VLA Test Memorandum 212

Measuring the Aperture Efficiency (η_a) of the VLA antennas

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Introduction

There are several ways to calculate the dish aperture efficiency (η_a) of the VLA antennas. [Note that throughout this memo, it will be assumed that the reader knows about η_a. If not, see Napier 1994, or Kraus 1986]. This memo will describe two ways which were initiated in the early days of the VLA, and which I have been using lately to try to guage the performance of the antennas. The first way is to observe a planet in single-dish pointing mode ("PA") and use the measured antenna temperature to infer η_a. Described by Herrero (1978), this technique was resurrected and modified by Doug Wood when he was here in order to check the Q-band efficiencies. The second way is to observe a source of known flux density, and use the measured visibilities to infer η_a. This is very similar to a method that Barry Clark used some time ago to measure antenna efficiencies (Clark 1977, 1978). I do it a bit differently, but it is essentially the same. This memo will describe only the technique, while results will come out in a subsequent memo.

Single Dish Observations of Planets

If we want to know η_a for an antenna, it is sufficient to look at a point source of known flux, and measure the true antenna temperature. This will yield η_a through the relation (see Kraus 1986, eqns. 6-225, and 6-227, where his ϵ_{ap} is what I'm calling η_a):

$$\eta_a = \frac{2\,k\,T_A}{A_p\,S} \quad , \tag{1}$$

where k is Boltzmann's constant, T_A is the measured antenna temperature, A_p is the physical area of the antenna, and S is the source flux density. A 10 Jy point source produces an antenna temperature of only about 0.9 K for VLA antennas (using $\eta_a = 0.5$). At the higher frequencies, sources of this strength are few and far between. Since the measurement of system temperatures is only accurate to about 1% (see below and discussion in Bagri 1994), sources with larger flux densities are needed. Several of the planets have flux density much greater than 10 Jy during opposition or conjunction (e.g., the flux density of Venus at 0.5 AU distance is roughly 500 Jy at 43 GHz). For a planet of brightness temperature T_p, ignoring primary beam effects, the source flux density is given by:

$$S = \frac{2\,k\,T_p}{\lambda^2}\,\Omega_p \quad , \tag{2}$$

where λ is the observing wavelength, and Ω_p is the apparent size of the planet ($\Omega_p = \pi\,R^2/D^2$ for a planet of radius R at distance D). Substituting the value for S from equation 2 into

Table 1: Size and Antenna Temperature of some of the Planets

body	size† ($''$)	T_A (K) L	C	X	U	K	Q
Mercury	12.25	*	*	*	0.6	1.1	3.2
	4.63	*	*	*	0.1	0.2	0.4
Venus	63.41	0.3	4.4	11.4	26.5	46.2	117.2
	9.72	*	0.1	0.3	0.6	1.1	2.8
Mars	25.10	*	0.2	0.6	1.5	2.9	8.5
	3.50	*	*	*	*	*	0.1
Jupiter	49.86	0.7‡	1.3‡	2.2	4.1	7.3	23.4
	30.55	0.3‡	0.5‡	0.8	1.5	2.8	8.8
Saturn	20.64	*	0.1	0.3	0.7	1.4	3.9
	15.04	*	*	0.2	0.4	0.7	2.1
Moon	-	104.4	129.6	122.8	98.8	83.6	66.6

$* \rightarrow T_A < 0.1$ K.

† max and min equatorial diameter for years 1995-2020, except for the Moon (see text).

‡ includes a non-thermal (synchrotron) source which is not confined to the disk.

equation 1 yields:

$$\eta_a = \frac{T_A\,\lambda^2}{A_p\,T_p\,\Omega_p} \quad . \tag{3}$$

However, when the planets are bright enough to contribute significantly to the antenna temperature, they must be quite close, and are hence quite large on the sky (e.g., at 0.5 AU the diameter of Venus is $\sim 30''$). This means that the primary beam effects must be considered, at least at high frequencies, and this will be covered subsequently. Table 1 shows several of the planets and the Moon, giving their apparent size range, and a rough estimate of the range of induced antenna temperature as a function of frequency. Note that the brightness temperature of all of these bodies is a function of wavelength. Also note that for the Moon, I have used a size which is equivalent to twice the FWHM of the primary beam at the wavelength in question (i.e., nearly out to the first null). I used the standard average values of η_a to calculate the values of T_A in Table 1: 0.51, 0.65, 0.63, 0.52, 0.42, and 0.35, for L, C, X, U, K, and Q-bands.

