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ATMOSPHERIC PHASE STABILITY AT THE VLA

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ABSTRACT

A series of 109 phase stability measurements were made at the Very Large Array (VLA) radio telescope during 1984 and 1985; the structure function of phase was calculated for each observing session. The power law index of this structure function was found to have a median value of $\beta = 0.68$. This agrees well with the value of 2/3 expected from Kolmogorov turbulence in the case where the antenna separation exceeds the scale height of the turbulent region. Although the power law index ranged from $\beta = 0$ to 1.8 during the various measurements, the median showed little variation with respect to time of day or season and did not depend on the strength of the turbulence.

The rms value of the phase variation at one kilometer antenna separation, expressed as excess electrical path length, was used to characterize each days measurement. On winter nights the rms was at its lowest with a median value of 0.62 mm and at its highest on summer days with a median of 1.63 mm. Summer nights and winter days had about equal phase stability with a median of 0.90 and 0.92 mm respectively.

INTRODUCTION

Much of the current research in radio astronomy, as in optical astronomy, involves forming images of cosmic objects. In both wavelength ranges, the images are usually formed after the wavefront has passed through the atmosphere. The wavefront is distorted by the varying index of refraction in the turbulent atmosphere; this gives a random component to the phase across the telescope aperture and therefore the image is blurred.

This problem is widely discussed in the context of optical imaging in astronomy under the subject of astronomical "seeing" (Young (1974), Woolf

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(1982), Coulman (1985) and references therein). However, there has been relatively little done, either theoretically or observationally, on the problem of seeing at radio wavelengths (Hinder and Ryle, 1971, Hamaker, 1978, Hargrave and Shaw, 1978, Armstrong and Sramek, 1982). This paper will present data on the atmospheric phase errors observed at the VLA over a two year period. This data is presented more fully in Sramek (1990).

The process of imaging the radio sky with an interferometer array is discussed in detail by Thompson, Moran, and Swenson (1986), and by Perley, Schwab, and Bridle (1989). For a specific and more detailed description of two modern astronomical synthesis arrays see Napier et al. (1983) for the VLA and Hogbom and Brouw (1974) for the WSRT.

THEORY OF ATMOSPHERIC SEEING

The problem of electromagnetic wave propagation through a turbulent medium has been treated in depth by many authors. See for example Tatarskii (1961, 1971), and Ishimaru (1978). Although much of the discussion is aimed at optical wavelengths, the theory is directly translated to radio wavelengths with two significant changes. First, the size of the apertures used in radio astronomy are much larger. While at optical wavelengths the apertures may be several meters in diameter, at radio wavelengths the apertures may be several kilometers or even several thousand kilometers in size (see e.g. Kellermann and Thompson (1985) for a discussion of VLBI). The aperture size can exceed the thickness of the atmosphere and also exceed the outer scale the of turbulence. Both of these effects will modify the spectral density of the fluctuations of phase.

The second difference is that at optical wavelengths the index of refraction variations are largely due to density fluctuations in oxygen and nitrogen in the dry air. At radio wavelengths changes in the index of refraction are dominated by fluctuations in the water vapor density. At longer radio wavelengths (>10 cm), fluctuations in the electron density in the ionosphere can also become important.

A radio interferometer array provides a very useful tool for measuring the characteristics of the turbulent atmosphere. When the array observes an unresolved radio source, for each pair of antennas, the phase of the measured complex visibility is simply the difference of the radio path length of the two rays reaching the antennas. If the antennas are separated by distance ρ, the observed interferometer phase is $\Delta\phi = \phi(x) - \phi(x+\rho)$. A useful statistical description of the phase fluctuations is provided by the spatial structure function $D_\phi(\rho) = \langle [\phi(x) - \phi(x+\rho)]^2 \rangle$, where $\langle\rangle$ indicates an ensemble average. An approximate measure of this structure function can be obtained from the temporal variance of the interferometer phase measured at a variety of antenna spacings.

