Global Ground VLBI Network as a Tied Array for Space VLBI

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Abstract

The minimum detectable flux density for interferometers between ground based ~~VLBI network~~ and space missions Radioastron and VSOP has been estimated for phasing a group of ground based antennas and for using the global fringe fitting scheme. It is shown that the both methods provide the same minimum detectable flux density.

1 The approach to the problem

The both now planned space VLBI (SVLBI) missions RADIOASTRON and VSOP have a small space antenna and as a result a poor sensitivity. In this situation in principle it is possible to combine all ground network in one huge radio telescope of Earth and provide the SVLBI between it and space antenna. Two definition of VLBI sensitivity can be considered:

- Minimum detectable flux density of a feature in the map.

The theoretical limit sensitivity is achieved just by averaging correlated signals from all baselines including the space radio telescope, if the delays and fringe rates for all base lines are known in advance.

- The minimum flux density for detection fringe lobes for all baselines including the least sensitive one.

If the correlated flux density is less than minimum flux density of the second definition the fringes will not be detected on the least sensitive baselines (baselines with space antenna) and such baselines will be excluded from the analysis. So we have to pay special attention on the second definition in the case of space VLBI. The sensitivity of the interferometer the space antenna with an individual ground based antenna is not enough to detect fringes from some sources. The sensitivity can be improved phasing all or group of ground based antennas. Two procedures of phasing ground based radio telescopes can be offered.

1. **Actual phasing of ground based radio telescopes.**
   It can be done applying global fringe fitting together with self-calibration to the group of ground based antennas. Having applied the obtained fringe rate, delay and phase of each antenna we can summarize signals of the ground based antennas and correlate this sum with the space antenna signal.
2. **Global fringe fitting.**

The comparison of the two procedures is given below.

2 Actual Phasing of Ground Based Radio Telescopes

The signals received of antenna-i of the Earth radio telescope and the space radio telescope can be described by the next equations:

$$\xi_i = \sqrt{P_{si}} \cdot \xi_0 \ + \ \sqrt{P_{ni}} \cdot n_i$$

$$\xi_s = \sqrt{P_{ss}} \cdot \xi_s \; + \; \sqrt{P_{ns}} \cdot n_s \tag{1}$$

where $P_{si} = \frac{1}{2}SA_i$ is a power of a source signal received by the ground based antenna-i
$P_{ss} = \frac{1}{2}SA_s$ is a power of a source signal received by the space antenna
S is the flux density of the source; A_i, A_s are the antennas effective areas.
$P_{ni} = kT_i$ is a total noise power of the antenna-i
$P_{ns} = kT_s$ is a total noise power of the space antenna
k is the Bolzman constant; T_i, T_s are noise temperatures of
the ground based antenna 'i' and the space one.
ξ_0, ξ_s, n_i, n_s are the normal random process'; $\overline{\xi_0} = \overline{\xi_s} = \overline{n_i} = \overline{n_s} = 0; \overline{\xi_0^2} = \overline{\xi_s^2} = \overline{n_i^2} = \overline{n_s^2} = 1$

Since the beginning we have supposed the source is a point one for ground based VLBI. That's why we have identical ξ_0 in the expression (1) for all ground based antennas. Now lets consider it is unresolved on the baseline Earth-Space as well. Then we can say that $\xi_0 = \xi_s$. The mathematical expectation of the random magnitude in the output of correlator is directly proportional to the correlation coefficient of input signals and its deviation depends upon the type of digitizers.

$$\overline{\eta_i} = \frac{\frac{1}{2}S\sqrt{A_iA_s}}{k\sqrt{T_iT_s}}; \qquad \sigma(\eta_i) = \frac{\alpha}{\sqrt{2\tau\Delta f}} \tag{2}$$

where $\alpha \simeq 1.57$ for TWO-TWO levels digitizers [1]
$\alpha \simeq 1.133$ for FOUR-FOUR levels digitizers [2]
$\alpha \simeq 1.333$ for TWO-FOUR levels digitizers [3]
τ is an averaging time
Δf is bandwidth of signals at the correlator input.

