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VLA TEST MEMO 210

BENEFITS OF USING FULL COMPLEX CORRELATIONS

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ABSTRACT

We have used recently developed online software to measure full complex correlations for 50 MHz continuum observations on the whole array. This gives about 8% improvement in the signal to noise ratio over what is achieved using normal correlation measurements. Another benefit, and perhaps more significant one, is that when complex correlation measurements are used the average closure errors are considerably smaller than what is achieved using the normal correlation measurements. With phasing the array the phase closure errors increase somewhat for normal correlations (over what is observed for not phasing the array) but for the complex correlations they reduce to very low values. Quadrature phase errors, in the quadrature networks used in the samplers, explain this behaviour.

INTRODUCTION

The VLA correlator has twice the number of multipliers than are necessary for continuum observations. This allows the measurement of full complex correlations. Advantage of this is that the sampling theorem for the complex signals requires that they must be sampled only at the bandwidth and not at twice the bandwidth rate. Thus with 50 MHz signals if you have complex correlation measurements using sampling at 100 MSamples/sec, then you get advantage in the signal to noise ratio (SNR) of twice over sampling. This improves the SNR by about 8% over what is achieved by using a 50 MHz bandwidth signal in 100 MSamples/sec sampler and using normal correlation. This improvement in the SNR was demonstrated last year for measurements using observations with one baseline (VLA TEST MEMO. 206).

Recently Ken Sowinski has completed required online software to calculate complex correlations for the whole array. We have used the new online software to make measurements and compare the results with the presently used normal correlation measurements. The results are described below.

TEST RESULTS

We made 50 MHz continuum observations at X-band of 3C84 and a blank field 5° north of it using normal online software to measure visibilities (correlation values). We repeated the observations with the new online software which provides full complex correlation measurements. The blank field visibility data are calibrated using the 3C84 data in usual way in AIPS and used to generate Stoke V images in each case. The rms noise in the Stoke V images are listed in table

1 and are about 8% smaller in complex correlation measurements than those using the normal correlations as expected.

The average closure errors for observations of 3C84 using full complex correlations and normal correlations for 50 MHz bandwidth are shown in table 2. Table 3 gives the number of times various antennas exceeded the closure errors of 0.3% in amplitude and 0.3° in phase during these observations. From these results it is seen that the closure errors with complex correlation measurements are generally smaller by a factor of about 2 to 3 for all antennas than the values obtained using normal correlation measurements. Table 4 compares amplitude and phase closure errors between complex and normal correlation measurements for different bandwidths. Both amplitude and phase closure errors are much larger in case of normal correlation measurements than in case of the complex correlations. Also the closure errors increase with decreasing the bandwidth almost linearly, especially for lower bandwidths. This suggests that the dominant cause of the closure errors is at low frequency end at the baseband.

Table 5 gives average closure errors for different bandwidths for normal and complex correlation measurements for (normal) not phasing the array and compares it with phasing the array. Comparing the closure errors between phasing and not phasing the array for normal correlation measurements it is seen that amplitude closure errors are smaller and phase closure errors are larger when the array is phased than it is not. Note that hardware phasing can only be done for two IFs and in these observations it was done for IFs A and D. Next thing to notice is that closure errors for complex correlation measurements, both in amplitude and phase, are smaller than normal correlation measurements without phasing the array. Also closure errors in complex correlation case are smaller with phasing the array than without phasing the array. This is especially noticable for narrower bandwidths. A possible explaination for this performance is given in the next section.

It may be worth mentioning here that if we want to take full advantage of achieving reduced closure errors for all four IFs, then we have to phase the array in hardware in all four IFs. At present we can phase the array in hardware in only two IFs. A relatively inexpensive scheme to allow phasing of the other two IFs in hardware, using computer controlled phase shifters in the F8 offset LOs, suggested in VLA Electronics Memo 221, will be useful for this.

WHY ARE CLOSURE ERRORS SMALLER FOR COMPLEX CORRELATIONS ?

