VLBA Correlator Memo No.102

Polynomials for use in the VLBA Correlator
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January 1995

The VLBA correlator evaluates fifth order polynomials to estimate the *a priori* fringe phase and rate. ~~These polynomials are based on~~ the output of the "calc" system of estimating wavefront delays to the various stations. This note concerns itself with getting from the "calc" outputs to the coefficients of the polynomials.

I investigated the goodness of fits using sets of calc 8.1 outputs supplied to me by John Benson, with the source BL Lac rising at both HN and SC on September 22, 1980, with the delay and delay rate calculated every 10s.

calc is used in data reduction in producing estimates of the various parameters which are self-consistent to perhaps as small as 1ps. In order to avoid confusing things, we should fit the calc outputs to a somewhat better accuracy. I suggest a criterion of 0.3ps (10d at 86 GHz) is appropriate. This can perhaps be relaxed a bit at low elevations, because the atmospheric delays have a much higher uncertainty than the fitting accuracy, so I consider a fit acceptable if it fits better than 0.3ps above 20d elevation, better than 1ps between 5d and 20d elevation, and better than 3ps between 2d and 5d elevation. Discontinuities in phase are more annoying than a more smoothly varying error, and I suggest a criterion that discontinuities at model change times should not exceed 0.1ps.

This investigation was sparked by problems with the size of the coefficient of the fifth order term, which, if too large, can lose precision in the case of too long a time from the base time. In particular, the spline fit algorithm (discussed below) has a tendency to impart a bit of ripple to the polynomial, with a larger than necessary value of the fifth order coefficient, which can lose precision at about 24h of job time (that is, 6h of clock time with a speedup factor of 4). Other fitting algorithms seem to keep this coefficient within bounds, so that problems should occur at times such that resetting the reference time every day of clock time (a pretty reasonable requirement, really) would keep us out of trouble. However, a crude estimate for the VSOP orbiter suggested that a similar loss of precision might occur near perigee after only about 10h of job time, which is starting to become unreasonable. Therefore, other solutions to this problem are being explored.

Choice of fitting algorithm

The algorithm currently in use in the correlator fits a fifth order natural spline polynomial to a set of ten delays and delay rates output by calc. The program used is the CACM collected algorithm program quindf(). The coefficients evaluated at the fifth of these data is used for the model interval between the fifth and sixth calc outputs. This has the nice property that there is no discontinuity at model change time. Hereafter, this will be referred to as the "spln" algorithm.

In addition to this algorithm, I investigated three others. First, and simplest, is to simply fit a fifth order polynomial exactly to six delays (ignoring the delay rates), and using it between the third and fourth data. Again, there is no discontinuity at model change time. I shall call this the "6pt" algorithm.

One can also fit a fifth order polynomial exactly to three delay, delay rate pairs, and use it for half a model interval on either side of the center point. This algorithm saves a substantial amount of computing, since two additional calc delays must be provided at the beginning and end of observation for the other algorithms. I shall refer to this as the "3pt" algorithm.

I also looked at fitting a sixth order polynomial to seven delays, and then discarding the coefficient of the sixth order term. I did this because I wanted to compare the coefficients in detail with the spln algorithm, and this comes up naturally with coefficients evaluated at the data points (like the spln algorithm). It is a pretty silly thing to want to do, but, having written the program, I shall discuss it. This I will refer to as the "7pt" algorithm.

For comparing these algorithms I used a two minute model interval in all cases. Below, the maximum error given by the fitted polynomial, compared with the actual calc delay at the 10 second points, within the interval in which the polynomial is to be used. The error is given for high ($> 20^o$) elevations, low ($\sim 5^o$), and very low ($\sim 2^o$) elevations.

Algorithm	Max err abv 20°	Max err at 5°	Max err at 2°
spln	0.31	3.18	21.27
6pt	0.00	0.00	0.30
3pt	0.02	0.40	2.90
7pt	0.01	0.01	0.69

It seems to me that the spln algorithm is unacceptable at low elevations. The 3pt algorithm is marginally acceptable, if we really want to save compute resources. It does not have the nice property that there is no jump at model change time, and this jump exceeds 0.1ps at elevations below 10d. Although the deficiencies could probably be made up by decreasing the model interval slightly, I recommend the 6pt algorithm.

Model interval

For the 6pt algorithm I looked at the errors in the polynomials as a function of the model interval for a model interval of 2 minutes, 3 minutes, 4 minutes, 5 minutes, and 6 minutes, with the results below.

Modl int	Max err abv 20°	Max err at 5°	Max err at 2°
2 min	0.00	0.00	0.30
3 min	0.00	0.03	3.79
4 min	0.00	0.17	25.3
5 min	0.00	0.68	74.0
6 min	0.01	2.21	116.

It appears to me that a three minute model interval is adequate.

VSOP

For the VSOP orbiter, I believe that the properties of the polynomial fitting are determined by the gravitational attraction of the earth, and that the perturbations of the sun, the moon, and the non-sphricity of the earth, while quite significant in their effect on the exact orbit, are sufficiently slowly varying, compared to the influence of the earth, that they may be neglected for current purposes. I think it unlikely that non-gravitational forces are important here also; in any case they are much too difficult for me to look into now. So I have taken a classical elliptical orbit, and calculated delays to a source lying in the plane of the orbit, at a one radian angle to the line connecting the earth and perigee. I have analysed only half the orbit, in expectation that the other half would not be very different.

The parameters of the orbit that I have used are:

semimajor axis	59550 microseconds
ellipticity	0.5877
period	396 minutes

Running them through the same polynomial fitting routines, I verified that loss of precision of phase accuracy occurs at about 7h of job time.

Because the highest operating frequency is only 22 GHz, a somewhat more relaxed criterion for fitting accuracy is reasonable. One ps is certainly adequate, perhaps two ps could be tolerated near perigee. A one minute model interval suffices for most of the orbit, except for the hour centered on perigee. A thirty second interval is required for times near perigee. (Indeed, I hesitate to predict that a thirty second interval is adequate without a more thorough exploration of the parameter space, though twenty seconds is certainly adequate.)