Having calibrated the output to the flux density we obtain the next expression for mathematical expectation and rms of the correlator calibrated output:

$$\overline{\eta_i} = S; \qquad \sigma(\eta_i) = \frac{\alpha 2k\sqrt{T_iT_s}}{\sqrt{2\tau\Delta f}\sqrt{A_iA_s}} \tag{3}$$

The final estimation of the correlated flux density can be done having summarized estimated fluxes for all baselines with appropriate weights.

$$S_f = \frac{\sum_i^n w_i\,\eta_i}{\sum_i^n w_i} \tag{4}$$

where i is the antennas number in ground based VLBI
n is number of antennas in ground based VLBI
w_i weight of interferometer antenna-i - space antenna

It is clear from equations (3) and (4) that mathematical expectation of the random magnitude S_f equals flux density S and its rms depends on the weights. It is known that there is an optimal values of the weights which provides the minimum rms. This optimal weights equal $w_i = \frac{1}{\sigma(\eta_i)^2}$ and relevant minimum rms is determined by the next formula:

$$\sigma(S_f)_{opt} = \frac{1}{\sqrt{\sum_i^n \frac{1}{\sigma(\eta_i)^2}}} \tag{5}$$

Having substituted $\sigma(\eta_i)$ from equation (3) we obtain the final expression for rms of measured flux density i.e. for sensitivity of our VLBI system to flux density:

$$\sigma(S) = \frac{\alpha 2k}{\sqrt{2\tau\Delta f}\sqrt{\sum_i^n \frac{A_iA_s}{T_iT_s}}} \tag{6}$$

Now we can determine the ratio $\left(\frac{A}{T}\right)_{eq}$ and the Source Equivalent Flux Density (SEFD) of equivalent radio telescope of the Earth:

$$\left(\frac{A}{T}\right)_{eq} = \sum_{i}^{n} \frac{A_i}{T_i}, \quad (SEFD)_{eq} = \frac{2k}{\left(\frac{A}{T}\right)_{eq}} \tag{7}$$

In the case of identical ground based radio telescopes

$$\left(\frac{A}{T}\right)_{eq} = n\frac{A}{T}, \quad (SEFD)_{eq} = \frac{2kT}{An} \tag{8}$$

Using this determination we obtain the next expression for the sensitivity in flux density:

$$\sigma(S) = \frac{\alpha}{\sqrt{2\tau\Delta f}}\sqrt{(SEFD)_{eq}\cdot(SEFD)_s} = \frac{\alpha}{\sqrt{2\tau\Delta f}}\frac{2k}{\sqrt{\left(\frac{A}{T}\right)_{eq}\left(\frac{A}{T}\right)_s}} \tag{9}$$

Now lets estimate the sensitivity of the Earth - Space VLBI to the brightness temperature. The sensitivity to the brightness temperature of a source T_b is related with the sensitivity to the flux density for solid angle Ω_s and wavelength λ by Rayleigh-Jeans law:

$$\sigma(S) = \frac{2k\,\sigma(T_b)}{\lambda^2}\Omega_s \tag{10}$$

The solid angle is determined by the width of the VLBI beam and equal approximately square of ratio of wavelength to maximum baseline $\left(\frac{\lambda}{D_m}\right)^2$. Combining equations (9) and (10) we obtain the next expression for sensitivity of the VLBI to brightness temperature:

$$\sigma(T_b) = \frac{\alpha {D_m}^2}{\sqrt{2\tau\Delta f}\sqrt{\left(\frac{A}{T}\right)_{eq}\left(\frac{A}{T}\right)_s}} \tag{11}$$

