VLB ARRAY MEMO No. 487

PHASE CALIBRATION TECHNIQUES

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1. INTRODUCTION

The phase part of the complex visibility measured by an interferometer may be considered to determine the difference in arrival times of a wavefront at the two antennas. (For a distributed source, there exists a collection of wavefronts, and these may be sorted out by Fourier synthesis; here we consider only a single component of the source.) From measurements of this time difference for various sources, the directions of arrival (source positions) and/or the baseline vector may be deduced. It is useful to work in time units rather than phase angles, because errors in the time measurement are directly related to direction and baseline errors, independent of observing frequency.

There are two rather different ways to determine the arrival time from the visibility measurement: one can measure the phase at one frequency, $\phi(f)$, or one can measure the slope of the phase as a function of frequency, $d\phi/df$. Dividing the first by the angular frequency $2\pi f$ gives the time as the "phase delay," except for an additive ambiguity of an integral number of periods at frequency f. The second (over 2π) is the "group delay." In principle, both will give the same result; but they are subject to very different errors. When the measurement errors are dominated by thermal noise, the phase delay is usually very much preferred. This is because determining the group delay requires phase measurements at two separated frequencies; if the total bandwidth is fixed, it must be split between the two frequencies, giving a factor of 2 more noise in the phase difference than in the single measurement for the phase delay. Furthermore, the group delay error is scaled by the reciprocal of the frequency separation, whereas that of the phase delay is scaled by the reciprocal center frequency, and the latter is usually much smaller.

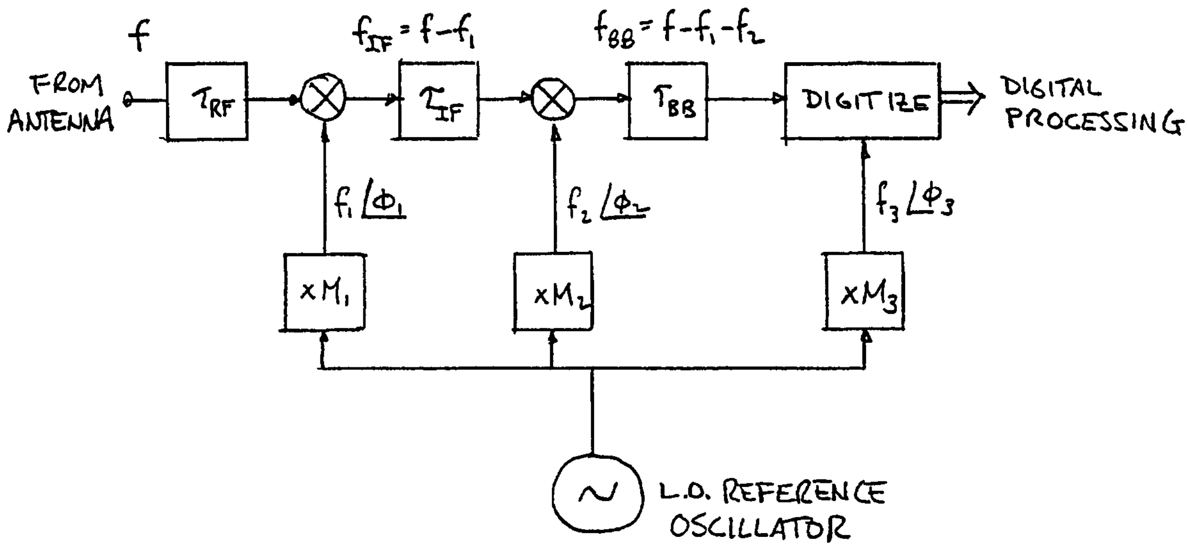

Figure 1: Receiving system model.