So, measuring the antenna temperature induced by a planet may be used to infer η_a.

The process occurs in 3 steps:

1. find $T_{\rm sys}$
2. find T'_A, the modified antenna temperature
3. calculate η_a

Each of these steps will now be discussed in more detail.

Finding $T_{\rm sys}$

The total system temperature on a VLA antenna can be recovered from the so-called "nominal sensitivity", which is recorded on tape. At every source change, the on-line system calculates the quantity:

$$I_{\rm sens} = \frac{21.59\ \eta'_a}{T'_{\rm cal}\ g} \tag{4}$$

for each antenna and IF, where η'_a is the *assumed* dish efficiency at the observed band, $T'_{\rm cal}$ is the *assumed* noise tube temperature (in K) for that antenna/IF, and g (the so-called "peculiar gain") is a fudge factor (see below). The 21.59 is a constant that subsumes the area of the dish, Boltzmann's constant, the front end gain, and other radiometric constants (note that for observations done prior to 1989, this value was 24.32). Now, every 10 seconds, the on-line system calculates the following quantity (the "nominal sensitivity"):

$$I_{\rm corf} = \frac{3}{V_{\rm sd}\ I_{\rm sens}} = \frac{3}{V_{\rm sd}} \left(\frac{1}{21.59}\ \frac{T'_{\rm cal}\ g}{\eta'_a} \right) \quad , \tag{5}$$

where $V_{\rm sd}$ is the front end synchronous detector voltage for each antenna/IF. For each correlated visibility, the geometric mean of $I_{\rm corf}$ for the two antennas/IFs is used as a multiplicative factor to convert correlation coefficient to 10's of Janskys. This value is what is written to the archive tape. The values of $T_{\rm cal}$, η'_a, and g are retrieved from files on the on-line system.

The front-end (FE) system temperature is given by:

$$T_{\rm sys} = \frac{15\ T_{\rm cal}\ V_{\rm TP}}{V_{\rm sd}} \quad , \tag{6}$$

where $T_{\rm cal}$ is the *actual* (as opposed to assumed) noise tube temperature (in K) for a given antenna/IF, and $V_{\rm TP}$ is the total power voltage input to the correlator. The ALC's constrain $V_{\rm TP}$ to be near 3 V, so this is nearly a constant value. The factor of 15 is strictly an electronics gain factor. This factor is different for the back-end (BE) system temperature. I have ignored offsets in the total power and sync detector voltages here. So,

$$T_{\rm sys} \sim \frac{45\ T_{\rm cal}}{V_{\rm sd}} \quad , \tag{7}$$

or,

$$V_{\rm sd} \sim \frac{45\ T_{\rm cal}}{T_{\rm sys}} \quad . \tag{8}$$

Substituting this into equation 5 yields:

$$I_{\rm corf} \sim \frac{3\ T_{\rm sys}}{45\ T_{\rm cal}} \left(\frac{1}{21.59} \ \frac{T'_{\rm cal}\ g}{\eta'_a} \right) \quad , \tag{9}$$

or,

$$T_{\rm sys} \sim 323.85\ \frac{\eta'_a\ T_{\rm cal}}{g\ T'_{\rm cal}}\ I_{\rm corf} \quad . \tag{10}$$