If we assume that the variations in the index of refraction in the atmosphere are described by a Kolmogorov spectrum, the expected structure function of phase is a power law of form $D_\phi(\rho) \propto \rho^\beta$ (Tatarskii, 1961). When the antenna separation is less than the thickness of the turbulent region, L, and we have 3-dimensional isotropic turbulence, than

$$D_\phi(\rho) \propto \rho^{5/3} \qquad \text{for } \rho < L.$$

When the antenna separation exceeds the thickness, we expect the structure function to flatten (Stotskii, 1973, Dravskikh and Finkelstein, 1979),

$$D_\phi(\rho) \propto \rho^{2/3} \qquad \text{for } \rho > L.$$

Treuhaft and Lanyi (1987) arrive at the same result and illustrate the transition using a numerical integration. When the antenna separation exceeds the outer scale of the turbulence, L_0, then increasing ρ does not increase the phase fluctuations and $D_\phi(\rho)$ is constant.

THE OBSERVATIONS OF PHASE VARIATIONS

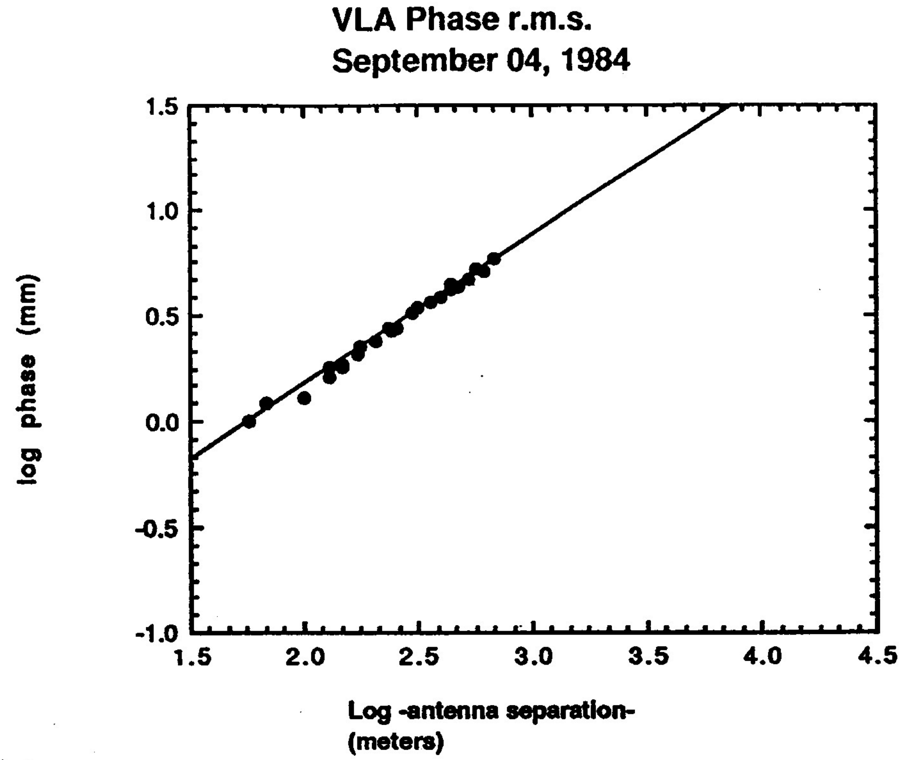
The data on phase stability was taken at the NRAO Very Large Array (VLA) located in central New Mexico at an altitude of 7000 ft. The array consists of 27 antennas each 25 m in diameter. All are moveable and can be positioned on a Y shaped array; the array usually occupies one of four standard configurations with a maximum antenna spacing of 36, 11, 3, and 1.1 km. In any configuration there is a forty to one range in antenna spacings. Some of the data were taken during reconfigurations of the array which gave a wider range of antenna spacings.

OBSERVING PROCEDURE

The phase stability observations were made by continuously observing an unresolved radio source for a few hours as the turbulent atmosphere drifted across the array. One hundred and nine such observing runs were made at random times between December 9, 1983 and December 30, 1985.

The sources observed were north of declination +39 degrees and observed near transit; during the two hours of tracking, there was little change in source elevation. The elevation for the observations was above 60°. The observing frequency was either 15.0 GHz or 4.9 GHz. One antenna was arbitrarily chosen as the reference antenna and the phase difference with respect to that antenna was obtained for 26 interferometer pairs. The complex visibilities were vector averaged for 50 seconds before going into a database for further analysis.