However, often the errors are not dominated by thermal noise, but rather by errors in knowledge of the delays in the signal processing electronics. To study this in detail, consider the receiving system model of Figure 1. This is representative of the VLBA electronics, and is typical of many other radio telescopes. The model includes two frequency conversions prior to digitization, with each conversion being from the upper sideband, but this could be generalized in an obvious way. The signal frequencies will be called RF, IF, and baseband (BB). Three local oscillators are required (including the sampling clock), and they are assumed to be derived from the same reference oscillator. When used as half of an interferometer, this receiving system produces a phase shift for input frequency f of

$$\phi(f) = 2\pi[f\tau_{RF} + (f - f_1)\tau_{IF} + (f - f_1 - f_2)\tau_{BB}] - \phi_1 - \phi_2 - \phi_3$$

and thus it affects the phase delay by

$$T_\phi = \frac{\phi(f)}{2\pi f} = \tau_{RF} + \frac{f - f_1}{f}\tau_{IF} + \frac{f - f_1 - f_2}{f}\tau_{BB} - \frac{\phi_1 + \phi_2 + \phi_3}{2\pi f}$$

and the group delay by

$$T_g = (1/2\pi)\frac{d\phi}{df} = \tau_{RF} + \tau_{IF} + \tau_{BB}.$$

Notice that the phase delay is much less sensitive than the group delay to instrumental delays at IF and baseband, provided that $f_{IF} = f - f_1$ is much less than f. On the other hand, the group delay is insensitive to the local oscillator phases.

2. ASSUMPTIONS

To proceed further we will need to make some assumptions. First, assume that the total values of T_ϕ and T_g will be determined by astronomical calibration at regular intervals of time. Errors are made only to the extent that these instrumental delays are unstable, so that they vary between calibration observations. Since the latter are time consuming, we desire not to do them too often. We can then consider how to design the equipment to be sufficiently stable; or alternatively, how to include a means of rapidly measuring the variation of instrumental delays so as to correct for them.

Next, we make some reasonable estimates of the relative values of the instrumental variables. Let us neglect variations in the RF delay, on the ground that it is likely all in a stable, cryogenic receiver. Let the IF delay include a long transmission line of delay τ_ℓ whose temperature coefficient of delay is 58ppm/C (value for coaxial lines with PTFE dielectric); and bandpass electronics with P_2 poles, bandwidth B_2, and temperature coefficient (23ppm/C)P_2/B_2 (value for aluminum). Then let the temperature of the transmission line vary by 20 C and that of the electronics by 2 C, and suppose that temperature is the main perturbation. Similarly, let the baseband delay have temperature coefficient (23ppm/C)P_3/B_3 with temperature changes of 1 C. This gives

$$\text{std}(\tau_{RF}) = 0,$$
$$\text{std}(\tau_{IF}) = .00116\tau_\ell + 4.6 \times 10^{-5} P_2/B_2,$$
$$\text{std}(\tau_{BB}) = 2.3 \times 10^{-5} P_3/B_3.$$

A reasonable estimate of the state of the art in local oscillator multiplier chains is that the output phase is stable to .01 radian/GHz. To this must be added the phase variation of the reference oscillator, scaled to the LO frequency. The latter is best expressed in terms of the Allan standard deviation $\sigma(t)$, and depends on the time interval t between measurements. The result is

$$\text{std}(\phi_i) = [.01\text{rad/GHz} + 2\pi t\sigma(t)]f_i.$$

3. VALUES FOR THE VLBA

In order to make some quantitative estimates, we now substitute values for the parameters that correspond roughly to those of the VLBA. We expect an IF transmission line of about 30 m, or $\tau_\ell = 100$ nsec.

Assume P_2=10 poles in the IF bandpass and P_3=7 poles at baseband, with bandwidths of 500 MHz and 8 MHz respectively. For concreteness, take f=8.4 GHz and f_{IF}=840 MHz. This gives

$$\mathrm{std}(\phi_{LO}) = 0.15\mathrm{rad} + 2\pi f t\sigma(t)$$

$$\mathrm{std}(\tau_{IF}) = 116\mathrm{psec}$$

$$\mathrm{std}(\tau_{BB}) = 20\mathrm{psec}.$$

where ϕ_{LO} is the total local oscillator phase, and the phase variations of the multiplier chains have been combined as the RSS. The errors in the measured delays are then

$$\mathrm{std}(T_\phi) = 11.9\mathrm{psec} + t\sigma(t)$$

$$\mathrm{std}(T_g) = 118\mathrm{psec}$$

where all variations have been combined as RSSs except (pessimistically) the reference oscillator stability. For small values of t, both the group delay and phase delay errors are dominated by the IF delay variation, but the phase delay error is ten times smaller. At larger t, the group delay error remains the same but the phase delay gets worse at a rate depending in the stability of the reference oscillator, which eventually dominates.