Consider the cosine and sine outputs of the quadrature networks in the samplers as C1 = $cos(\omega t)$ and S1 = $sin(\omega t - \delta_1)$ for antenna 1, and C2 = $cos(\omega t + \phi)$ and S2 = $sin(\omega t + \phi - \delta_2)$ for antenna 2. Then we can write the two normal correlation outputs as: $< C1 * C2 - i.S1 * C2 >$ and $< S1 * S2 + i.C1 * S2 >$. The complex correlation output can be written as: (1/2) $< (C1 * C2 + S1 * S2) + i.(C1 * S2 - S1 * C2) >$.

We can write

$$\begin{aligned} < C1 * C2 - i.S1 * C2 > &= < cos\omega t.cos(\omega t + \phi) - i.sin(\omega t - \delta_1).cos(\omega t + \phi) > \\ &= cos\phi + i.sin(\phi + \delta_1). \end{aligned} \tag{1}$$

$$< S1 * S2 + i.C1 * S2 > = < sin(\omega t - \delta_1).sin(\omega t + \phi - \delta_2) + i.cos\omega t.sin(\omega t + \phi - \delta_2) >$$
$$= cos(\phi + \delta_1 - \delta_2) + i.sin(\phi - \delta_2). \qquad (2)$$

As δs are small, terms with second and higher powers of δs can be neglected. Also for the phased array $\phi = 0$, and we can simplify to write

$$< C1 * C2 - i.S1 * C2 > \simeq 1 + i.sin\delta_1$$

$$< S1 * S2 + i.C1 * S2 > \simeq 1 - i.sin\delta_2$$

$$(1/2). < C1 * C2 + S1 * S2 + i.(C1 * S2 - S1 * C2) > \simeq 1 + i.(sin\delta_1 - sin\delta_2)/2$$

In this case the closure phase error for a three antenna system in normal correlation will be

$$\text{Phase closure} = \phi_{12} + \phi_{23} + \phi_{31} \simeq sin\delta_1 + sin\delta_2 + sin\delta_3 \simeq \delta_1 + \delta_2 + \delta_3$$

Here ϕ_{12},ϕ_{23} and ϕ_{31} are phases for interferometer baselines with antennas 1-2, 2-3 and 3-1 and δ_i is quadrature phase error for antenna i.

In complex correlation case the closure phase error can be written as

$$\text{Phase closure} = \phi_{12} + \phi_{23} + \phi_{31} \simeq \frac{sin\delta_1 - sin\delta_2}{2} + \frac{sin\delta_2 - sin\delta_3}{2} + \frac{sin\delta_3 - sin\delta_1}{2} = 0$$

In normal observations where phasing is not done (i.e. $\phi \neq 0$), the output of the interferometer, in the two cases for the normal correlation can be approximated by

$$< C1 * C2 - i.S1 * C2 > \simeq cos\phi + i.(sin\phi + \delta_1.cos\phi) \qquad (3)$$

and

$$< S1 * S2 + i.C1 * S2 > \simeq cos\phi - (\delta_1 - \delta_2).sin\phi + i.(sin\phi - \delta_2.cos\phi) \qquad (4)$$

For complex correlation case the output can be approximated as

$$(1/2). < C1 * C2 + S1 * S2 + i.(C1 * S2 - S1 * C2) > \simeq [cos\phi - \frac{\delta_1 - \delta_2}{2} sin\phi] + i.[sin\phi + \frac{\delta_1 - \delta_2}{2} cos\phi] \quad (5)$$

Eqs. (3)-(5) and the fact that closure errors for complex correlation measurements are in general $\lesssim 1/3$ closure errors for normal correlation (Tables 4 and 5) suggest that δs should be repeating well on different antennas. As closure errors increase linearly with decreasing bandwidth it seems the dominant cause of the closure errors is at frequencies below about 1 MHz at baseband.

The theoretical phase error curve for the quadrature networks used in the samplers is given by Mauzy and Escoffier (1976, VLA Electronics Memo 132) and is reproduced here as Fig. 1. The quadrature phase error exceeds 1° at about 0.8 MHz and increases rapidly with decreasing frequency. The phase errors below about 1 MHz, which are expected theoretically, should repeat well from one sampler to another. The reduction of closure errors for complex correlation and the observation that closure errors increase linearly with decreasing bandwidth (Table 4)and how they change with phasing the array (Table 5) are consistant with the phase quadrature deviations causing the closure errors.

