4-Element Frequency Sequences

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When the CDP first started using the VLBA antenna at Pie Town, we were concerned about achieving reasonable bandwidth synthesis delay resolution functions (DRF) for both S- and X-band with a total of only 8 video converters, especially for the increased bandwidths which we wanted to use for the extended R&D experiments (ERDE). The standard CDP sequences use 6 and 8 channels at S- and X-band, respectively. These sequences give us good performance in terms of spanned bandwidth and ambiguity spacing. If fewer frequency channels are used, there has to be some trade-off between spanned bandwidth, ambiguity spacing, and sidelobe level of the delay resolution function.

Our original approach for Pie Town was to create a few extra channels by frequency switching some of the video converters. While I still think this is (theoretically!) the optimum solution, in practice it verges on dismal failure. The switching seems to overwhelm the computer at Pie Town (the new computer may help), and I surmise that it is not very popular at the correlators, either. It also causes us to lose some data, too.

So that we can get more data, more reliably, especially as we use more of the VLBA antennas, I have investigated the matter of frequency sequences again. I think there are some 4-frequency sequences that we should try experimentally. The drawbacks are smaller ambiguity spacings and/or higher sidelobes. The sequences that I propose we try have total lengths of 24 and 26 units. At S-band, they result in ambiguity spacings comparable to those we have already used for the ERDE and R&D90 experiments. (For these experiments, the 6-frequency sequence that we used had a length of 25.) At X-band, to span 700+ MHz, the new sequences have to be multiplied by factors of 30 or so, giving ambiguity spacings of 30 to 35 nanoseconds, compared to 50 nanoseconds for ERDE/R&D90. I think we can use such narrow spacings as long as we are careful.

I started from several assumptions:

1. There will never be more than 8 video converters at a VLBA site.

2. It is desirable to span 120-125 MHz at S-band and >700 MHz at X-band. These are the spans we used for the ERDE and R&D90 experiments.

3. The maximum tolerable sidelobe in the delay resolution function is about 90%, relative to the peak. This seems very high compared to our usual sequences (the Doug Robertson sequences: DSR6 and DSR8) which have sidelobe levels around 55%. However, for good SNRs (>20 or so, and >30 may be desirable), even sidelobe levels like these should not lead FRNGE astray, such that it finds the wrong

peak in the DRF. High SNRs should generally be easy to achieve with the VLBA antennas.

The 8 channels available at VLBA sites could be allocated 3 and 5 or 4 and 4 to S- and X-band. The longest sequences with only 3 frequencies that have sidelobes less than about 90% are only 6 or 7 units long. To span 120 MHz at S-band with such a sequence requires a large multiplier: between 17 and 20. The ambiguity spacing would be between 50 and 60 nanoseconds. At S-band, this is too small for easy, reliable ambiguity resolution. Therefore, we are driven to a 4 and 4 split between the two bands.

In order to keep the ambiguity spacings large, at S-band in particular, I have targeted multipliers of 5 or 6 for the S-band sequence. To span 120-125 MHz, this means sequences of length of order 20 to 25. Sequences with these lengths will also be useful at X-band, where higher multipliers can be tolerated because the effects of the ionosphere go down as frequency squared.

To search all possible sequences, I wrote a small computer program that calculated the DRF for all possible 4-frequency sequences of a given length. This is really straightforward and not too much of a computational burden: the end frequencies are fixed at 0 and the maximum, while the inner two frequencies step over all possible remaining pairs. Only half the DRF needs to be calculated, since it is symmetric about zero delay. I then searched each DRF for the peak sidelobe, and then sorted by sidelobe level for the various sequences. This procedure took less than a minute on my PC for each sequence, up to length 26. From the sorted list, I then looked for "good" sequences.

