VLBA ACQUISITION MEMO #218

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To: VLBA Data Acquisition Group

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Subject: The effect of DC offsets on the complex correlation amplitudes

VanVleck

The 2- and 4-level quantization used by the VLBA produces signal distortion. In the case of 2-level quantization the distortion of the spectrum is given by the well-known VanVleck correction. This relates the digital correlation function $P(\tau)$ to the analog correlation $R(\tau)$

$$P(\tau) = (2/\pi)\sin^{-1} R(\tau)$$

This non-linear function can be expanded as a series

$$\sin^{-1} R(\tau) = R + \frac{R^3}{6} + \frac{3R^5}{40} + ---$$

and in the case of 4-level there is a similar expansion with different coefficients.

Fringe rotation at the processor and fringe phase errors

Fortunately the fringe rotation at the processor greatly simplifies the effects of distortion introduced by the quantization non-linearity. For a continuum source and a rectangular bandpass equal to half the sample rate in width (see Thompson, Moran and Swenson, Equ. 9.19)

$$R(t,\tau) = a\cos(\phi + ft + \pi\tau/2)\ sinc\ (\pi\tau/2)$$

where τ is in unit of the Nyquist sample interval and f is the fringe frequency in radians/sec. Harmonics of the fringe frequency are generated by the quantization and become significant for highly correlated signals since R ~ 1 for this case. However, harmonics of the fringe frequency are filtered out in the processor and we are only interested in the fundamental. For the N^{th} harmonic we have complex frequency terms given by

$$(e^{i(\phi+ft)} + e^{-i(\phi+ft)})^N$$

for which the terms at the fundamental are of the form

$$b(N)(e^{i(\phi+ft)} + e^{-i(\phi+ft)})$$

where b(N) is a real function and hence the interferometer phase ϕ is unchanged and is independent of the size of the coefficients in the expansion of the non-linearity.

Since the interferometer phase is unchanged the closure phases for non-zero fringe rates are not affected by the quantization except in systems with high dispersion, in which case, second order closure errors can result from the amplitude errors.

Fringe amplitude errors

For the continuum source with a flat bandpass the normalized fringe amplitude is given by

$$A = \int_{-\infty}^{+\infty} \int_{0}^{\pi/2} \sin^{-1}[a \cos(\theta+\pi\tau/2)\, sinc(\tau/2)]\, sinc\,(\pi\tau/2) \cos\theta \; d\theta \; d\tau$$

where a = normalized analog correlation coefficient

A = measured correlation coefficient

This integral has been numerically computed and is plotted as the solid curve in Figure 1. The curve shows the error relative to a perfectly linear relationship as a function of the analog correlation. Amplitude errors are under 1% for 30% correlation and we only have to be concerned about very strong continuum sources on the shortest baselines and certain spectral line cases. In the high correlation cases corrections will have to be made for accurate closure amplitudes. Alternately 4-level quantization could be used for which the maximum amplitude error (at 100% correlation) is only about 5%. The dotted curve of Figure 1 shows the "modified" VanVleck correction

$$P(\tau)=(2/\pi)(4/\pi)\int_{0}^{\pi/2} \sin^{-1}(R(\tau)\cos\theta)\cos\theta d\theta$$

needed for correction of spectral line data. Again the relation is shown as a deviation from the linear one for small correlation. In the spectral line case a simple adjustment of the amplitudes will be inaccurate. In the case of the FX correlator the cross-spectrum should be transformed to the cross-correlation, the modified ("rotated") VanVleck correction applied, and then transformed back to the cross-spectrum. For comparison, Figure 1 also shows the standard VanVleck correction required when fringe rotation is done before quantization.

The effect of D.C. offsets

A D.C. offset does affect the quantization function but as we have seen this only affects the fringe amplitudes and not the phases. Post quantization fringe rotation eliminates phase errors and results only in amplitude errors. For a D.C. offset error of 10% the effective system temperature will be increased by 1% and the resulting quantization errors from a 1% correlation (10% D.C. at each site) is entirely negligible. The most serious consequence of large D.C. offsets will be the D.C. fringe rate sidelobes which will produce spurious signals at very low fringe rates.

