VLBA ACQUISITION MEMO #228

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To: VLBA Acquisition Group

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Subject: Tape winding and tape thickness non-uniformity

A variation of the tape thickness across the tape can lead to severe packing/winding problems. A severe packing instability can arise if the tape is thicker at the edges. In this case the tape pack will curl up at the edges and eventually reach an unstable condition. Figure 1 shows the basic geometry. The radius of the $(i+1)_{th}$ turn is given by

$$r_{i+1} \approx r_i + t\,(1 - P_r/Y_r - \sigma S_t + \mu)\,(1 + \theta_i^2/2) \tag{1}$$

where	r_i	=	radius of i^{th} turn
	t	=	tape thickness
	P_r	=	interlayer or radial pressure
	Y_r	=	Young's modulus in radial (thickness) direction
	S_t	=	Strain in tangential (along tape) direction
	σ	=	Poisson's ratio
	Θ_i	=	slope of the i^{th} turn
	μ	=	fractional increase in thickness at edge of tape

Away from the edge of the tape $\Theta_i = 0$ and $\mu = 0$, so that

$$y_{i+1} \approx y_i + t(\mu - p_r / Y_r - \sigma s_t + \theta_i{}^2/2) \tag{2}$$

where

y	=	increase in radius at edge
p_r	=	increase in radial pressure at edge
s_t	=	increase in tangential strain at edge

The curling of the edge will increase with an increasing number of turns and become unstable unless

$$\mu - p_r/Y_r - \sigma s_t + \theta_i{}^2/2 < 0 \tag{3}$$

In order to carry the analysis further, I assume the simple shape of the edge curl shown in Figure 1. In this case

$$y \sim \theta\ l \tag{4}$$

where

l = scale length for edge curl

Also

$$S_t \sim y/r \tag{5}$$

and if we neglect the bending torque and assume the radial pressure results in tangential tension of **n** turns

$$p_r \sim Y_t\ (y/r)\ (t/r)\ n \tag{6}$$

so that the stability criteria of (3) becomes

$$\mu - \left(\frac{Y_t}{Y_r}\right)\left(\frac{\theta l t n}{r^2}\right) - \left(\frac{\theta l \sigma}{r}\right) + \theta^2/2 < 0 \tag{7}$$

or

$$\mu \leq \theta l \frac{\sigma}{r} + \frac{Yt}{Yr}\ \theta l t \frac{n}{r^2} - \theta^2/2 \tag{8}$$

which is of the form

$$\mu \leq K\theta - \theta^2/2 \tag{9}$$

This can be satisfied for some range of Θ if

$$\mu \leq K^2/2 \tag{10}$$

Substituting values of

(Yt/Yr)	= 8 (for smooth PET film)
l	= 0.05" (observed)
n	$\approx$ r/t
r	= 6"
σ	= 0.3

gives $\mu \leq 0.2\%$

Returning to the question of the restoring force exerted by bending. For a deflection Θ over length l the pressure is

$$(\theta Y_t t^3) \,/\, (4.\, l^3) \tag{11}$$

which is smaller than the radial pressure from tangential tension. If the radial restoring pressure due to bending is equated to that due to the tangential tension in one turn the scale length for the edge curl is given by

$$l = \sqrt{tr/2} \tag{12}$$

$\sim$ **1000 μm for t = 16 μm, r = 15 cm**

which is close to the observed scale. Substitution into equation (10) gives

$$\mu \leq ((Y_t \,/Y_r)^2 \; t \;/(2r)) \tag{13}$$

The limit of $\mu < 0.2\%$ for stability may be somewhat less stringent in practice because at low interlayer pressures the effective radial modulus is often greatly reduced by surface roughness and air entrapment (see Willet and Poesch, Journal of Applied Mechanics, June 1988, Vol. 55/365). For $(Y_t/Y_r) = 40$, the non-uniformity limit becomes 6%. Because of the extreme uncertainty in the effective radial modulus it is hard to make accurate predictions from the theory, but the predicted trends are:

1] Thicker edges can result in packing instability.

2] For a given non-uniformity the instability can be avoided with a lower radial modulus from more air entrapment or rougher backcoat.

3] It is more difficult to attain stability at large pack radii.

4] An induced edge curl due to an uneven splice, dirt, or other causes can produce a "propagated" instability via the $-\Theta^2/2$ term of inequality (8) even with a uniform tape.

It is suspected that some tapes have thicker edges due to rough slitting and/or debris on the edges and as the above analysis suggests, this then leads to an unstable pack or "cinching" as it is sometimes called.

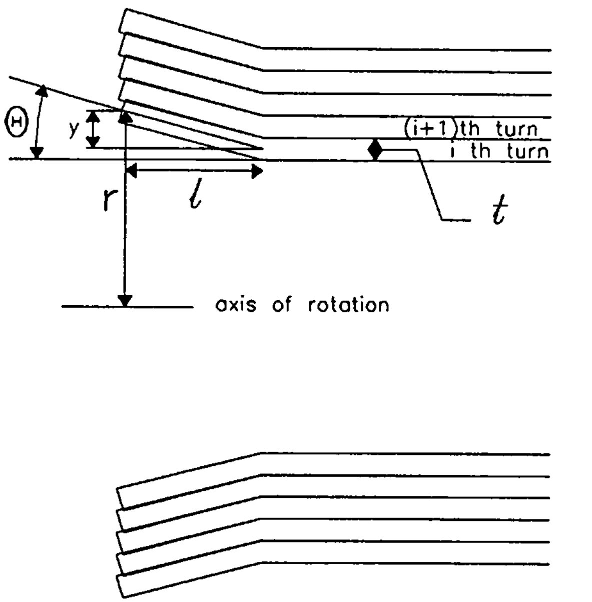

Figure 1. Geometry of pack

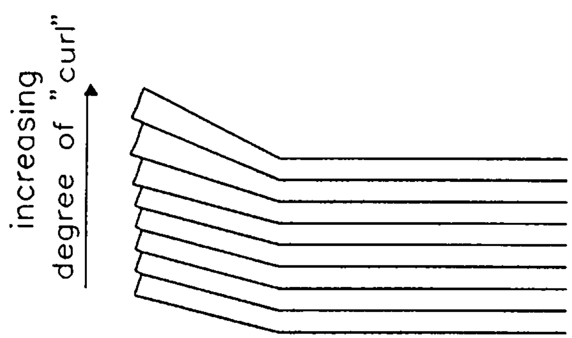

Figure 2. Edge instability

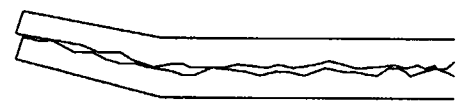

Figure 3. Illustration of how surface roughness can prevent edge curl instability by increasing the effective modulus ratio – see below

Theory:

For stability $\mu \lesssim \left[(Yt/Yr)\ l/r\right]^2$

where

μ = non-uniformity

(Yt/Yr) = ratio of tangential to radial elastic modulus

l/r = curl scale / reel size $\simeq \sqrt{t/(2\,r)}$

A MECHANISM FOR TAPE PACK INSTABILITY