4. CORRECTION METHODS

It has been suggested (VLBA EM#36, AM#37) that the group delay through the entire receiver might be measured by injecting stable, periodic pulses into the input and coherently detecting them after digitization. Indeed, it is easily shown that for a pulse power equal to 1% of the total power the phase can be measured to .01 radian in only 1 sec at 4 MHz bandwidth. However, the method is limited by the accuracy to which the pulse timing can be maintained. Such a system is used in the Mark III VLBI terminals, and that design provides our best information about the achievable accuracy. It relies on a very stable pulse generator and on transmission of a 5 MHz reference signal via coaxial cable, where the length of the cable is continuously monitored by a round-trip phase measurement. The quoted errors (Mark III Documentation Manual) are

Pulse generator: temperature coefficient $\leq$ 6 ps/C

Cable measuring electronics: stability $\leq$ 10 psec/hour; linearity $\leq$ .05%.

Assuming that the pulse generator temperature can be controlled to 0.5 C, and that the linearity error refers to one period of the 5 MHz signal being measured, the latter dominates for periods up to several hours. The RSS error is then about 100 psec. Note that this corresponds to a phase accuracy of .003 radian at 5 MHz.

Since the main variation in delay is in the IF transmission cable, other methods of monitoring this cable length should be considered. The VLBA IF cables will be identical to the LO reference cable, and all will be bundled together. The LO cable will transmit a 500 MHz reference signal, and a round trip phase measurement on this signal will monitor the line length variations. At this frequency, a measurement accuracy of only .02 rad (1 deg) corresponds to 5.6 psec, and this is easily achievable. The length variations of the IF cables may not accurately track that of the LO cable; to allow for this, let us assume a tracking error of 10% of the total variation, or 11 psec (based on 20 C temperature change for 30 m of PTFE coax).

The corrections thus have the following estimated results:

	Uncorrected	*Pulse Cor.*	*LO Cor.*
IF cable	110psec		12psec
IF electronics	9.2		9.2
BB electronics	20		20
RSS	118	100	25

Combining this with the results of the previous section gives:
Pulse correction:

$$\mathrm{std}(T_\phi) = 100\mathrm{psec} + t\sigma(t)$$

$$\mathrm{std}(T_g) = 100\mathrm{psec}$$

LO line correction:

$$\mathrm{std}(T_\phi) = 3.2\mathrm{psec} + t\sigma(t)$$

$$\mathrm{std}(T_g) = 25\mathrm{psec}$$

Notice that the pulse correction makes the phase delay error worse. This is because the phase delay is measured at a single frequency, where the correction must be applied as a phase shift; at the level of accuracy assumed here, the apparent phase shift will be mostly attributable to errors in measurement of the pulse generator reference line rather than actual phase variations in the signal path. In the group delay error, the pulse correction produces only a small improvement. Correction based on measurements of the LO line produces considerable improvement in both delay errors, except for the effect of the reference oscillator on the phase delay.

5. REFERENCE OSCILLATOR

The reference oscillator for the VLBA will be a hydrogen maser, and we expect it to meet the stability requirements of Specification A53308N001. The maximum Allan standard deviation curve from this specification is reproduced in Figure 2. Using this along with the results of the last section gives the delay error curves plotted in Figure 3. For times greater than 10,000 sec (not covered in the maser specification), two possible extrapolations are shown: the first assumes that the Allan standard deviation remains at 2E-15, and the second assumes that it degrades to 1E-14 at 100,000 sec.