Good sequences have low sidelobe levels and a large rms frequency span. It is the rms frequency span which determines the formal error on the group delays (σ_τ). Larger spans give smaller errors:

$$\sigma_\tau = \frac{1}{2\pi \ \nu_{rms} \ \mathrm{SNR}}$$

The ratio of rms frequency span to actual maximum frequency span ("frequency ratio" or *r* hereafter) ranged from a bit less than 0.36 to a bit more than 0.44. For comparison, a uniformly filled "aperture" (to use the spatial frequency domain terminology) has a frequency ratio 0.289, while an array consisting of only two elements has a frequency ratio of 0.5. I used the Quattro Pro spreadsheet program to calculate the frequency ratio and to plot the DRF. Sequences with large ratios have the frequencies concentrated towards the two ends of the sequence, *i.e.* the sequence 0-1-19-20 has the best ratio of 0.476 for all sequences of total length 20 (but it has a truly lousy sidelobe pattern: sidelobes larger than 98% of the peak), while 0-7-12-20 has a small ("poor") ratio of 0.365 (but about as good a sidelobe pattern as exists for length 20: maximum sidelobe is ~84% of the peak).

I searched maximum lengths from 20 to 26. At all of these lengths, I concentrated on sequences with frequency ratios larger than 0.4. The best sequences I found are 0-1-19-24 (r = 0.444 with 88% sidelobes) and 0-3-22-26 (r = 0.438 with 89% sidelobes).

Figures 1 and 2 show the DRFs for these two sequences. In the figures, except as noted, the delay scale assumes 1 MHz unit cell spacing for the frequency sequences. With 1 MHz unit cells, the delay ambiguities are 1000 nanoseconds. For a given multiplier, the delay ambiguities are reduced by a factor equal to the reciprocal of the multiplication factor. *i.e.* for a factor of 5 multiplication, the delay ambiguity becomes 1000/5 = 200 nanoseconds.

At S-band, these sequences could be multiplied by 5 to yield total spans of 120 or 130 MHz, similar to the 125 MHz span used for ERDE/R&D90, with 200 nanosecond ambiguities. The rms frequency spans would be 53.3 and 56.9 MHz, respectively. For comparison, the 6-frequency sequence we used for ERDE had an rms span of 50.7 MHz. Either of the 4-frequency sequences would give slightly smaller formal group delay errors. For stations with 6 video converters, the 0-1-19-24 sequence could be filled in to give the sequence 0-1-8-19-22-24. This is a zero redundancy sequence, with a frequency ratio of 0.407 and maximum sidelobes of about 65%. (See Figure 3.) The 0-3-22-26 sequence can be filled in to give the sequence 0-1-3-10-22-26. This is also a zero redundancy sequence, with a frequency ratio of 0.394 and maximum sidelobes of about 65%. (See Figure 4.) The rms spanned bandwidth for the 6-frequency length 24 sequence multiplied by 5 would be 48.84 MHz, compared to 51.22 MHz for the 6-frequency length 26 sequence and 50.7 MHz for the sequence we used for ERDE. The differences in effective bandwidth are minimal. The relative effect on the formal error of ionosphere-corrected group delays would be less than one picosecond.

At X-band, the 4-frequency sequences could be multiplied by 30 (times 24) for a total length of 720 MHz (same as for ERDE) and rms bandwidth of ~320 MHz, and 28 (times 26) for a total length of 728 MHz and rms bandwidth of ~319 MHz. The rms bandwidth for the ERDE sequence (DSR8 multiplied by 20) is only 280 MHz. Thus, the 4-frequency sequences actually result in more precise group delays (at the penalty of much higher sidelobe levels in the DRF)! The ambiguity spacings would be 33.3 and 35.7 nanoseconds.

For use at stations with all 8 video converters, the 4-frequency sequences can be augmented to give sequences of 0-1-2.5-7-15-19-23.5-24 and 0-1-3-9-17-22-25-26. The first of these really has unit cells of 0.5 MHz. This means that its ambiguities are 2000/30 = 66.666... nanoseconds. The second sequence, if multiplied by 28, has an ambiguity spacing of 35.7 nanoseconds. Since the ambiguity at X-band for ERDE was 50 nanoseconds, the first of these sequences is no problem in that respect. An ambiguity spacing of 35.7 nanoseconds should not be too much trouble, either, as long as care is taken that good *a prioris* are used in CALC. Figures 5 and 6 show the DRFs for these sequences. The frequency ratios are 0.393 for the first sequence, and 0.394 for the second. These are comparable to the 0.389 ratio for the ERDE sequence.