Again, the values of $T'_{\rm cal}$, η'_a, and g are retrieved from files on the on-line system (the SYSnIF files where n is the band, e.g. SYSQIF for Q-band). Note that uncertainties in these values are unimportant, as long as the same ones are used which were used by the on-line system. Errors are due to fluctuations in $T_{\rm cal}$, and in $V_{\rm TP}$. Of these, fluctuations in $T_{\rm cal}$ should dominate. There is no good knowledge of how these values fluctuate over short or long time scales, however, current wisdom is that the values are relatively stable (to $\sim 10\%$, see Bagri and Lilie 1993, and Lilie 1992). Therefore, estimating the value of $T_{\rm sys}$ from the nominal sensitivity, without a good value for $T_{\rm cal}$, should be accurate to $\sim 10\%$ for a given antenna. A more accurate estimate may be obtained if a measured value of $T_{\rm cal}$ is available. These values may be deduced via a TIP scan (Butler 1996). Note also that if the BE $T_{\rm sys}$ correction is used (by turning the right switch in the OBSERVE file), then the multiplicative number (323.85) will be different than presented above.

For data which was written to VLA archive tapes after some time in winter of 1996/97, the on-line system estimate of the true system temperature (both FE and BE) are written on the tape. In this case, it is much easier to recover the true system temperature via:

$$T_{\rm sys} = T'_{\rm sys}\ \frac{T_{\rm cal}}{T'_{\rm cal}} \quad , \tag{11}$$

where $T'_{\rm sys}$ is the on-line estimate of the system temperature. Again, the $T_{\rm cal}$ values may be deduced via a TIP scan.

Finding T'_A

Given the total system temperature, it is then necessary to find the portion of $T_{\rm sys}$ which is due to the planet. I need to inject a short discussion on how the VLA operates in PA mode here. During PA mode operation, there is a cycle of positions which is observed by the antennas which looks like:

submode	position
1	*
2	OFF
3	*
4	+EL
5	*
6	-EL
7	*
8	+AZ
9	*
10	-AZ
11	*
12	OFF
13	*
14	ON
15	*
16	OFF

*	$\rightarrow$	invalid data during move to next position
ON	$\rightarrow$	antennas pointing to nominal on-source position
+/-AZ	$\rightarrow$	antennas pointing off source by a half beamwidth in azimuth
+/-EL	$\rightarrow$	antennas pointing off source by a half beamwidth in elevation
OFF	$\rightarrow$	antennas pointing off source by +2.5 beamwidths in azimuth

When a source change is detected in PA mode, the first submode recorded will be submode 7, but valid data may not be considered to be present until submode 10 is observed (because of system settling time). So, for each full cycle, 1 pointing is made directly at the source position, 4 are made at the nominal half power points (calculated *apriori* by the on-line system as: HWHM = 1440 / ν_o arcsec, where ν_o is the center frequency in GHz, e.g., at 43 GHz, HWHM = 33.49 arcsec), and 3 are made at a presumed blank region of sky. By definition (in the on-line system), each of these pointings is 10 seconds (as are the moves), but individual samples are collected at intervals specified by the integration time (as small as $1\frac{2}{3}$ sec). The measured quantity of interest here is $V_{\rm sd}$, which is sampled at a rate of once each 1.25 sec. In addition there is a folded in electronic time-constant of 1 second for the value itself. The software then takes those samples and performs an exponential smoothing which has a time constant of 5 sec (4 samples) [the on-line system takes each sample of $V_{\rm sd}$ and modifies the current estimate of that value by adding $V_{\rm sd}/N \cdot N/(N-1)$ to it, with $N = 4$, which is the software equivalent of an RC circuit with a time constant of 4 samples, or 5 sec]. The implication of this sampling and smoothing is that the right way to do this sort of observation is to specify an integration time of $1\frac{2}{3}$ sec, then take the last recorded value of $V_{\rm sd}$ for each submode (there will be 6 per pointing) as the best estimate of the value for that pointing. There is another complication which must be mentioned here. The on-line system does not label the records as they are written to the archive tape with the correct submode. As the system stands currently, the correct way to pick off the last recorded value for each submode is to actually pick the *second* (of 6) of the values. The submode mislabeling is a result of making sure that the submode labels for the survey are correct, and there is currently no plan to fix this "feature". Note that the accuracy of an *individual* sample of $V_{\rm sd}$ is determined by the quantization of the value in the A/D conversion. This accuracy is about 1 part in 600 (about .2%), because 3 V is set to be 600 A/D counts. The accuracy of the smoothed values will be somewhat better, but the overall accuracy of values of $T_{\rm sys}$ for a single antenna/IF is probably only about 1% (Bagri 1994).