1(a)



1(b)

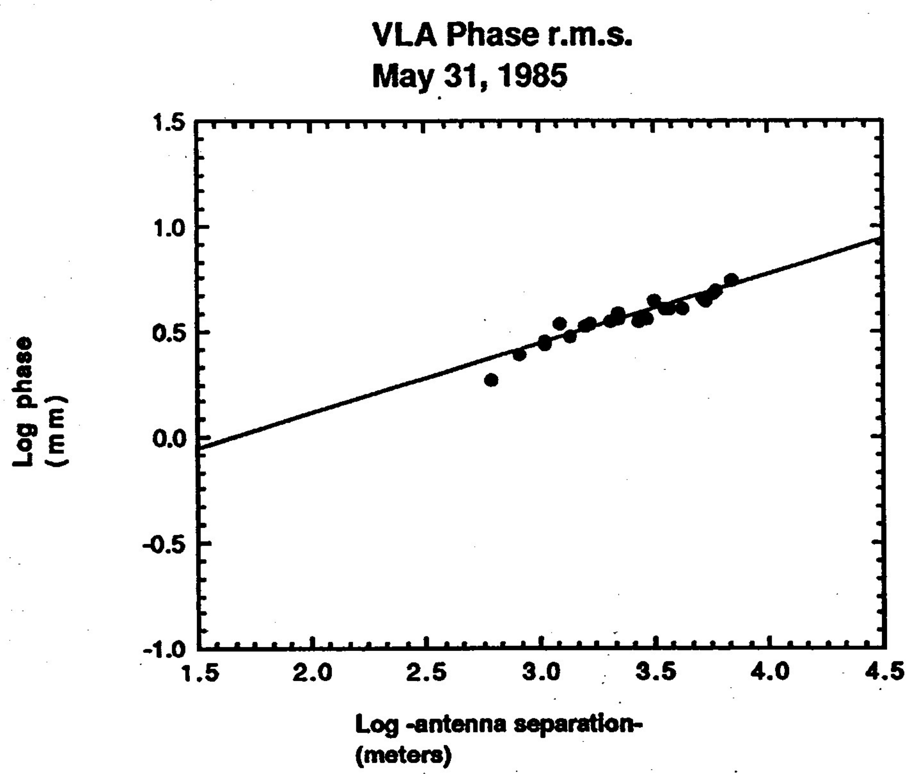


Fig. 1 Observed values of phase r.m.s., $\sqrt{D'_\phi(\rho)}$, in mm of excess path length vrs. antenna separation in meters. (a) a very turbulent day with S_{1km} = 7.53 mm, and a slope of $\beta/2$ = 0.72. (b) S_{1km} = 2.70 mm and $\beta/2$ = 0.34

All of the atmospheric phase fluctuations observed here are probably due to the none dispersive neutral atmosphere; the contribution from the ionosphere is assumed to be negligible. During 1984 and 1985, near the minimum of solar activity, this is a reasonable assumption.

THE STRUCTURE FUNCTION ANALYSIS

Before calculating the structure function, the mean value was removed from each of the 26 time series of interferometer phases, $\Delta\phi(\rho)$. An estimate of $D_\phi(\rho)$ is given by

$$D'_\phi(\rho) = \lambda^2\, 1/n\, \Sigma\, [\phi(x) - \phi(x+\rho)]^2 = \lambda^2\, 1/n\, \Sigma\, [\Delta\phi(\rho)]^2,$$

where λ is the observing wavelength in millimeters, and ϕ is expressed in cycles of phase (or lobes). The sum is over the couple hundred 50 second data points from each interferometer pair. The calculated values of $\sqrt{D'_\phi(\rho)}$ were then fitted by a power law in ρ

$$S(\rho) = \sqrt{D'_\phi(\rho)} = S_{1km}\, \rho^{\beta/2}$$

where ρ is measured in kilometers and S_{1km} is the one kilometer intercept, measured in mm of excess path length. Figures 1 shows the data and the best fit curves for two observing sessions.

Over the two year period, the observations were made in all four array configurations with a wide range of antenna separations. However all configurations have the one kilometer spacing, so S_{1km} offers a consistent measure of the phase stability of the array. The slope β and the intercept S_{1km} are used to parameterize the atmospheric phase stability of each observing session.