I suggest that we try the 0-1-19-24 sequence and its derivatives for the last R&D experiment this year. The sidelobes of the DRF are a little better for this sequence, so FRNGE will not be quite as susceptible to finding an incorrect peak in the DRF as it would be with the length 26 sequence.

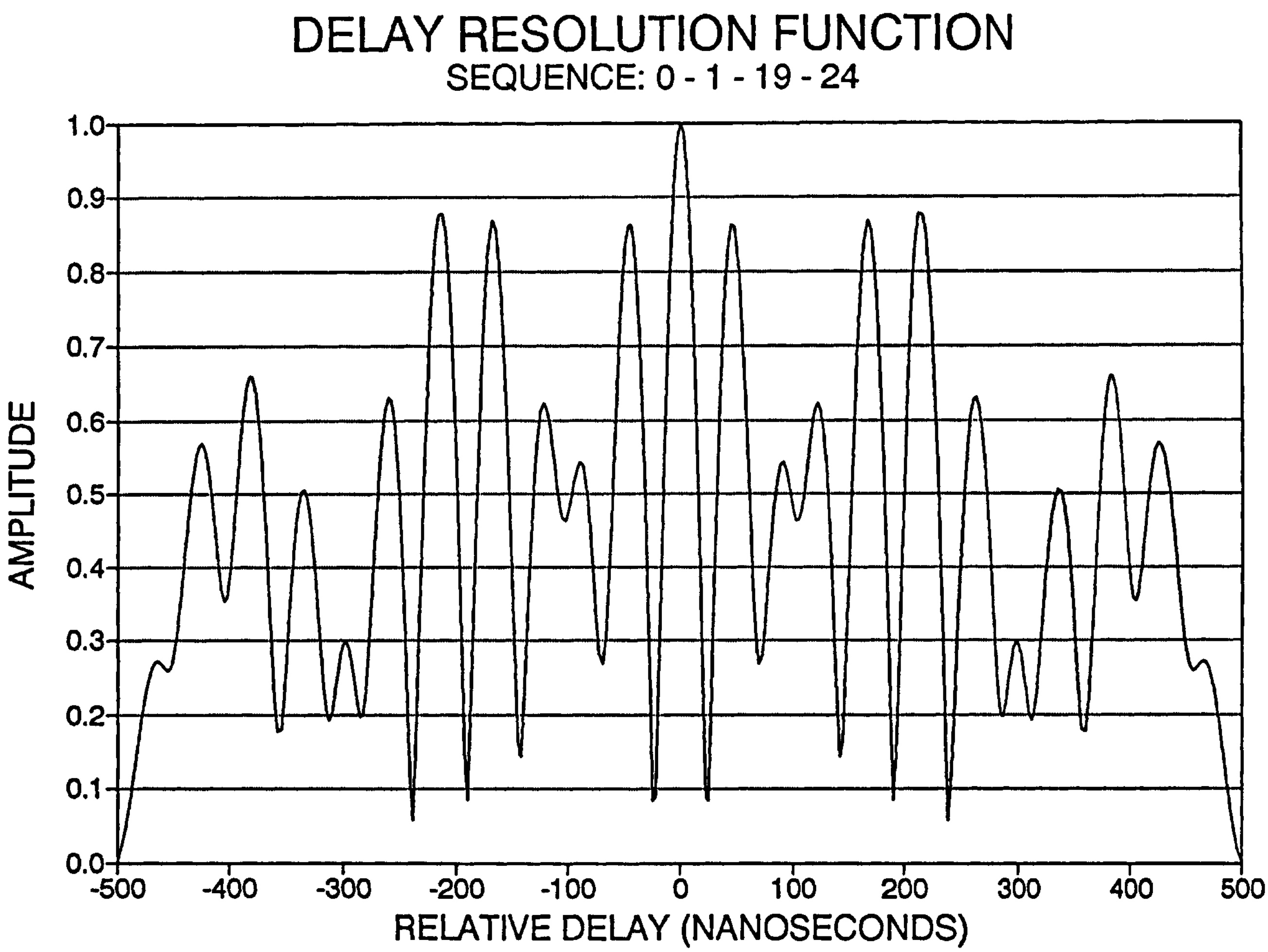

Figure 1. Delay resolution function for sequence 0-1-19-24. Frequency ratio = 0.444.