Now, making the assumption that $T_{\rm sys}$ may be broken into 2 components: one due to things above the top of the atmosphere (T_1), and one due to all things below ($\rm T_o$), most of the contribution to $T_{\rm sys}$ not related to the planet may be subtracted out by simply subtracting the average value of the OFF pointings. For the moment, let us ignore pointing errors, which we will then reconsider later. In this case, we would only need observations on planet and off planet, from which we could immediately derive the planetary contribution. For the on-planet pointings we have:

$$T_{\rm sys} = T_o + K \frac{\int\int_{\rm on} T_b(x,y)\, A(x,y)\, dx\, dy}{\int\int_{\rm on} A(x,y)\, dx\, dy} + K \frac{\int\int_{\rm off} T_{\rm MB}(x,y)\, A(x,y)\, dx\, dy}{\int\int_{\rm off} A(x,y)\, dx\, dy} \quad , \tag{12}$$

where T_b is the brightness temperature of the planet being observed, $T_{\rm MB}$ is the microwave background temperature, A is the normalized antenna reception pattern, x, y are the sky coordinates, "on" indicates that the integration is to be done over the portion of the sky on the planet, "off" indicates that the integration is to be done over the portion of the sky off the planet, and K is a term which takes into account the opacity, and the conversion from planet brightness temperature to antenna temperature (see equations 1-3 above):

$$K = \frac{\eta_a\, A_p\, \Omega_p}{\lambda^2}\, e^{\tau_o \csc E} \quad , \tag{13}$$

where τ_o is the zenith opacity, and E is the elevation of the planet when observed. For the off-planet pointings we have:

$$T_{\rm sys} = T_o + K \frac{\iint_\infty T_{\rm MBG}(x,y)\, A(x,y)\, dx\, dy}{\iint_\infty A(x,y)\, dx\, dy} \quad . \tag{14}$$

So, by subtracting out the off-planet pointings, we are left with the quantity:

$$T'_A = K \frac{\int\int_{\rm on} (T_{\rm planet}(x,y) - T_{\rm MB})\, A(x,y)\, dx\, dy}{\int\int_{\rm on} A(x,y)\, dx\, dy} \tag{15}$$

(making the assumption that $T_{\rm MB}$ is constant). This equation can be directly inverted to solve for η_a.

However, recall that this has ignored pointing errors, which complicate the calculation of the exact value of T'_A. If we assume that the elevation and azimuth portions of the antenna reception pattern (the "primary beam") can be considered separately, then for each of these two beams, we have 3 measurements. These 3 measuerements must be used to solve for the 3 unknowns: the amplitude at the center of the beam, some measure of the width of the beam, and the offset of the true beam from the assumed pointing center. If we assume a gaussian beam, then:

$$A(u) = A_o\, e^{-\ln 2\,(u-\Delta\, u)^2/\sigma^2} \quad , \tag{16}$$

where A_o is the amplitude at the center, σ is the half-width, and $\Delta\, u$ is the offset. In this case, the 3 measurements are:

$$A_i = A_o\, e^{-\ln 2\,(u_i-\Delta\, u)^2/\sigma^2} \quad . \tag{17}$$

This set of equations can be analytically solved for the 3 unknowns (e.g. McKay 1997). Of course, in a general sense, the fit to the beam is simply a nonlinear least squares problem, given any arbitrary beam shape. These sorts of problems can be solved, given the equation for the shape of the beam, and its derivative with respect to the unknowns (see e.g. Press *et al.* 1988, section 14.4). In the case of the gaussian beam, the derivatives are:

$$\frac{\partial\, A_i}{\partial\, A_o} = e^{-\ln 2\,\phi^2} \quad , \tag{18}$$

$$\frac{\partial\, A_i}{\partial\, \Delta\, u} = \frac{2\,\ln 2\,\phi\, A_o}{\sigma}\, e^{-\ln 2\,\phi^2} \quad , \tag{19}$$