Since $D_\phi(\rho)$ has the form of a power law, the calculated value of $D'_\phi(\rho)$ will depend very much on how the lowest frequency components of the time series are treated. First, the observation must last long enough to sample the the largest relevant components of the turbulent spectrum, those with length ρ or L_o, whichever is smaller. Assuming that the turbulence is frozen-in and moves with speed v, the observation should last several time ρ/v. Our two hour observation meet this criteria for $\rho = 10^4$ m and v = 10 m/s.

Second, the removal of a mean, a slope, or a higher order polynomial in time must be done consistently with all the data. This is especially important when comparing different observing sites using different databases. Here we have only removed the mean. Although residual baseline or source position errors may thus contribute to the phase variance, removing higher order polynomials would risk artificially lowering the measured turbulence.

A single power law slope fits the data of any one day quite well. No one single observing session shows the expected change in slope, β, from 5/3 to 2/3 to 0. This is probably due to the small range in ρ available in any one observing session. It is expected that there will be a slow transition between these regimes extending for at least three decades in ρ (Treuhaft and Lanyi, 1987). Also, the atmosphere is probably more complicated than a model with a single outer scale of turbulence or a single scale height. An integration through several turbulent regions will also tend to produce a slow rollover of the structure function as ρ increases.

Even on the best days for the atmosphere, there should be a minimum $D'_\phi(\rho)$ due to instrumental phase instabilities. Since the instrumental term is likely to be nondispersive like the atmosphere and may vary with baseline and time, it is difficult to separate from atmospheric effects. Based on the minimum observed values of D'_ϕ , the sum of instrumental, quantization, and residual baseline errors is probably about 0.2 millimeter.

STATISTICS OF SEEING

The distribution of the values of S_{1km} and $\beta/2$ are shown in Figures 2 and 3. The data are grouped into night/day and summer/winter observations. Night time observations refer to the time period from 1800 to 0600 MST. Observations made April through September are labeled summer, and those made October through March are called winter observations. The median values of these quantities are given in Table 1. The value of the ten kilometer intercept, S_{10km} , was also calculated from S_{1km} and $\beta/2$ for each observing session; its median is also shown in Table 1. This calculation was only done for observing sessions with antenna separations greater than five kilometers.

Table 1 Summary of VLA phase stability statistics

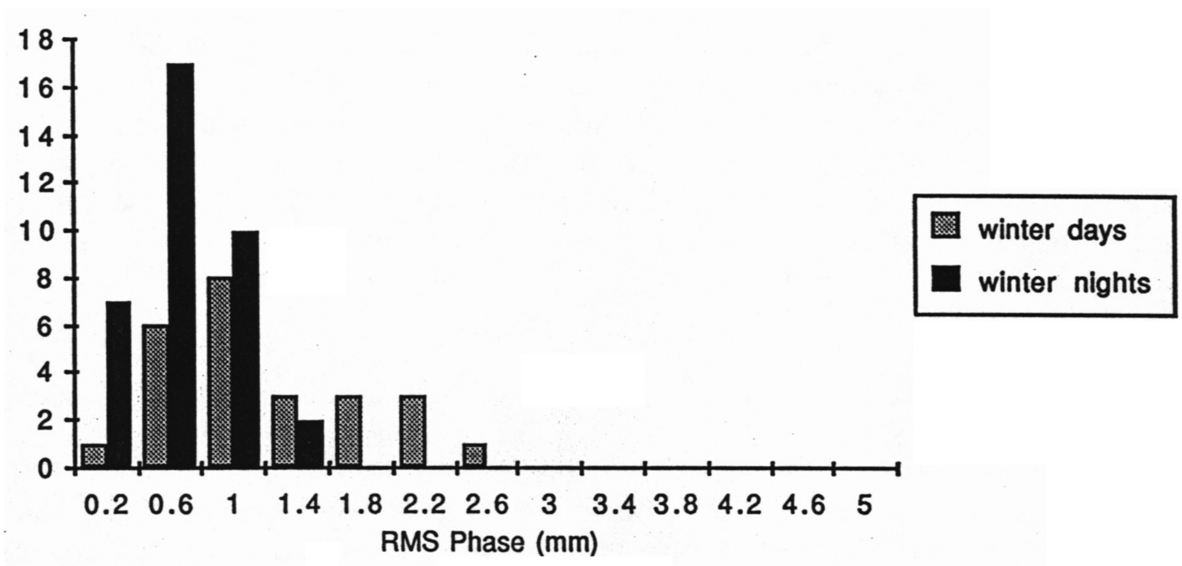
Median values of excess path length in mm and the power law slope of the root variance of phase.

