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The role of antenna size in selfcalibration

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In recent discussions concerning the VLBA there has been some uncertainty over the optimum choice of antenna size for selfcalibration. Of course, in the lack of any constraints we would choose sufficiently big antennas. Suppose that, instead, we are limited to a given total collecting area of A, say. An interesting question concerns the number of antennae, N, that comprise this area. There are two aspects to this question. First, how does the noise behaviour of selfcalibration vary with N ?, and secondly, how does the u,v coverage vary with N ? The second point is somewhat obvious and I will not consider it further. To proceed with the first aspect we will have to describe the noise behaviour of selfcalibration.

The basic equation of selfcalibration [2] expresses the relationship between the observed and true Fourier components $V_{i,j}$,$T_{i,j}$ between the antennae i and j. If the complex gain error of the i'th antennae at a given time is g_i then :

$$V_{i,j} = g_i . g^*_j . T_{i,j} + e_{i,j} \qquad (1)$$

where $e_{i,j}$ is an error term due to the receiver noises in each of the antennae.

The noise behaviour of selfcalibration is extremely complicated to calculate, mainly because of the use of the CLEAN algorithm to deconvolve the image and to reject those parts of the trial image which we do not believe. Some simple statements can be made when the object of interest is reasonably unresolved [1]. The r.m.s. gain error per antenna is:

$$\sigma_g = \sigma_e/(F.(N-2)^{1/2}) \qquad (2)$$

The antennae are assumed to be identical. F is the flux of the point source and σ_e is the receiver noise (Jansky) in the integration time. Selfcalibration can only work if σ_g is much less than one in the characteristic time for changes in the gains. The numerical factor 2 should be replaced by 3 if gain amplitudes as well as phases are estimated. The asymptotic behaviour is as the inverse of the square root of the number of telescopes; again, this agrees with intuition. The corresponding expression for an extended source is much more complicated and depends in part upon the unknown noise behaviour of the CLEAN algorithm.

In terms of the total collecting area we have that :

$$\sigma_e \propto N/A \qquad (3)$$

$$\sigma_g \propto N/(A*(N-2)^{1/2}) \qquad (4)$$

which, for large N, goes as the square root of N. Selfcalibration is possible if this is much less than unity. Therefore, the antenna size has an effect upon the selfcalibration of a point source, large antennae being preferable. Note however that this dependence is equivalent to the inverse of the antenna diameter. This means that the gain on increasing from 25m to 32m diameter antennae is only about 28%.

The final noise in a dirty map of this nearly unresolved source is approximately that which would be obtained with perfect calibration and one less antenna [1] :

$$\sigma_{map} \alpha \ (N/(N-2))^{1/2}.A^{-1} \qquad (5)$$

Again, this is easy to understand. Conceptually, we can simply calculate the calibration relative to one antenna, number 1, say :

$$g_i \sim V_{i,1}/F \qquad i \neq 1 \ , \ g_1 = 1 \qquad (6)$$

Baselines involving antenna 1 contribute nothing to decreasing the noise and therefore (N-1) baselines are missed.

For extended sources this whole story gets much, much more complicated since the simple calculations made here are not applicable [1]. Fortunately, some points seem obvious : in the VLA extended structure can, to some extent, be bypassed by only making gain solutions from a restricted data set. Thus, given optimum control of selfcalibration, the results above should be reasonably relevant. The relative sparsity of u,v plane coverage of the VLBA makes this selection of a range of baselines to be used rather more difficult. For a large (> 10) telescopes a very simple argument can be used to establish another approximate result. Suppose that the fifty percentile correlated flux is F, then, with proper selfcalibration (i.e. with a signal to noise sensitive algorithm [2,3] and a well chosen model) and in non-pathological situations, the error in the gain estimate for a typical antenna should be :

$$\sigma_g \leq 2^{1/2}.\sigma_e/(F.(N-2)^{1/2}) \qquad (7)$$

Given that this error is much less than unity then the noise amplification should be similar to that in the quasi unresolved case.

Finally, it may seem that antenna size has some indirect effect on selfcalibration in that for small antennae and, therefore, large fields of view, confusion may become a problem. However, both the field of view, measured in beams, and the number of independent visibility measurements go as the square of N. Therefore the number of points to be estimated keeps level with the number of constraints. Note that non-isoplanicity may well become a problem if the antenna size is too small (e.g. as occurs in the proposed 75MHz addition to the VLA). In the VLBA bandwidth smearing will dominate both these effects anyway.

It seems, therefore, that for optimum signal to noise in selfcalibration the total collecting area should be shared amongst a small number of antennae. This would, however, provide very poor u,v plane coverage and would hinder successful selfcalibration. The trade off point therefore depends on the emphasis given dynamic range as opposed to sensitivity.

References :

1. Cornwell,T.J.,VLA scientific memorandum 135, 1981.
2. Schwab,F.R.,SPIE,231,18-24, 1980.
3. Cornwell,T.J. and Wilkinson,P.N.,M.N.R.A.S.,196,1067-1086, 1981.