and,

$$\frac{\partial\, A_i}{\partial\, \sigma} = \frac{2\,\ln 2\,\phi^2\, A_o}{\sigma}\, e^{-\ln 2\,\phi^2} \quad , \tag{20}$$

where $\phi = (u - \Delta\, u)/\sigma$. In reality, however, the beam is not a gaussian, but rather has Besselian form (Napier 1994):

$$A(u) = A_o \left[\Lambda_1\left(\frac{u-\Delta\, u}{w}\right)\right]^2 \quad , \tag{21}$$

where Λ_k is the Lambda function of order k : $\Lambda_k(z) = \Gamma(k+1)\left(\frac{1}{2}z\right)^{-k} J_k(z)$ (so that $\Lambda_1(z) = J_1(z)/(z/2)$), $\Delta\, u$ is again the offset, and w is a measure of the beam width. In this case, the derivatives are given by:

$$\frac{\partial\, A_i}{\partial\, A_o} = \left[\Lambda_1(\psi)\right]^2 \quad , \tag{22}$$

$$\frac{\partial A_i}{\partial \Delta u} = \frac{A_o \psi}{2w} \Lambda_1(\psi)\, \Lambda_2(\psi) \quad , \tag{23}$$

and,

$$\frac{\partial A_i}{\partial w} = \frac{A_o \psi^2}{2w} \Lambda_1(\psi)\, \Lambda_2(\psi) \quad , \tag{24}$$

where $\psi = (u - \Delta u)/w$. The true half-width at half-max is given by: $\sigma = 1.61634\, w$, where the 1.61634 comes from the fact that $[\Lambda_1(1.61634)]^2 = 0.5$.

So, given the 3 values of the beam measurements yields through the fit the value of A_o, which is actually the value of T'_A, given the scaling. Values are derived independently for each of the IF's of the antennas, and for the elevation and azimuth (8 values in all). Checks of the widths and offsets for both azimuth and elevation provide information on the fit validity.

Calculating η_a

Finally, given the estimate of T'_A for an antenna, we must solve equation 15 for η_a. This requires knowledge of the distribution of brightness temperature of the planet ($T_b(x, y)$), and the normalized antenna reception pattern (the "beam", $A(x, y)$) over the size of the planet. We've already made the assumption that we know the beam shape, as described in the above section. Note, however, that if there are significant deviations from the theoretical beam shape (as was probably the case when the Q-band receivers were first installed), none of the following really makes much sense, and values of η_a derived from use of this technique will be highly suspect. Also note that we *never* know the beam response accurately out past the second null or so. This implies that the Moon is not a very good target to use for this type of determination, since it is simply too large. To a good approximation, the brightness distributions on the planets useful for this technique (Venus, Jupiter, and possibly Saturn, Mercury and Mars) are slightly limb-darkened disks of the form:

$$T_b(u) = T_{b_o} \cos^n \theta \quad , \tag{25}$$

where T_{b_o} is the brightness temperature at the subearth point, and θ is the incidence angle, given by: $\theta = \sin^{-1} r = \sin^{-1}(u/u_{max})$, with $u_{max} = R/\lambda$, for a planet of apparent radius R. The values of n are usually $\lesssim 0.2$. Since both the beam and the planet are circularly symmetric, equation 15 reduces to:

$$T'_A = K \left[\frac{T_{b_o} \int_0^1 A(\rho)\, (1 - \rho^2)^{n/2}\, \rho\, d\rho}{\int_0^1 A(\rho)\, \rho\, d\rho} - T_{MB} \right] \quad . \tag{26}$$

Unfortunately, in general the integrals must be evaluated numerically, but that isn't a particularly hard thing to do. Assigning the variable f to the ratio of the integrals, the value of η_a can then be directly calculated:

$$\eta_a = \frac{T'_A \, \lambda^2}{A_p \, e^{-\tau_o \csc E} \, (T_{b_o} f - T_{MB})} \quad . \tag{27}$$