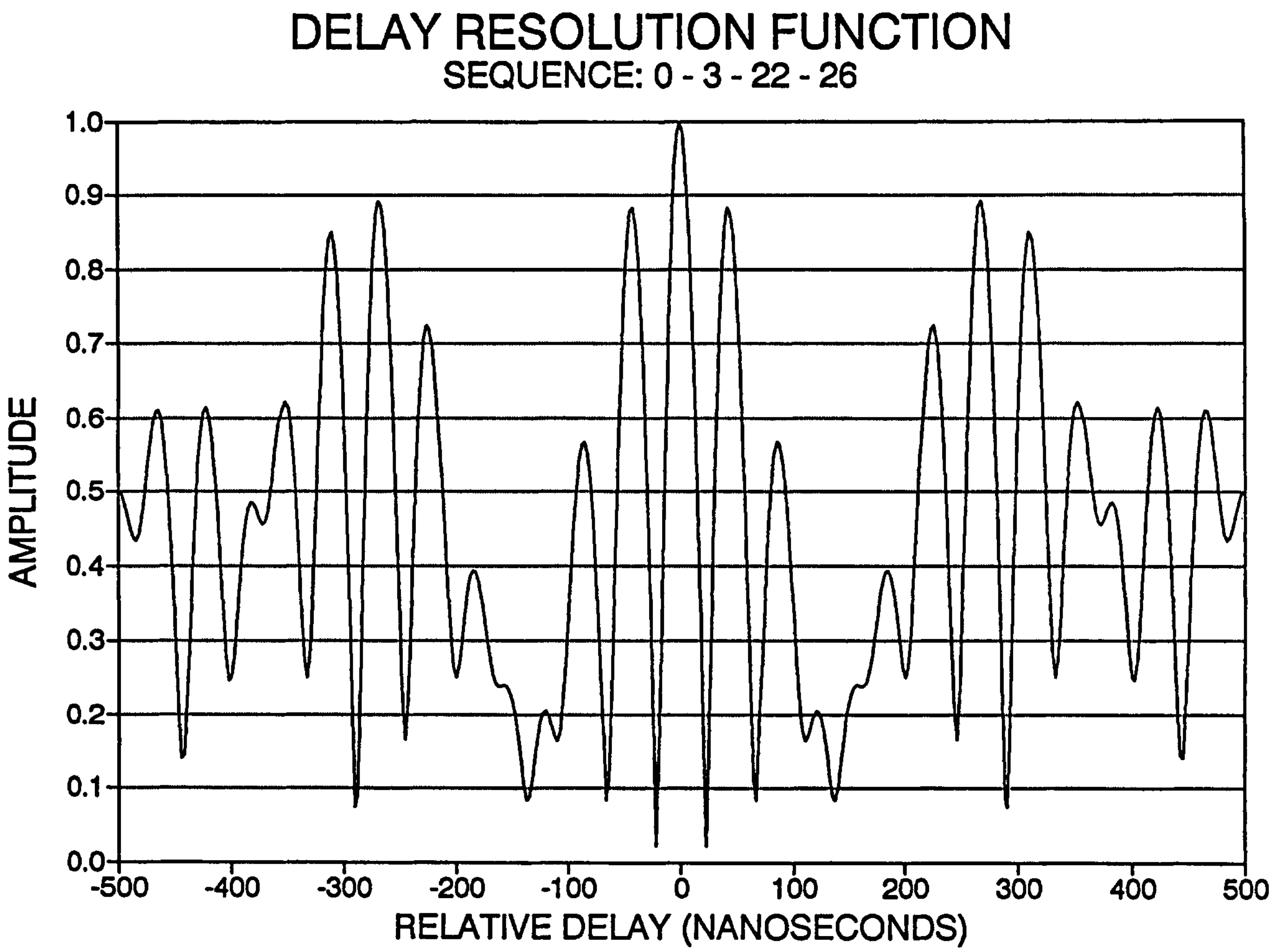

Figure 2. Delay resolution function for sequence 0-3-22-26. Frequency ratio = 0.438.