	Summer		Winter	
	Day	Night	Day	Night
S_{1km} (mm)	1.63	0.90	0.92	0.62
S_{10km} (mm)	3.40	1.75	1.82	1.22
slope ($\beta/2$)	0.42	0.37	0.30	0.29

Both Table 1 and Figure 2 show clearly that winter nights offer the best phase stability; summer nights and winter days are not as good and have similar stability. Summer days have the worst phase stability. There is about a 3 to 1 ratio for the median phase stability of summer days to winter nights.

The power law slope ($\beta/2$) shows little change throughout the year, with a median value of 0.34. Figure 4 shows the power law slope ($\beta/2$) plotted against the phase rms, S_{1km}. There is no indication that the slope is a function of the strength of the turbulence.

RMS Phase at I km



RMS Phase at I km

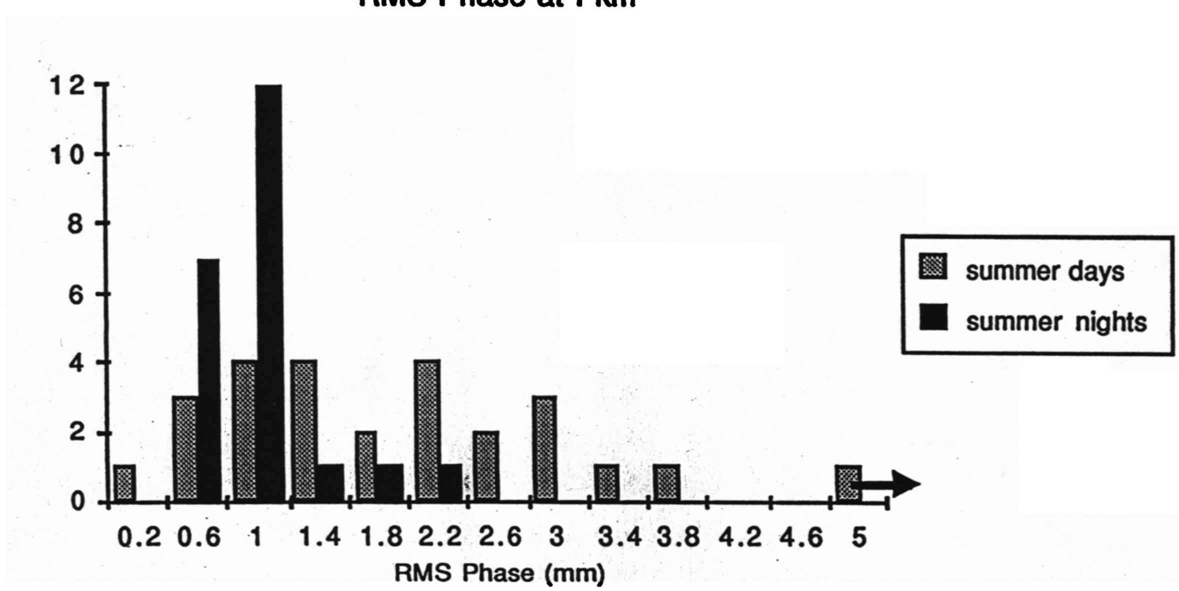
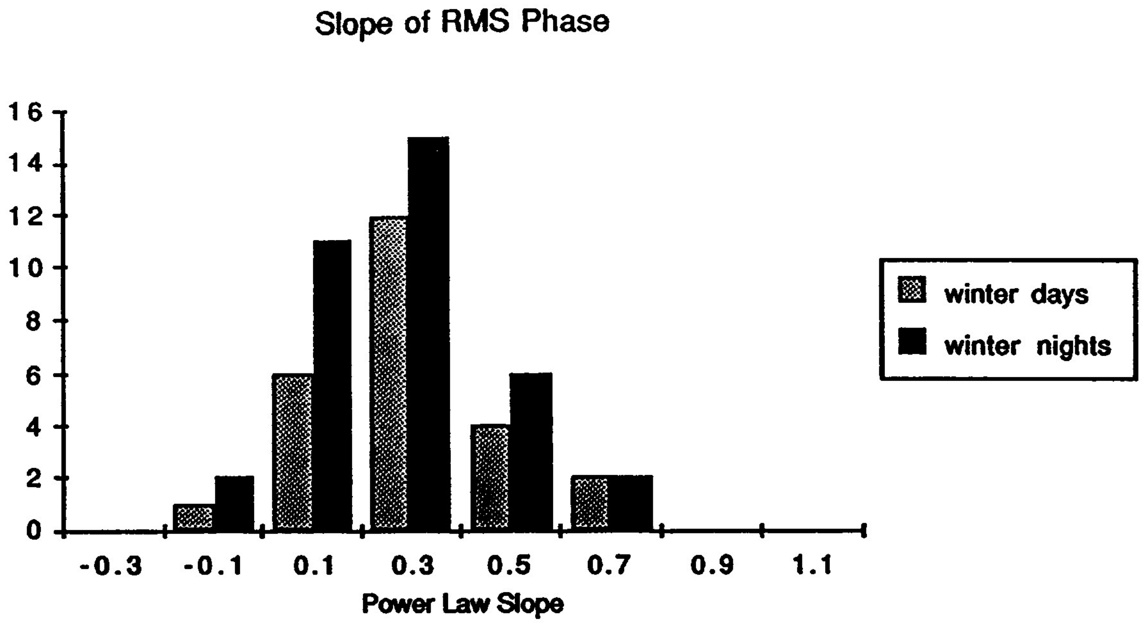


