Delay and Delay Rate and Their Errors for an Orbiting Antenna.

L. Kogan

National Radio Astronomy Observatory, Socorro, New Mexico, USA

January 20, 1997

1 The formulae obtaining.

The projections of an Earth satalite position and velocity on the equatorial coordinate system axises are determined by the formulae:

$$\begin{aligned} X &= R\cdot[\cos\Omega\cos(\omega+f)-\cos i\sin\Omega\sin(\omega+f)] \\ Y &= R\cdot[\sin\Omega\cos(\omega+f)+\cos i\cos\Omega\sin(\omega+f)] \\ Z &= R\cdot\sin i\sin(\omega+f) \qquad (1)\\ V_x &= V\cdot[\cos\Omega\cos(\omega+f+\omega_v)-\cos i\sin\Omega\sin(\omega+f+\omega_v)] \\ V_y &= V\cdot[\sin\Omega\cos(\omega+f+\omega_v)+\cos i\cos\Omega\sin(\omega+f+\omega_v)] \\ V_z &= V\cdot\sin i\sin(\omega+f+\omega_v) \qquad (2)\end{aligned}$$

where i is the orbit plane inclination,
Ω is right ascension of the ascending mode,
ω is the angle at the orbit plane from the ascending node to the perigee,
f is the true anomaly,
R is the distance from the Earth center to the satellite,
V is the length of velocity vector,
ω_v is the angle between the vector connecting the Earth center and satellite and vector of velocity.

Delay and delay rate projected on the source direction are determined by the following eqations:

$$\tau = X\cos\delta\cos\alpha+Y\cos\delta\sin\alpha+Z\sin\delta \qquad (3)$$

$$\dot\tau = V_x\cos\delta\cos\alpha+V_y\cos\delta\sin\alpha+V_z\sin\delta \qquad (4)$$

where α, δ are right ascension and declination of the source.

Having substituted expressions for X, Y and Z from equations (1) we can obtain the following equation for the delay:

$$\tau = R\left\{\cos\delta\left[\cos\omega_f\cos(\Omega-\alpha)-\cos i\sin\omega_f\sin(\Omega-\alpha)\right]+\sin i\sin\omega_f\sin\delta\right\} \qquad (5)$$

where $\omega_f=\omega+f$
We can rewrite the expression (5) introducing a new angle ψ determined by the following expression:

$$\sin\psi=\frac{\cos i\sin\omega_f}{\sqrt{\cos^2\omega_f+\cos^2 i\sin^2\omega_f}};\ \cos\psi=\frac{\cos\omega_f}{\sqrt{\cos^2\omega_f+\cos^2 i\sin^2\omega_f}} \qquad (6)$$

Using equations (6) we can obtain another equation for delay:

$$\tau = R\left(\sqrt{1-\sin^2\omega_f \sin^2 i}\,\cos\delta\cos(\psi+\Omega-\alpha)+\sin i\sin\omega_f\sin\delta\right) \tag{7}$$

Having provided the same type of analysis we can obtain the following two type of expressions for the delay rate:

$$\dot{\tau}= V\left\{\cos\delta\left[\cos\omega_{vf}\cos(\Omega-\alpha)-\cos i\sin\omega_{vf}\sin(\Omega-\alpha)\right]+\sin i\sin\omega_f\sin\delta\right\} \tag{8}$$

where $\omega_{fv} = \omega + \omega_v + f$.

$$\dot{\tau}= V\left(\sqrt{1-\sin^2\omega_{fv}\sin^2 i}\,\cos\delta\cos(\psi_v+\Omega-\alpha)+\sin i\sin\omega_{fv}\sin\delta\right) \tag{9}$$

ψ_v is determined by the following expression:

$$\sin\psi_v = \frac{\cos i\sin\omega_{fv}}{\sqrt{\cos^2\omega_{fv}+\cos^2 i\sin^2\omega_{fv}}};\ \cos\psi = \frac{cos\omega_{fv}}{\sqrt{\cos^2\omega_{fv}+\cos^2 i\sin^2\omega_{fv}}} \tag{10}$$