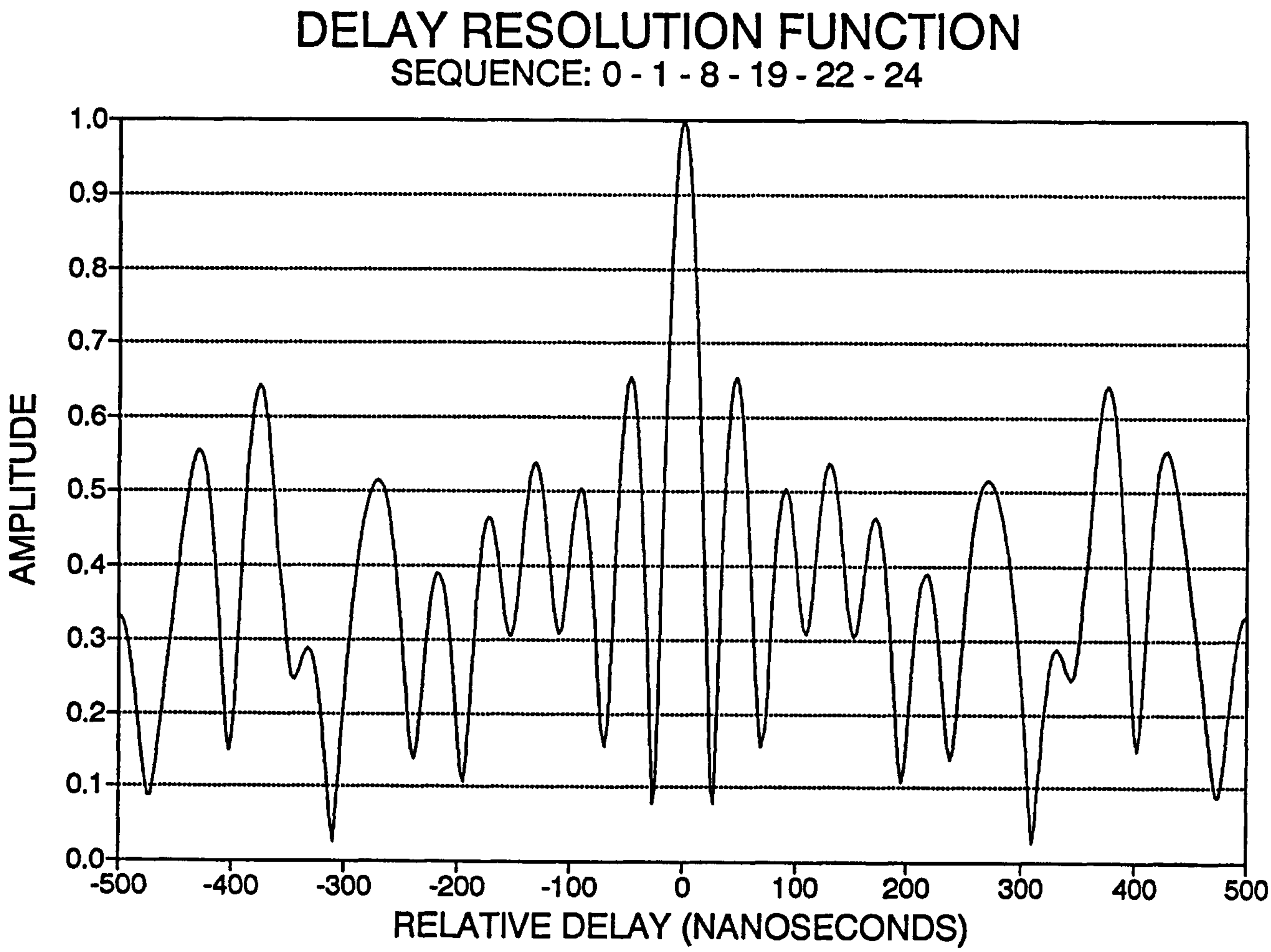

Figure 3. Delay resolution function for sequence 0-1-8-19-22-24. Frequency ratio = 0.407.