Comparison with Correlator Memos #68 and 75

These more quantitative results are consistent with the concepts expressed in Correlator Memo #68. Fred Schwab has examined the best linear fit for the 4-level quantization and gets an error of 6.4% at 100% correlation. For a 100% correlation with rotating fringes only about half the time is the correlation at 100% and hence the maximum amplitude error will be about 3% in this case. Clearly, if we wish to avoid the complexity of quantization corrections, 4-level is much superior to 2-level. In either case, D.C. offsets should be no problem unless they are extremely large.

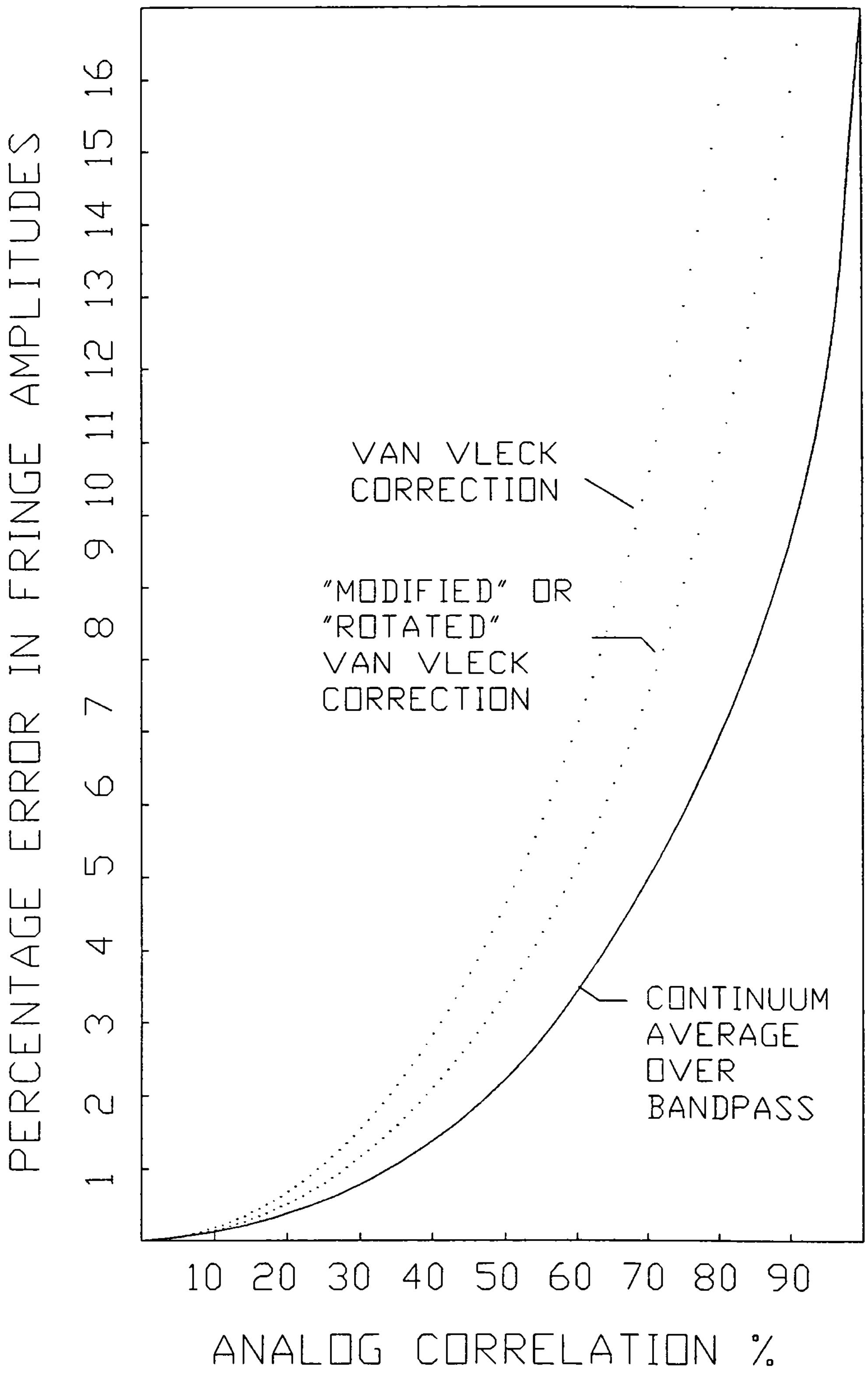

FIGURE 1

AMPLITUDE ERRORS INTRODUCED FOR 2-LEVEL QUANTIZATION WITHOUT VAN VLECK CORRECTION