Fig 2. The distribution of the phase r.m.s expressed as mm of excess path length at one kilometer antenna spacing, S_{1km}, for 109 observing sessions.



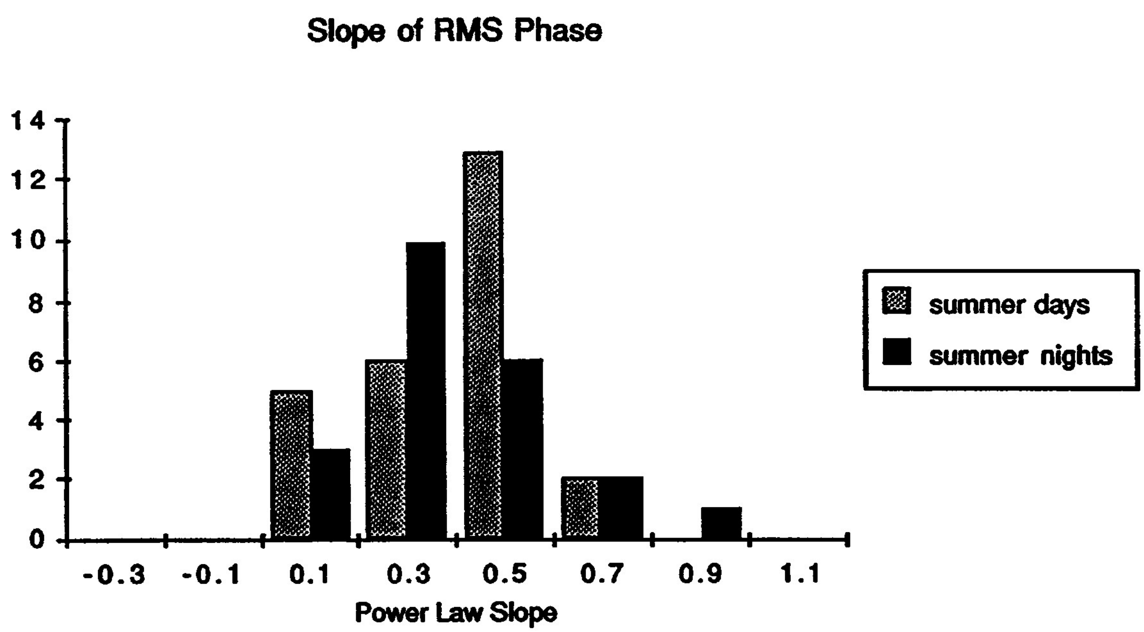


Fig. 3 The distribution of $\beta/2$, the slope of the phase r.m.s. Isotropic, three-dimensional turbulence would give a slope of $\beta/2 = 5/6$. The results here are in agreement with non-isotropic turbulence with $\beta/2 = 1/3$.