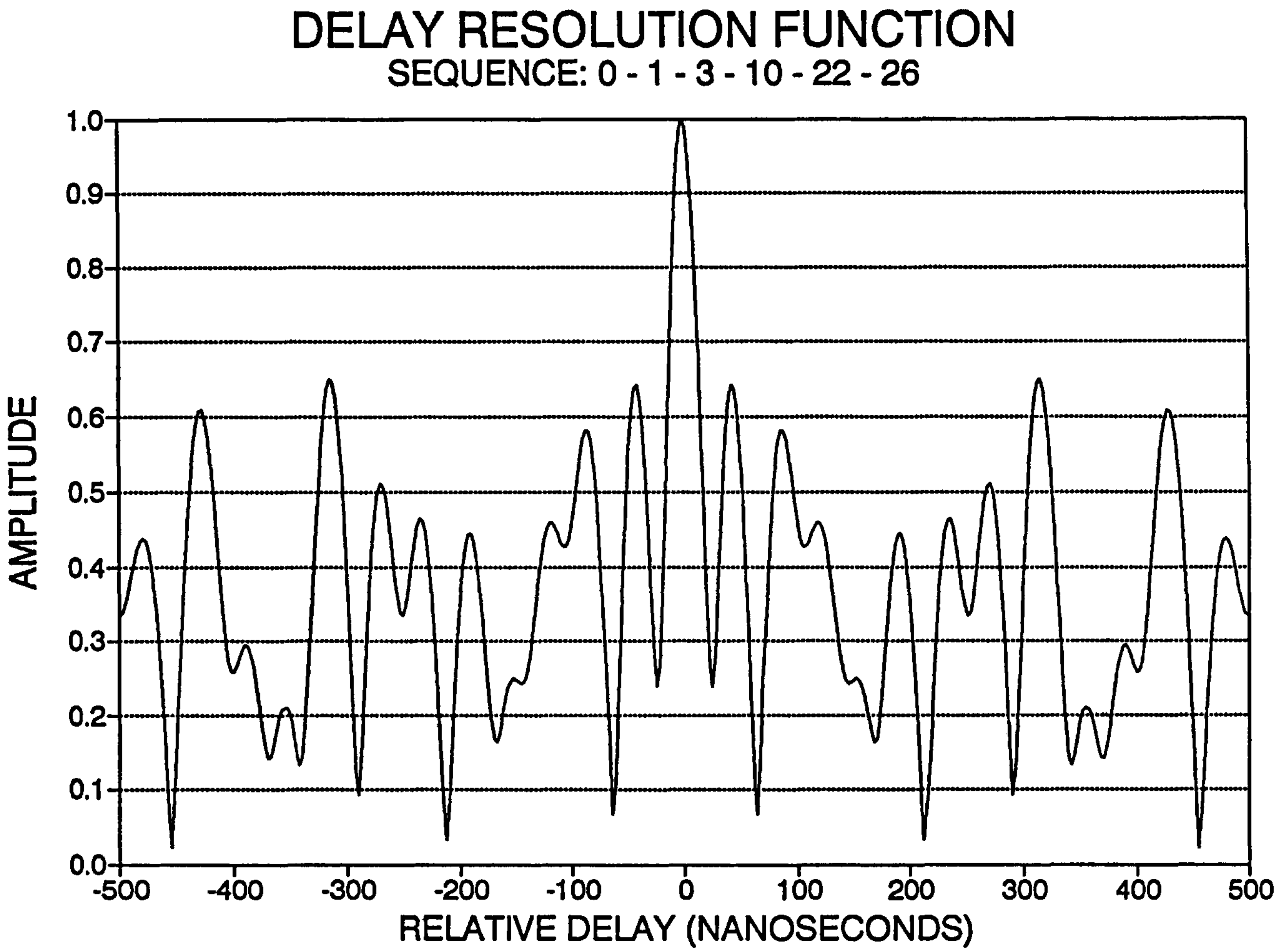

Figure 4. Delay resolution function for sequence 0-1-3-10-22-26. Frequency ratio = 0.394.