CONCLUSION

With full complex correlation measurements we gain about 8% in the SNR. The closure errors are much smaller for complex correlation measurements than they are for measurements with normal correlation. This is explained by the fact that closure errors in complex correlation case are dependent on difference of quadrature phase errors between various antennas unlike quadrature phase error for a given antenna in case of normal correlation measurements. As dominant cause of phase quadrature errors are contributed by the phase errors below 1 MHz in the quadrature networks and they repeat from unit to unit, and therefore we get effective cancellation of closure errors due to this effect in the complex correlation case.

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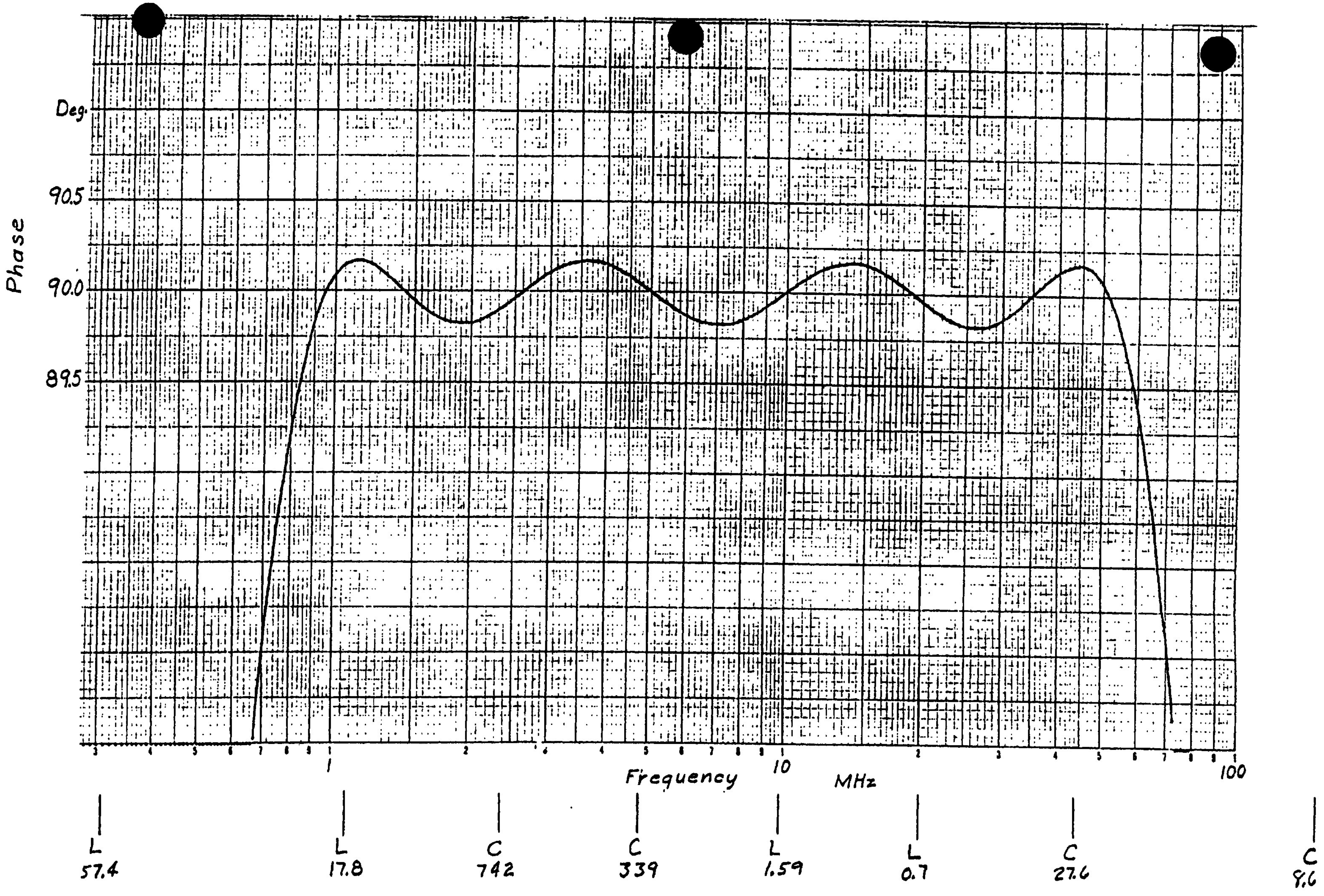

FIG. 1 THEORETICAL CURVE FOR PHASE RESPONSE OF THE QUADRATURE NETWORK IN SAMPLERS (FROM VLA ELECTRONICS MEMO 132)

TABLE 1: RMS NOISE IN STOKE V IMAGES FOR A BLANK FIELD FOR FULL COMPLEX CORRELATION AND NORMAL CORRELATION.