3 Global Fringe Fitting

Cotton and Schwab [4], [5] and Rogers [6] analyzed the relation for reduction of minimum detectable flux density with increasing number of identical antennas in the global fringe fitting scheme. They assumed that the detection threshold is set by a minimum SNR and that the SNR is inversely proportional to the error in the determination of the phase. We will use this conceptual framework to estimate the reduction of the detection threshold for the case of different antennas especially for the case of small one antenna (a space one). In a global fringe fitting scheme an antenna phase, delay and rate are determined for each antenna from all the data; as the number of baseline increases rapidly with increasing number of antennas, the minimum detectable point source flux density decreases. The uncertainty of the determination of each antenna phase with respect to the reference antenna can be used as a measure of the SNR. Lets suppose we have 'n' large ground based antennas: 1,2..i..k..n and one small space antenna: s. (Fig. 1). The phase difference on baseline ks ϕ_{ks} can be estimated just from this baseline φ_{ks} or using two baselines ki and is φ_{kis}. The final estimation of ϕ_{ks} can be done having averaged (with appropriate weights) the one-baseline estimation and the all two-baseline estimations:

$$\phi_{ks} = \frac{1}{W}\left[\frac{1}{\sigma_{ks}^2}\varphi_{ks} + \sum_{i=1}^{n-1}\frac{1}{\sigma_{ki}^2+\sigma_{is}^2}\varphi_{kis}\right] \tag{12}$$

where σ_{ki}^2 is a phase variance of a ground based interferometer; antennas k,i
$\sigma_{is}^2, \sigma_{ks}^2$ are the phase variance of interferometers between
the space antenna s and the ground antennas i,k
W is a sum of the all measurements weights; $W = \frac{1}{\sigma_{ks}^2} + \sum_{i=1}^{n-1}\frac{1}{\sigma_{ki}^2+\sigma_{is}^2}$

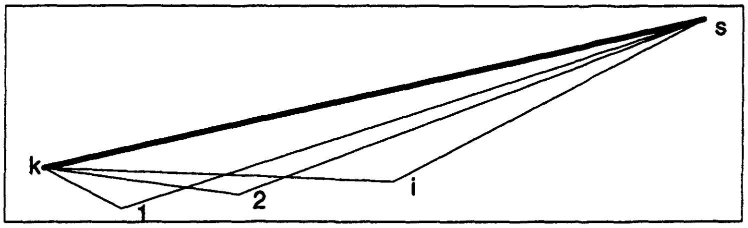

Figure 1: One- and two-baseline combinations for estimation of phase of interferometer: ground based antenna 'k' - space antenna 's'

Using of three- and more- baselines data for estimation of ϕ_{ks} does not improve it because the noises of such data are determined by the last arm: the interferometer with the space antenna and therefore they correlated with two-baseline data. The threshold reduction factor (TRF) which determines the minimum detectable flux density on the interferometer ks due to using the global fringe scheme can be evaluated as a square root of ratio of the variances:

$$TRF = \frac{\sigma_{\varphi ks}}{\sigma_{\phi ks}} \tag{13}$$

The variance of the one-baseline phase φ_{ks} is equal σ^2_{ks} by definition. The variance of final estimation of ϕ_{ks} can be found from equation (12) as $\frac{1}{W}$. Having substituted these expression for the variances in equation (13) and using expression for W in equation (12) we can obtain the following expression for TRF:

$$TRF = \sqrt{1 + \sum_{i=1}^{n-1} \frac{\sigma^2_{ks}}{\sigma^2_{ki} + \sigma^2_{is}}} \tag{14}$$

The variance of an interferometer phase is inversely proportional to the SNR. Therefore the following expressions have a place.

$$\sigma^2_{ks} \sim \frac{T_k T_s}{A_k A_s}; \quad \sigma^2_{is} \sim \frac{T_i T_s}{A_i A_s}; \quad \sigma^2_{ki} \sim \frac{T_k T_i}{A_k A_i} \tag{15}$$

where T_k, T_i, T_s are the noise temperatures of the corresponded radio telescopes;
A_k, A_i, A_s are the effective areas of the corresponded radio telescopes;