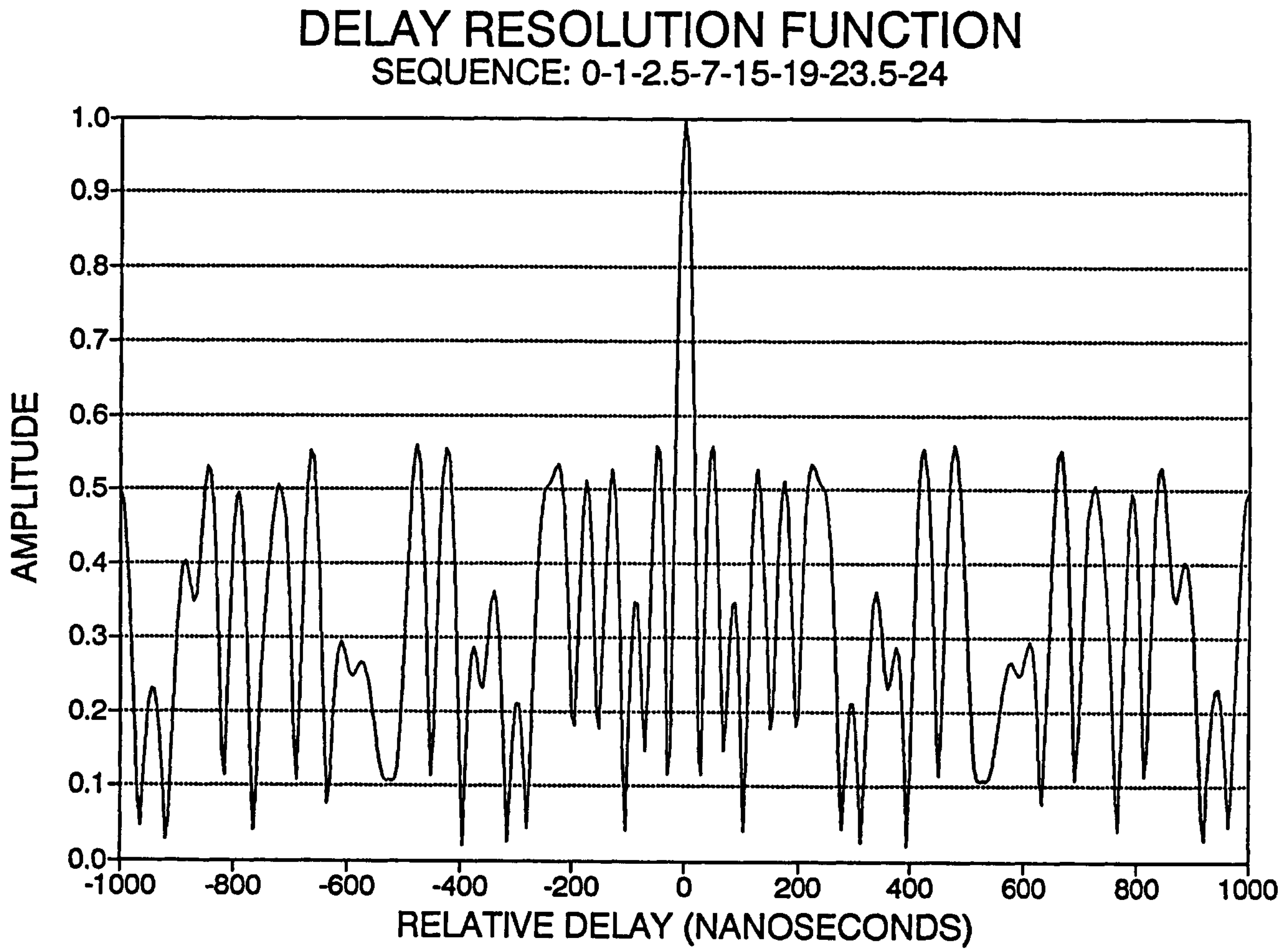

Figure 5. Delay resolution function for sequence 0-1-2.5-7-15-19-23.5-24. Frequency ratio = 0.393.