	STOKE V MAP NOISE (micro Jy)	
IF	AC	BD
FULL COMPLEX CORRELATION	39.9	44.3
NORMAL CORRELATION	43.0	48.1

BLANK FIELD (5deg North of 3C84 at about 70d El)
CALIBRATOR 3C84,BW=50MHz

TABLE 2: AVERAGE CLOSURE ERRORS FOR COMPLEX CORRELATION AND NORMAL CORRELATION MEASUREMENTS FOR OBSERVATIONS ON 3C84 AT X-BAND for BANDWIDTH = 50 MHz

	COMPLEX CORRELATION				NORMAL CORRELATION			
IF	A	B	C	D	A	B	C	D
AMPLITUDE (%)	0.32	0.38	0.31	0.32	0.46	0.49	0.58	0.37
PHASE (deg)	0.10	0.14	0.10	0.10	0.30	0.28	0.34	0.23

TABLE 3: NUMBER OF TIMES CLOSURE ERRORS EXCEEDED AMP=0.3%,PH=0.3deg FOR 50 MHz OBSERVATIONS OF 3C84 AT X-BAND FOR COMPLEX CORRELATION AND NORMAL CORRELATION MEASUREMENTS

	COMPLEX CORRELATION				NORMAL CORRELATION			
IF ANTENNA NO.	A	B	C	D	A	B	C	D
1	9	13	6	13	21	21	17	18
2	6	19	4	6	20	25	22	19
3	19	12	12	13	20	12	24	24
4	7	8	10	11	19	18	22	16
5	15	13	18	13	18	14	26	16
6	9	10	10	15	12	13	19	17
8	18	8	5	4	23	18	23	11
9	16	21	3	7	25	23	23	8
10	9	15	9	10	22	19	20	11
11	5	18	15	5	15	22	26	12
12	15	19	8	7	17	22	23	14
13	16	xx	12	19	20	xx	22	22
14	9	8	10	9	19	22	25	15
15	15	16	12	18	22	22	21	17
16	7	5	8	7	15	16	23	13
17	12	10	18	16	22	19	24	17
18	10	18	6	10	15	20	21	14
19	9	11	10	13	16	13	24	17
20	7	11	10	9	18	21	23	17
21	13	17	12	13	23	23	22	22
22	19	15	21	15	23	24	17	18
23	7	6	5	1	18	14	23	12
24	9	7	11	8	15	16	26	16
25	14	15	13	16	21	21	21	16
26	8	13	13	14	24	17	19	16
27	15	20	17	13	25	22	18	19
28	2	12	16	15	12	21	24	19

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xx = Data flagged

TABLE 4: AVERAGE CLOSURE ERRORS FOR OBSERVATIONS OF 3C84 FOR MEASUREMENTS USING FULL COMPLEX CORRELATION AND NORMAL CORRELATION USING DIFFERENT BANDWIDTHS.

BAND	CORRELATION	AVE AMP CLOSURE ERROR (%)				AVE PH CLOSURE ERROR (DEG)			
WIDTH	TYPE IF	A	B	C	D	A	B	C	D
50 MHz	COMPLEX	0.343	0.402	0.313	0.316	0.12	0.17	0.11	0.10
	NORMAL	0.446	0.552	0.586	0.416	0.27	0.30	0.33	0.22
25 MHz	COMPLEX	0.216	0.297	0.293	0.317	0.09	0.12	0.12	0.11
	NORMAL	0.525	0.583	0.765	0.577	0.59	0.59	0.72	0.58
12 MHz	COMPLEX	0.314	0.311	0.328	0.373	0.14	0.14	0.15	0.15
	NORMAL	0.713	0.662	0.939	0.820	0.50	0.44	0.66	0.55
6 MHz	COMPLEX	0.450	0.438	0.468	0.426	0.26	0.23	0.27	0.20
	NORMAL	1.150	1.066	1.409	1.245	0.85	0.72	0.96	0.85
3 MHz	COMPLEX	0.816	0.841	0.880	0.736	0.48	0.47	0.54	0.44
	NORMAL	2.226	2.010	2.557	2.194	1.69	1.38	1.75	1.57
1.5NHz	COMPLEX	1.685	1.661	1.766	1.515	0.98	1.01	1.13	0.97
	NORMAL	4.777	4.478	5.315	4.438	3.44	3.11	3.55	3.42