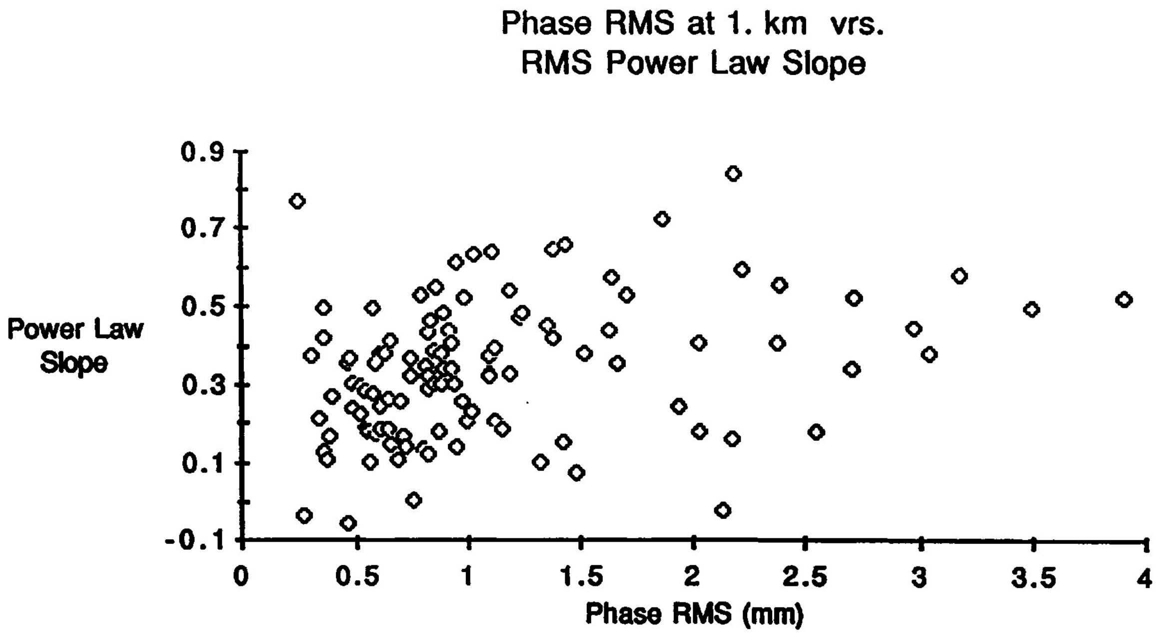


Fig. 4 The power law slope, $\beta/2$, vrs. the r.m.s. phase, S_{1km}. The slope of the structure function does not depend on the magnitude of the turbulence.

The intrinsic instrumental phase variations of the array are small, but they still introduce a bias which tends to lower the structure function slope. This is most severe for those observing sessions with $\rho_{max} < 1$ km, and $S_{1km} < 1$ mm. There were 23 observing sessions with $\rho_{max} < 1$ km. These had a median slope of $\beta/2 = 0.37$. If we exclude the 15 sessions with $S_{1km} < 1$ mm, the median increases to 0.59. If we do the same test for the 75 sessions with $\rho_{max} > 2.5$ km, the median $\beta/2$ only moves from 0.33 to 0.36. This bias in the small ρ, small S_{1km} data does not significantly alter the medians in Table 1, but does indicate that the structure function is steeper for $\rho < 1$ km.

CONCLUSION

The median phase stabilities in Table 1 show that winter nights are about three times better than summer days. However statistically the "seeing" obtained during summer nights is as good as that during winter days.

In general, the root variance phase errors increase as $\rho^{0.34}$. This is in good agreement with the prediction for Kolmogorov turbulence with the antenna separation exceeding the thickness of the turbulent region. When we restrict $\rho_{max} < 1$ km, and reduce the sample size to decrease the instrumental bias on the slope, we find that $\beta/2$ steepens to 0.59. This is also as expected from the Kolmogorov theory.

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