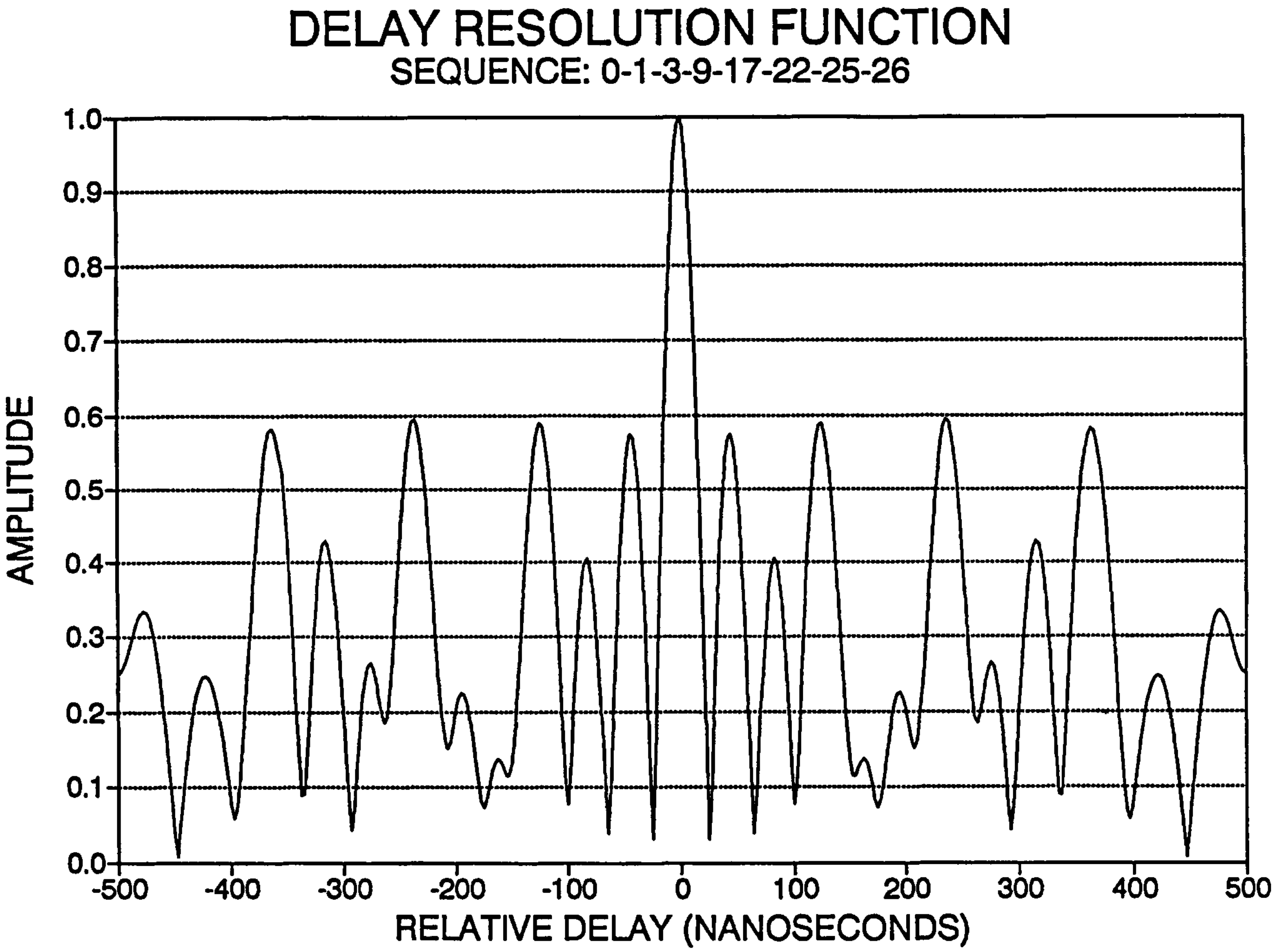

Figure 6. Delay resolution function for sequence 0-1-3-9-17-22-25-26. Frequency ratio = 0.394.