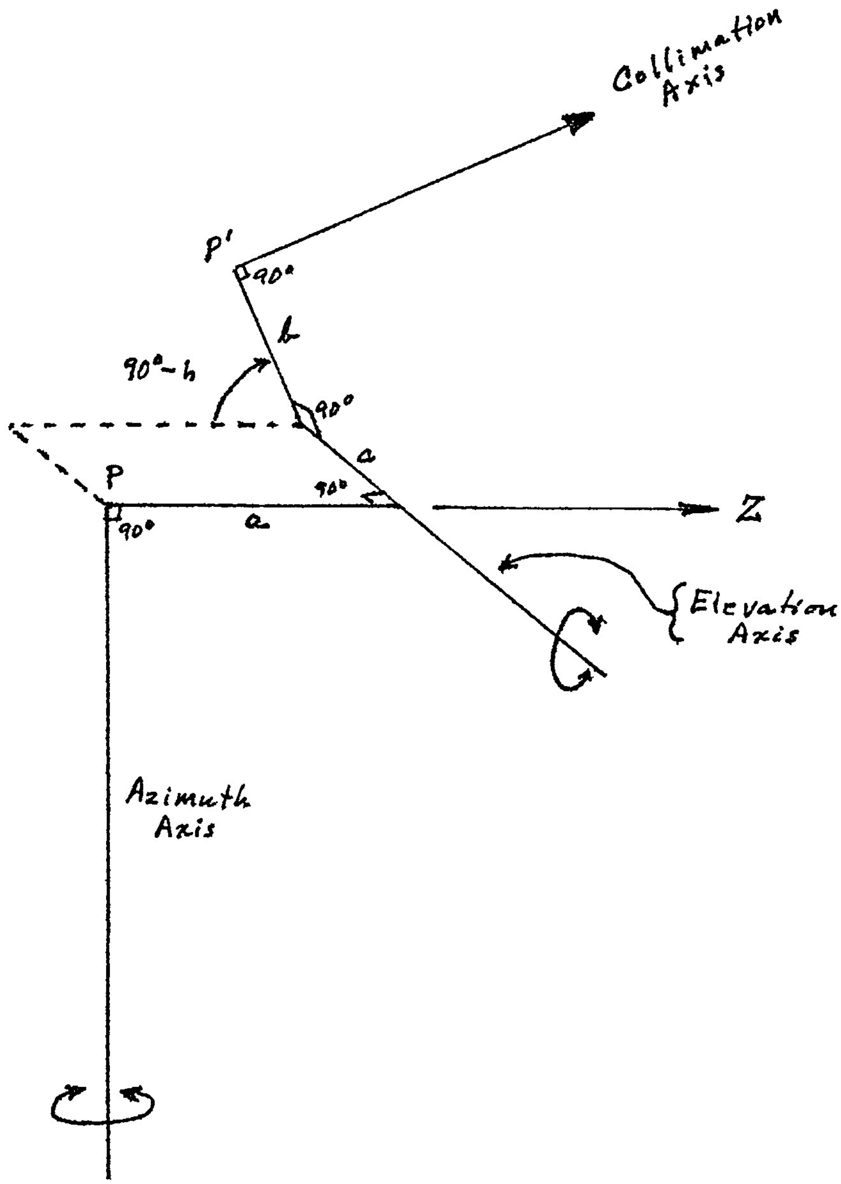
Collimation Axis
P'
90°
b
90°-h
90°
a
P
90°
90°
a
Z
Elevation Axis
Azimuth Axis

Figure 1