We consider that the space antenna is small comparatively with any antenna of the ground based VLBI network. Therefore $\sigma^2_{is} \ll \sigma^2_{ki}$. Having taken into account this consideration and using equations (15) we can rewrite equation (14)

$$TRF = \sqrt{1 + \frac{T_k}{A_k}\left(\frac{A}{T}\right)_{eq}} \tag{16}$$

where $\left(\frac{A}{T}\right)_{eq} = \sum_{i=1}^{n-1} \frac{A_i}{T_i}$ is the ratio of effective area to the noise temperature of an equivalent radio telescope of the Earth. This definition was used early (See equation (7))

For identical antennas of the ground based VLBI network equation (16) provides $TRF = \sqrt{n}$ which coincides with the result of phasing the 'n' identical antennas (see the previous section).
Now lets finally estimate the minimum detectable flux density on the baseline between the ground based antenna 'k' and the space antenna 's'. Having used equation (3 and definition of TRF we can obtained the next expression for the minimum detectable flux density realized in global fringe fitting.

$$\sigma(S)_{gl} = \frac{\sigma(S)}{TRF} = \frac{\alpha 2k}{\sqrt{2\tau\Delta f}} \frac{1}{\sqrt{\left(\frac{A}{T}\right)_k \left(\frac{A}{T}\right)_s}\sqrt{1 + \left(\frac{T}{A}\right)_k \left(\frac{A}{T}\right)_{eq}}} = \frac{\alpha 2k}{\sqrt{2\tau\Delta f}} \frac{1}{\sqrt{\left(\frac{A}{T}\right)_s \sum_{i=1}^{n} \left(\frac{A}{T}\right)_i}} \tag{17}$$

This expression coincides with the equation (6) of the previous section. Therefore we have come to the conclusion:

The global fringe fitting procedure and phasing of ground based antennas provide the same threshold reduction factor of minimum detectable flux density for the interferometer Earth-Space. The global fringe fitting procedure has an advantage of phasing of ground based antennas because it realizes the baselines between the space antenna and individual antenna of ground based VLBI network.

We have provided a test to check the sensitivity gain in global fringe fitting. Ten VLBA antennas have been used to observe the source 1751+096 at X band. This source is known as unresolved at VLBA at this band (private communication by Craig Walker). Two antennas - PT and MK were dispointed to imitate a small antenna of diameter $\sim 10m$. So PT and MK imitated an orbital RADIOASTRON or VSOP antenna. Antenna in FD failued in the observation. So finally we had 7 ground based antennas and two 'orbital' ones. We used 8 antennas in the test - 7 ground based antennas and one 'orbital'. In accordance with the theory given here we could expect the sensitivity gain in threshold ratio factor (TRF) term equal $\sqrt{7}$. We used AIPS global fring fotting task FRING to estimate TRF. Adverb DPARM(1) was used to switch number of baseline combination in estimation of phases for a given baseline. The case DPARM(1) = 1 corresponds to the only baseline (two antennas way) and DPARM(1) = 3 corresponds to the global fringe fitting including three and four antennas ways as well for estimation of phase for the given baseline (see Fig. 1). The TRF can be estimated as a ratio of SNR in the case of DPARM(1) = 3 (SNR3) and DPARM(1) = 1 (SNR1). The result is given in the table 1 for different solution intervals.

Table 1: Experimental Threshold Reduction Factor in Global Fringe Fitting

Solution, sec	10	30	40	60	$\sqrt{7}$
TRF=SNR3/SNR1	17/6=2.8	32/12=2.7	38/13.5=2.8	46/16=2.9	2.7

It is seen the experiment result is in a good agreement with the theory.