2 Analysis of errors

It is natural that the orbit parameters are not known ideally in advance. As a result the delay and delay rate calculated by formulae (5), (7) and (8), (9) differ of the real values. The errors of the prediction can be evaluated depending on parameters errors. The list of the parameters affecting the delay and delay rate error is the subject of the consideration. The first aproach to the list is all six parameters of the orbit at the reference time. This aproach is not realistic because the time of estimating can be far away from the reference time and as a result the error of orbit prediction can be rather big. At the same time some orbit parameters such a distance to the satallite, its angular position on the orbit are probably measured often. So instead of six original parameters of the orbit we can offer to use four following parameters:
1. Distance from the Earth center to the satalite - R. For the delay rate it is the length of the velocity vector - V.
2. Inclination of the orbit plane to the equator - i.
3. Right ascension of the ascending node - Ω.
4. Angular position of the satallite on the orbit - ω_f as a sum of angle of the peregee to the ascending node ω and true anomaly f. In the case of delay rate, the third term ω_v is added in ω_f.
Using these parameters we can write the following expressions for the errors:

$$\Delta\tau = \frac{\partial\tau}{\partial R}\Delta R + \frac{\partial\tau}{\partial i}\Delta i + \frac{\partial\tau}{\partial\Omega}\Delta\Omega + \frac{\partial\tau}{\partial\omega_f}\Delta\omega_f \tag{11}$$

$$\Delta\dot{\tau}= \frac{\partial\dot{\tau}}{\partial R}\Delta R + \frac{\partial\dot{\tau}}{\partial i}\Delta i + \frac{\partial\dot{\tau}}{\partial\Omega}\Delta\Omega + \frac{\partial\dot{\tau}}{\partial\omega_{fv}}\Delta\omega_{fv} \tag{12}$$

The relevant partitial derivatives can be found using equations (5) and (8).

$$\frac{\partial \tau}{\partial R} = \cos\delta\left[\cos\omega_f \cos(\Omega-\alpha) - \cos i \sin\omega_f \sin(\Omega-\alpha)\right] + \sin i \sin\omega_f \sin\delta \tag{13}$$

$$\frac{\partial \tau}{\partial i} = R\left\{\cos\delta \sin i \sin\omega_f \sin(\Omega-\alpha) + \cos i \sin\omega_f \sin\delta\right\} \tag{14}$$

$$\frac{\partial \tau}{\partial \Omega} = -R\cos\delta\left[\cos\omega_f \sin(\Omega-\alpha) + \cos i \sin\omega_f \cos(\Omega-\alpha)\right] \tag{15}$$

$$\frac{\partial \tau}{\partial \omega_f} = R\left\{\cos\delta\left[-\sin\omega_f \cos(\Omega-\alpha) - \cos i \cos\omega_f \sin(\Omega-\alpha)\right] + \sin i \cos\omega_f \sin\delta\right\} \tag{16}$$

$$\frac{\partial \dot{\tau}}{\partial V} = \cos\delta\left[\cos\omega_{fv} \cos(\Omega-\alpha) - \cos i \sin\omega_{fv} \sin(\Omega-\alpha)\right] + \sin i \sin\omega_{fv} \sin\delta \tag{17}$$

$$\frac{\partial \dot{\tau}}{\partial i} = V\left\{\cos\delta \sin i \sin\omega_{fv} \sin(\Omega-\alpha) + \cos i \sin\omega_{fv} \sin\delta\right\} \tag{18}$$

$$\frac{\partial \dot{\tau}}{\partial \Omega} = -V\cos\delta\left[\cos\omega_{fv} \sin(\Omega-\alpha) + \cos i \sin\omega_{fv} \cos(\Omega-\alpha)\right] \tag{19}$$

$$\frac{\partial \dot{\tau}}{\partial \omega_{fv}} = V\left\{\cos\delta\left[-\sin\omega_{fv} \cos(\Omega-\alpha) - \cos i \cos\omega_{fv} \sin(\Omega-\alpha)\right] + \sin i \cos\omega_{fv} \sin\delta\right\} \tag{20}$$