Implementation

Originally, the intent was to implement all of this into AIPS, and to that end, I wrote two AIPS tasks to do the above derivation of η_a. The first, called TYCVP, takes the values of nominal sensitivity and converts them to $T_{\rm sys}$. The second, called TYPTG, takes the resultant values of $T_{\rm sys}$ and calculates T'_A and hence η_a. Note that in order to do the fitting, the program must have access to the submode information. In current AIPS, FILLM does nothing with the submode. So, Gustaaf van Moorsel put together a special version of FILLM which puts the submode into the TY table along with the nominal sensitivity. The above tasks deal with these non-standard TY tables. However, it became quite cumbersome to maintain a non-standard version of FILLM, and it turned out that this version didn't work right on all PA mode data anyway. Because of this, I have gone to an entirely non-AIPS implementation. The values of nominal sensitivity (or $T'_{\rm sys}$ for data written in 1997 or later) are read directly from the archive with a program written by Wes Young (called `sfill`), and the derivation of T'_A and η_a are done with a program I wrote (called `typtg`).

Problems

Problems with this technique (i.e., sources of error) that I can see include:

1. For the higher frequencies, the Rayleigh-Jeans approximation breaks down, and the value of $T_{\rm sys}$ should be modified like:

$$\hat{T}_{\rm sys} = \frac{h\,\nu}{k} \, \frac{1}{e^{h\nu/kT_{\rm sys}} - 1} \quad , \tag{28}$$

   where h is Planck's constant, and ν is the frequency. The error introduced by this is about 1% at 43 GHz for a $T_{\rm sys}$ of 100 K.

2. There is an implicit assumption that the atmosphere is constant as a function of time and position. This will only be true if the weather is good.

3. The *true* values of the noise tube temperatures are not known exactly. It is not clear how accurate the estimation of these values from TIP scans is (Butler 1996). I suspect that they are no better than a few percent (relative), and can easily be worse in bad weather.

4. The derived opacity has some uncertainty associated with it. I suspect that it is no better than a few percent (relative).

5. Uncertainty in the true flux density (brightness temperature) of the observed planet propagates directly. At the higher frequencies, this may be a significant source of error.

6. Assuming that the beam shape is known, and deriving the beam parameters from only 3 pointings in each direction (elevation and azimuth) seems risky to me. The problem with obtaining many more samples of the beam shape is that it takes a significant amount of extra time. In this extra time, there is more chance for things to change (source elevation, atmosphere, etc. . .).

7. In the fit to the beam shape, it is assumed that the source is a point source. For the planets, which are certainly not point sources when bright enough, equation 17 is not right, as it should involve an integral of the sky brightness over the proper beam shape. It is not clear at this point how big an effect this is.

Observations of a Source of Known Flux Density

I base my estimates of aperture efficiency in this way on the assumption that the peculiar gains should really be 1.0, and that given the *true* values of the aperture efficiencies and cal temps, the conversion from estimated correlation coefficient to visibility value yields values which have real meaning on an absolute flux density scale.

The raw visibility that comes out of the on-line system for the cross product of antenna i with antenna j is constructed via (in the case of continuum observations):

$$\hat{V}_{ij} = 256\,\sqrt{I_{\mathrm{corf}_i}\,I_{\mathrm{corf}_j}}\;\hat{r}_{ij} \quad , \tag{29}$$

where $\hat{r}_{ij}$ is the estimate of the correlation coefficient. See equation 5 for a description of I_{corf}. The scale is supposed to be set such that a raw value of 0.1 implies a true flux density "close" to 1.0 Jy, where "close" is arbitrarily defined. Substituting equation 5 (with proper antenna subscripts) into equation 29 gives:

$$\hat{V}_{ij} = 35.57\,\sqrt{\frac{T'_{\mathrm{cal}_i}\,g_i}{V_{\mathrm{sd}_i}\,\eta'_{\mathrm{a}_i}}\,\frac{T'_{\mathrm{cal}_j}\,g_j}{V_{\mathrm{sd}_j}\,\eta'_{\mathrm{a}_j}}}\;\hat{r}_{ij} \quad . \tag{30}$$

