NATIONAL RADIO ASTRONOMY OBSERVATORY

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PHASE AND AMPLITUDE CLOSURE

C. Bignell and N. Vandenberg

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1.0 INTRODUCTION

The VLA software, particularly on the MODCOMP computer systems, has been developed under the assumption that a complex gain and complex cross-polarization can be assigned to each IF chain such that the effective gain and cross-polarization for each correlator can be calculated from the correlator outputs and the complex gain and polarization assigned to the appropriate IF chains. This assumption will break down if the different IF's possess different bandpasses. A set of closure conditions can be developed from the above assumption and the violation of these conditions is a test of that assumption.

The phase closure condition is well known. Assume three parallel channels have complex gains of g_1, g_2, g_3 respectively. The complex correlator outputs for a source of flux density s are; $V_{12} = g_1 g_2^* s$, and $V_{13} = g_1 g_3^* s$, and $V_{14} = g_2 g_3^* s$. A closure parameter, Vclosure, can be defined to be:

$$\begin{aligned} \text{Vclosure}_{13} &= \log V_{12} + \log V_{23} - \log V_{13} \\ &= \log s + \log g_2^* + \log g_2 \end{aligned} \tag{1}$$

Vclosure is real and has zero phase. If ϕ_{ij} is the phase of the parallel hand correlator output of telescopes i and j, then equation (1) reduces to:

$$\phi_c = \phi_{12} + \phi_{23} - \phi_{13} \tag{2}$$

which is zero for phase closure.

The condition for amplitude closure can be determined from observations obtained with four antennas. We have:

$$\begin{aligned} \text{Vclosure}_{14} &= \log V_{12} + \log V_{34} - \log V_{13} - \log V_{24} \\ &= 0 \end{aligned} \tag{3}$$

If A_{ij} is the amplitude of the parallel hand correlator output of telescopes i and j, then equation (3) reduces to:

$$A_c = A_{12}\ A_{34}\ /\ (A_{13}\ A_{24}) \tag{4}$$

$A_c = 1$ for amplitude closure.

3C84 was observed at C-Band with 5 minute integrations at 50, 24, 12, 4, 1·5, and 0·5 MHz bandwidths.

2.0 PHASE CLOSURE

Equation (2) was used to calculate ϕ_c for each of the filters, channels and different baselines. The results are listed in Table 1. The worst bandpasses are the 50 MHz and 1·5 MHz ones; they are closed only to within 3 degrees. The closure on Channel A is worse than on Channel C.

3.0 AMPLITUDE CLOSURE

Equation (4) was used to calculate A_c for each of the bandpasses and channels. The results are listed in Table 2. The 50 MHz and 12 MHz bandpasses show the worst closure in that the amplitudes are off by as much as 5 percent. Again, Channel A shows worse closure than Channel C.

4.0 PHASE OFFSETS OF DIFFERENT FILTERS

It is possible to calculate phase offsets for each filter relative to one filter (in this case, the 50 MHz) and relative to one antenna (Antenna 1 was chosen). The values calculated $\{\phi(BW)-\phi(50\ MHz)\}$ from the parallel hand observations of 3C84 are listed in Table 3 and phase offsets calculated from the cross hands of 3C84 are given in Table 4. In principle, these phases could be used to correct the visibility phases when a change of bandwidths is made such that the phases are all in the same scale.

5.0 CORRELATION COEFFICIENTS

The correlation coefficients of the different IF's and baselines are another indicator of the different shapes of the various bandpasses. Table 5 lists the correlation coefficients for each of the antenna pairs and each of the IF pairs. These have been normalized by division of the appropriate 50 MHz IF coefficients. It is interesting to note that the normalized correlation coefficients at 1.5 MHz bandwidth for some of the baselines (notably 1A2A, 2A3A and 3A4A) are very low at approximately 33 percent of the other coefficients. These numbers indicate that there is possibly some hardware problem with the IF of Channel A on telescope 3.

Table 1

Phase Closure ϕ_c

Bandwidth (MHz)	Channels	Telescope Combination			
		1,2,3	1,2,4	1,3,4	2,3,4
50	AA	-3.0	-1.3	-1.0	-1.3
	CC	+0.7	+1.3	-0.3	-1.0
24	AA	0	-0.3	-0.3	0
	CC	-0.3	-0.8	-0.7	0
12	AA	-0.3	+0.3	+0.3	-0.3
	CC	0	-1.0	-0.7	+0.3
4	AA	+1.0	+1.0	0	0
	CC	0	-1.0	-0.7	0
1.5	AA	+1.3	+1.0	+2.5	+3.0
	CC	+0.5	-0.5	-1.5	-1.0
.5	AA	+0.5	0	-0.5	0
	CC	-1.0	-0.3	-0.5	-1.0

Table 2

Amplitude Closure A_c

Bandwidth (MHz)	Channels	$A_{12}A_{34}/(A_{13}A_{24})$
50	AA	1.046
	CC	.995
24	AA	1.003
	CC	1.001
12	AA	1.046
	CC	1.017
4	AA	1.012
	CC	1.003
1.5	AA	.998
	CC	1.017
.5	AA	.987
	CC	1.029

Table 3

Phase Offsets From Parallel Hands

Telescopes / Bandwidth (MHz)	2	3	4	2	3	4
	Channel A Relative to Channel A			Channel C Relative to Channel C		
24	3.7	0.7	-4.7	10.3	9	12.3
12	11.7	5.0	4.0	-9.0	-4.3	-2.4
4	18.2	1.7	-7.7	11.3	17.5	8.3
1.5	7.7	1.7	-6.2	1.7	2.5	1.3
0.5	15.2	0.7	-14.7	22.3	6	11.3

Table 4

Phase Offsets From Cross Hands

Telescopes / Bandwidth (MHz)	2	3	4	2	3	4
	Channel A Relative to Channel C			Channel C Relative to Channel A		
24	13	10	-13.7	-6	-17.3	18.3
12	18	15.7	-10.7	47.7	35.4	1.3
4	30	33	-28.7	-55	-51.8	46.8
1.5	18.5	12.5	10.2	67	46.7	-20.7
0.5	36.5	17.5	-27.7	-6	-50.3	60.8

Table 5

Correlation Coefficients

Bandwidths (MHz) \ Antenna Pairs	A Channels						C Channels					
	1,2	1,3	1,4	2,3	2,4	3,4	1,2	1,3	1,4	2,3	2,4	3,4
24	1.028	1.016	1.006	1.056	1.056	1.067	1.074	1.035	1.027	1.064	1.057	1.026
12	1.113	1.071	1.065	0.962	1.058	1.087	0.987	0.959	0.993	1.011	1.026	1.022
4	1.040	1.011	0.985	1.078	1.062	1.069	1.099	1.060	1.043	1.078	1.063	1.022
1.5	1.119	0.350	1.091	0.337	1.061	0.325	0.983	0.966	0.983	1.015	1.032	1.037
0.5	1.041	1.001	0.996	1.076	1.083	1.081	1.094	1.057	1.026	1.066	1.039	1.037