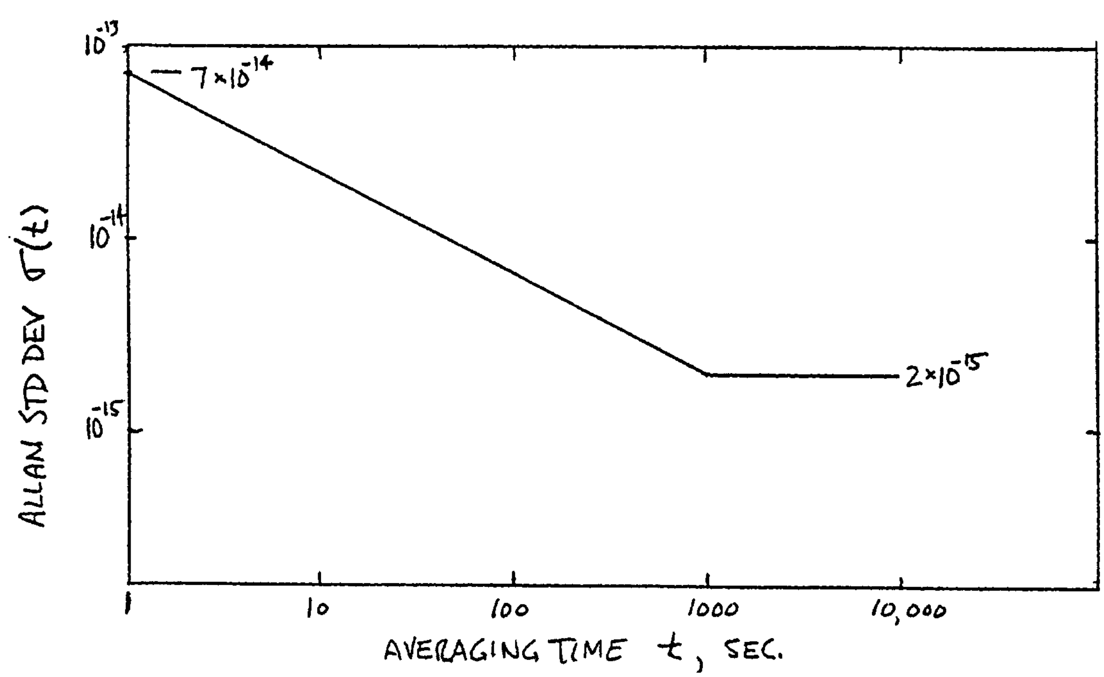

Figure 2: Maximum Allan standard deviation of maser oscillator, from VLBA Specification A53308N001.