What is the basis for equation 30? Given the *true* correlation coefficient ρ_{ij}, the conversion to visibility flux density (in Jansky) is (e.g. Moran and Dhawan 1995):

$$V_{ij} = \rho_{ij}\,\frac{2\,k}{A_{\mathrm{phys}}}\,10^{26}\,\sqrt{\frac{T_{\mathrm{sys}_i}\,T_{\mathrm{sys}_j}}{\eta_{\mathrm{a}_i}\,\eta_{\mathrm{a}_j}}} \quad , \tag{31}$$

where k is Boltzmann's constant, $A_{\rm phys}$ is the physical area of the antennas, $T_{{\rm sys}_i}$ is the system temperature of the i^{th} antenna, which has *true* aperture efficiency $\eta_{{\rm a}_i}$. For the VLA the system temperature is given by equation 6, so, using the physical size of the VLA antennas and putting in the constants gives:

$$V_{ij} = 253.13 \sqrt{\frac{T_{{\rm cal}_i}}{V_{{\rm sd}_i}\,\eta_{{\rm a}_i}} \frac{T_{{\rm cal}_j}}{V_{{\rm sd}_j}\,\eta_{{\rm a}_j}}}\ \rho_{ij} \quad . \tag{32}$$

Now, the question is: how are ρ_{ij} and $\hat{r}_{ij}$ related? The VLA calculates $\hat{r}_{ij}$ as (from Ken Sowinski):

$$\hat{r}_{ij} = \frac{c_{ij}}{\sqrt{c_{ii}\,c_{jj}}} \quad , \tag{33}$$

where c_{ij} is the cross-count output of the correlator for the ij correlator pair, and c_{ii} is the self-count for i. This normalization scheme is chosen to account for first-order errors in sampler 0-level and gain settings (Thompson 1973; D'Addario 1976). This is not the "standard" way of normalizing, which is (Cooper 1970; Clark 1978):

$$r_{ij} = \frac{c_{ij}}{N_{\rm max}} \quad , \tag{34}$$

where N_{max} is the total number of samples. In this case, and for small correlation coefficient, the true correlation coefficient is related to the digitally estimated 3 level correlation coefficient via: (e.g., see the sine approximation in Schwab 1979, using $v_5 = v_6 = 0.612$):

$$\rho_{ij} \sim 2.284\ r_{ij} \quad . \tag{35}$$

The difference in the normalizations is (see Table I-a of D'Addario 1976 with $V_i = 0.612$, and noting that his V_o is the same as what I call $N_{\rm max}$):

$$\sqrt{c_{ii}\,c_{jj}} \sim 0.541\ N_{\rm max} \quad , \tag{36}$$

implying that

$$r_{ij} \sim 0.541\ \hat{r}_{ij} \quad . \tag{37}$$

So, the relation between the true correlation coefficient and the VLA correlation coefficient is:

$$\rho_{ij} \sim 2.284\ r_{ij} \sim 2.284 * 0.541\ \hat{r}_{ij} \sim 1.236\ \hat{r}_{ij} \quad . \tag{38}$$

This agrees with D'Addario 1976, Table I-b (for $V_i = .612$ and small ρ). Substituting this back into equation 32 gives:

$$V_{ij} = 312.8 \sqrt{\frac{T_{{\rm cal}_i}}{V_{{\rm sd}_i}\,\eta_{{\rm a}_i}} \frac{T_{{\rm cal}_j}}{V_{{\rm sd}_j}\,\eta_{{\rm a}_j}}}\ \hat{r}_{ij} \quad . \tag{39}$$

This is very close to equation 30 (remembering that this equation is in Jy, and equation 30 is in 10's of Jy). Note that the difference (a factor of 1.14) is nearly the ratio of 24.32 to 21.59, and that 24.32 was the constant used in construction of the nominal sensitivities prior to 1989 (see equation 5).

