NORTHEAST RADIO OBSERVATORY CORPORATION
HAYSTACK OBSERVATORY

27 January 1984

TO: VLBA Correlator Group

FROM: A.E.E. Rogers

SUBJECT: Continuum SNR as a function of the number of correlation lags.

The optimal determination of interfermeter complex amplitude is obtained from the delay function† or integrated cross spectal function

$$D(\tau) = \int_0^B S_{xy}(\omega) e^{-i\omega\tau} \, d\omega/2\pi \tag{1}$$

or $D(\tau) \overset{FT}{\longleftrightarrow} Sxy(w)H(w) \otimes W(w) \overset{FT}{\longleftrightarrow} Rxy(\tau)W(\tau) \otimes (B_1+iB_2)$

where

$W(\tau)$ is a window in the cross-correlation

$B_1 = \int_0^B \cos\omega\tau \, d\omega/2\pi$

$B_2 = \int_0^B \sin\omega\tau \, d\omega/2\pi$

$B_1+iB_2 \longleftrightarrow H(w)$

B = bandwidth

For a continuum radio source with interferometric phase θ the signal portion of the cross-correlation funciton is

$$Rxy(\tau) = B_1 \cos\theta + B_2 \sin\theta \tag{2}$$

so that $D(\tau) = (B_1\cos\theta + B_2\sin\theta)W(\tau) \otimes (B_1+iB_2)$ (3)

If there is no error in apriori delay

$$D = \sum_{\substack{\text{all delays} \\ \text{within window}}} B_1^2\cos\theta + iB_2^2\sin\theta \tag{4}$$

since $B_1 B_2$ is zero for Nyquist sampling of a rectangular bandpass. The noise level is simple to calulate in the case of Nyquist sampling of a rectangular bandpass because each lag is uncorrelated from the next in which case the SNR, relative to the optimal case is

$$SNR = \left(\sum_{\text{WINDOW}} \left(B_1^{\,2} + B_2^{\,2} \right) \right)^{1/2} \Big/ \sqrt{2} \qquad (5)$$

where

$$\sum B_1^2 = 1$$

$$\sum B_2^2 = \frac{8}{\pi^2} \left[1 + 1/3^2 + 1/5^2 + 1/7^2 + \cdots 1/(M-2)^2 \right]$$

M = # lags in the window

$B_2^{\,2} = 0$ when M=1

i.e. for M=1 the degradation in SNR is $1/\sqrt{2}$

for M=3 5% degradation

for M=7 1.7% degradation

for M=15 0.7% degradation

† See Chapter 5 - methods of Experimental Physics vol 12, part C -