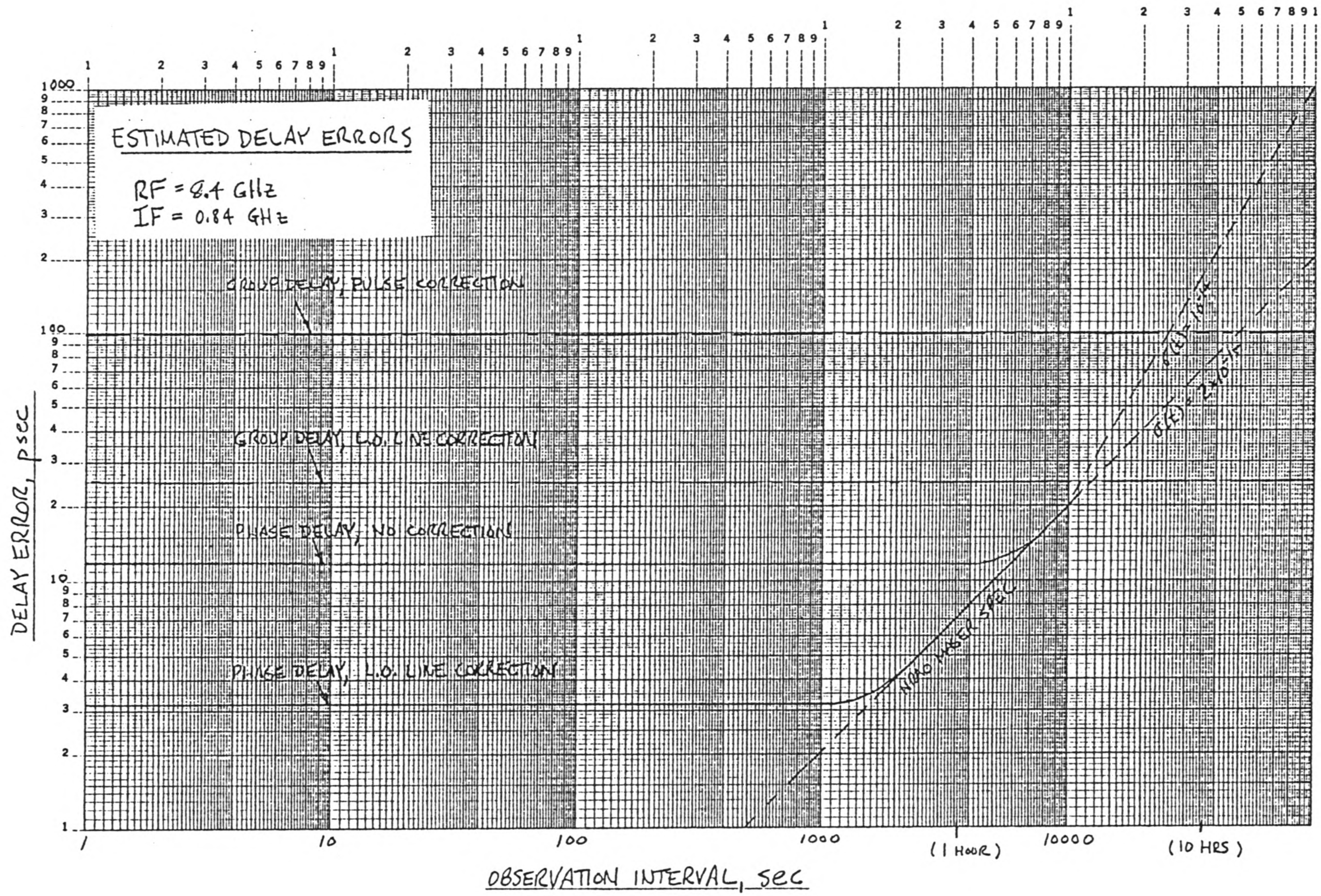

Figure 3: Estimated delay errors vs. observation interval.

6. CONCLUSIONS

For observations lasting 3 hours, and probably for observations lasting 10 hours, the VLBA will be sufficiently stable to allow phase delay measurements at the 100 psec level without corrections. Correcting for IF line length variations using measurements of the LO line should improve this to the 3 psec level for times up to 0.5 hour, degrading to 20 psec at 3 hours. In this time range, and for observing frequencies above 5 GHz, group delay measurements will be less accurate, with errors of about 25 psec after the same IF line correction. For longer observing times, the group delay error will remain the same but the phase delay error will continue to increase due to drift of the hydrogen masers.

It appears that phase delay measurements are preferred to group delay measurements for most observations with the VLBA. Of course, phase measurements are subject to lobe ambiguity. These results assume that such ambiguity will be resolved by coarser measurements, either at a lower center frequency or by using the group delay.

It also appears that delay corrections from measurements of pulses injected into the front ends will not significantly improve the group delay errors, based on the Mark III specifications. They certainly cannot improve the phase delay errors.

7. UNCERTAINTIES IN THIS ANALYSIS

Some of the assumptions used here are subject to question. The estimates of the intrinsic delay stability of the receiving system (without corrections) may be overly optimistic, in that some mechanisms of instability have not been explicitly considered. These include flexure of cables and the effects of mismatches. Nevertheless, it would be hard for the variations to be more than a factor of 10 larger, and that would not be enough to change the conclusions significantly.

The accuracy of the pulse injection scheme for delay correction was estimated from specifications in the Mark III documentation. This may be overly pessimistic, for a couple of reasons. First, the reference signal could be transmitted at a much higher frequency than the 5 MHz used in Mark III, resulting in a more accurate line length measurement. Second, if the range of variation of the reference line length is only about 100 psec, as calculated here, then the nonlinearity error may not be very significant. In that case, the error would be dominated by drift in the line measuring electronics, reducing the error from about 100 psec to a few times 10 psec. Assigning an error of 25 psec, we find that the method becomes competitive with using the LO cable length for corrections to the group delay, but that the phase delay measurement remains more accurate for observing times less than 3 hours.