TABLE 5: COMPARISION OF AVERAGE CLOSURE ERRORS USING COMPLEX CORRELATION AND NORMAL CORRELATION AND FOR WITH AND WITHOUT PHASING THE ARRAY ON 3C84 AT X-BAND FOR DIFFERENT BANDWIDTHS

CORREL. TYPE	ARRAY PHASING	AVE AMPLITUDE CLOSURE ERROR(%)				AVE PHASE CLOSURE ERROR (deg			
	IF	A	B	C	D	A	B	C	D
BANDWIDTH = 1.56 MHz									
NORMAL	NO PHASING	4.976	3.637	4.941	4.232	3.26	3.14	3.42	3.24
COMPLEX	NO PHASING	1.765	1.805	1.800	1.534	1.03	0.91	1.13	0.90
NORMAL	PHASING	0.952	4.508	5.084	0.855	4.22	3.05	3.54	4.07
COMPLEX	PHASING	0.302	1.653	1.709	0.249	0.05	0.99	1.02	0.05
BANDWIDTH = 3.125 MHz									
NORMAL	NO PHASING	2.335	1.626	2.392	1.951	1.55	1.35	1.73	1.47
COMPLEX	NO PHASING	0.900	0.918	0.891	0.730	0.47	0.41	0.54	0.44
NORMAL	PHASING	0.395	1.909	2.417	0.392	2.03	1.32	1.74	1.84
COMPLEX	PHASING	0.274	0.823	0.820	0.269	0.05	0.46	0.49	0.06
BANDWIDTH = 6.25 MHz									
NORMAL	NO PHASING	1.240	0.949	1.410	1.131	0.85	0.74	0.95	0.85
COMPLEX	NO PHASING	0.477	0.482	0.466	0.427	0.26	0.21	0.28	0.22
NORMAL	PHASING	0.276	1.024	1.367	0.345	1.10	0.75	0.97	1.07
COMPLEX	PHASING	0.250	0.457	0.445	0.302	0.05	0.22	0.24	0.08
BANDWIDTH = 12.5 MHz									
NORMAL	NO PHASING	0.788	0.625	0.932	0.803	0.55	0.43	0.62	0.55
COMPLEX	NO PHASING	0.302	0.314	0.319	0.369	0.15	0.13	0.17	0.15
NORMAL	PHASING	0.251	0.669	0.935	0.374	0.71	0.49	0.68	0.70
COMPLEX	PHASING	0.234	0.334	0.320	0.340	0.05	0.13	0.14	0.10
BANDWIDTH = 25 MHz									
NORMAL	NO PHASING	0.524	0.554	0.718	0.512	0.41	0.38	0.56	0.38
COMPLEX	NO PHASING	0.254	0.302	0.315	0.317	0.14	0.12	0.19	0.12
NORMAL	PHASING	0.241	0.562	0.745	0.318	0.50	0.41	0.56	0.45
COMPLEX	PHASING	0.233	0.314	0.301	0.305	0.06	0.11	0.13	0.10
BANDWIDTH = 50 MHz									
NORMAL	NO PHASING	0.428	0.519	0.541	0.437	0.28	0.25	0.31	0.22
COMPlEX	NO PHASING	0.325	0.382	0.326	0.324	0.15	0.13	0.13	0.12
NORMAL	PHASING	0.324	0.500	0.496	0.327	0.34	0.27	0.32	0.25
COMPLEX	PHASING	0.321	0.386	0.316	0.315	0.14	0.13	0.11	0.11