So, given an observation of a source of known expected flux density, and good estimates of the true T_{cal_i} values, I find the values of the η_{a_i} which yield the expected flux density for all cross correlation pairs. Given the original visibilities, the values of $\eta'_{\mathrm{a}_i}, T'_{\mathrm{cal}_i}$, and g_i, and new, *true* values of the η_{a_i} and T_{cal_i}, I modify the original visibilities to the true scale via:

$$V_{ij} = 8.79\,\hat{V}_{ij}\sqrt{\frac{\kappa'_i\,\kappa'_j}{\kappa_i\,\kappa_j}} \quad , \tag{40}$$

where 8.78 = 10 * 312.8 / 355.7, and

$$\kappa'_i = \frac{\eta'_{\mathrm{a}_i}}{T'_{\mathrm{cal}_i}\,g_i} \tag{41}$$

$$\kappa_i = \frac{\eta_{\mathrm{a}_i}}{T_{\mathrm{cal}_i}} \quad . \tag{42}$$

The steps in the process are:

1. Find the T_{cal_i} (from a TIP scan [Butler 1996]).

2. Do a phase-only self-cal on the raw visibilities.

3. Given the T_{cal_i}, and setting $\eta_{\mathrm{a}_i} = \eta'_{\mathrm{a}_i}$, construct an intermediate data set, where the visibilities (V'_{ij}) are obtained via equation 40.

4. Find the expected *attenuated* flux density of the source of known flux (currently only 3C286 at Q-band)
$$F_{\mathrm{expected}} = F_{\mathrm{true}}\,e^{-\tau_o A} \quad , \tag{43}$$
where τ_o is the zenith opacity (measured via a TIP scan [Butler 1996]), and A is the airmass.

5. Do an amplitude and phase self-cal on this data set, setting the model flux density to the expected value. The true aperture efficiencies are then:
$$\eta_{\mathrm{a}_i} = \frac{\eta'_{\mathrm{a}_i}}{S_i^2} \quad , \tag{44}$$
where S_i is the CALIB solution value for antenna i.

6. As a check, construct a final data set, using the true T_{cal_i} and η_{a_i} and equation 40, and make sure that the visibility curve is flat (for 3C286, at least, since we know it is basically unresolved by the VLA at Q-band in all configurations), and that the visibilities average to the correct (expected) flux density.

Problems

Problems with this technique (i.e., sources of error) that I can see include problems 1-5 listed for the planet observing determination above, and, in addition:

6. Second order sampler level and offset errors are not taken into account. Lore is that these errors are on the order of 0.1%.
7. If there is a pointing error which is not corrected by reference pointing (for example, if the reference pointing is done at X-band, and there is an error in the collimation for the band at which the observations are done), this will make the derived efficiency lower than the actual efficiency.
8. The linear conversion from r_{ij} to ρ_{ij} (equation 35) introduces some error. This error can be calculated given the expected true correlation coefficient by calculating a better conversion from r_{ij} to ρ_{ij} (e.g., the sine approximation of Schwab 1979) and comparing it to the linear conversion. For 3C286, which has about 1.5 Jy flux density at 43 GHz, the expected value of ρ is about 0.001, for which the error in the linear conversion is very small. Even for a 20 Jy source (e.g. 3C84) at 8.5 GHz, the expected value of ρ is only about 0.06, for which the error is about 0.02%.
9. There are real effects that the peculiar gains are supposed to take care of in the system. An examples is an incorrect delay for a given antenna/IF. Magnitude of the error caused by these effects is uncertain, but certainly should not exceed the rms fluctuation among the peculiar gains themselves, which is on the order of 10%.

Comparison of the 2 Methods

Because the second method uses the correlated visibilities, it does not produce a true value of the aperture efficiency, but rather a combination of aperture efficiency and other loss terms which occur between the front-end and the correlator. The efficiency derived in this way is therefore a "system" efficiency value. It is interesting in that it is this system efficiency which should be used in place of the true aperture efficiency when estimating what the thermal noise on the visibilities should be (e.g., equation 7-41 in Crane and Napier