4 Examples of Application of the Analysis to the ground based VLBI network.

Using 'n' identical ground based antennas decreases the minimum detectable flux density (temperature) in interferometer Earth-Space in $\sqrt{n}$ times. But in reality there is a saturation in the sensitivity gain. So instead of $\sqrt{n}$ gain we obtain real gain which equals:

$$G = \sqrt{\frac{n}{1 + n\frac{T_a}{T}}} \tag{18}$$

It is clear from equation (18) that the gain can not exceed the limit gain which equals $\sqrt{\frac{T}{T_a}}$ and there is not a sense to phase a number of antennas larger $n \simeq \frac{T}{T_a}$. Lets take an example. Suppose we phase n identical antennas with diameter 50m, efficiency 50% and noise temperature 30K and observe the source with a flux density 10Jy. Then antenna temperature equals 3K and so limit gain equals $\sqrt{\frac{30}{3}} \simeq 3$!
Just for example we calculated rms in flux density and brightness temperature of interferometers Radioastron/VSOP - a group of ground based antennas. We selected three of such groups: in Europe, North America and Australia. The calculation has been done for $\Delta f = 100MHz, \tau = 100sec$, and two levels digitizers ($\alpha = 1.57$). The parameters of the selected radio telescopes for the selected L-band ($\lambda \simeq 18cm$) for each above mentioned group are taken from [7], [8], [9] and shown in the tables 2, 3 and 4.

Table 2: Europe

Name	Diameter [m]	SEFD [Jy]	$\frac{A}{T}$ $\left[\frac{m^2}{K}\right]$
Effelsberg (D)	100	19	148
Lovell (UK)	76	44	64
Westerbork (NL)	93	57	49
Cambridge (UK)	32	212	13
Onsala(S)	25	394	7
Europe		10	281
Radioastron	10	3500	0.8
VSOP	8	14500	0.19

Table 3: Australia

Name	Diameter [m]	SEFD [Jy]	$\frac{A}{T}$ $\left[\frac{m^2}{K}\right]$
DSS43	70	40	70
Parks	64	145	19
Mopra	22	390	7
Narrabri	22	390	7
Australia		27	103
Radioastron	10	3500	0.8
VSOP	8	14500	0.19

The parameters of VSOP and Radiastron radio telescopes for the selected L-band ($\lambda \simeq 18cm$) are taken from [10], [11].

Table 4: North America. 8 Continental VLBA Antennas.

Name	Diameter [m]	SEFD [Jy]	$\frac{A}{T}$ $\left[\frac{m^2}{K}\right]$
VLBA(one antenna)	25	300	9.3
North America		37.5	75
Radioastron	10	3500	0.8
VSOP	8	14500	0.19

The maximum baseline between space and ground radio telescopes is supposed equal 50,000 km for RADIOASTRON and 20000 km for VSOP. The result of the calculation is shown in the table 5.

Table 5: The Sensitivity of Interferometer Radioastron/VSOP - Group of Ground Based RT.

$\Delta f = 100MHz$, $\tau = 100sec$, $\alpha = 1.57$,
$D_m = 50,000km$, for RADIOASTRON, $D_m = 20,000km$ for VSOP

Name	σS [mJy]	σT_b [K]
Effelsberg - Radioastron	2.8	$2.6 \cdot 10^9$
Europe - Radioastron	2.1	$1.9 \cdot 10^9$
Effelsberg - VSOP	5.8	$8.3 \cdot 10^8$
Europe - VSOP	4.2	$6.0 \cdot 10^8$
VLBA(one ant.) - Radioastron	11.5	$10.3 \cdot 10^9$
North America - Radioastron	4.1	$3.6 \cdot 10^9$
VLBA(one ant.) - VSOP	22.9	$3.3 \cdot 10^9$
North America - VSOP	8.1	$1.6 \cdot 10^9$
DSS43 - Radioastron	4.2	$3.7 \cdot 10^9$
Australia - Radioastron	3.4	$3.0 \cdot 10^9$
DSS43 - VSOP	8.4	$12.0 \cdot 10^8$
Australia - VSOP	6.9	$10.0 \cdot 10^8$

5 Conclusion

The minimum detectable flux density for interferometers between ground based VLBI network and space missions Radioastron and VSOP has been estimated for phasing a group of ground based antennas and for using the global fringe fitting scheme. It is shown that the both methods provide the same minimum detectable fluxdensity. The unlimited decreasing of detectable flux density can not be achieved by unlimited increasing of phasing antennas numbers. The gain is limited by saturation effect because of the noise of the source itself. It is difficult to achieve the gain larger than 3.

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