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VERY LARGE ANTENNAS FOR THE COSMOLOGICAL PROBLEM

II. A REFLECTING CROSS ANTENNA

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ABSTRACT

An antenna is suggested which reaches 3×10^5 sources per steradian as demanded in a previous paper. Radiation from the zenith is focused and horizontally reflected by two arms of a slightly curved cross. Two secondary plane reflectors bring both focal points to one receiver site where two feeds are located. All reflectors are fixed. One of the feeds is movable horizontally 30 meters in order to give a sky coverage of about one degree in declination. Even under unfavorable conditions, a total of 3000 sources can be observed with high accuracy (coma $\leq 1/5$ of beamwidth) and 6000 additional sources with lower accuracy (coma $\leq 1/2$ of beamwidth). The beamwidth is 35 sec. of arc.

An analysis of costs is carried out together with a rough outline of the design. All degrees of freedom are used to minimize the price, but many safety factors are included. The best solution is a wooden structure with a wire mesh surface; the cross arms are 630 m long and 23 m wide: the effective collecting surface is 13000 m^2, which is one third of the total geometrical surface of all the reflectors. This solution is for a maser of 50° noise temperature, operating at 7.4 cm wavelength. The price of the antenna is estimated to be \$510,000. The total price of \$890,000 includes antenna, maser, feeds, high precision survey, shielding, equipment and building.

I. INTRODUCTION

1. The Task

In a previous paper (v. Hoerner, 1961, referred to as Paper 1) we have estimated that an antenna should reach a limit of

$$N_{lim} = 3 \times 10^5 \text{ sources per steradian}$$

in order to detect reliably the distinctions between different cosmological models. We have calculated the properties of an antenna needed for this task, leaving two degrees of freedom: the noise temperature of the receiver and the wavelength. Once both are chosen the collecting surface of the antenna is given by the brightness limit, and the base area over which the system must be spread is given by the resolution limit. Both limits have been rediscussed in Paper 1.

The present paper is concerned with the question "What is the cheapest way to realize such an antenna?" A reflecting cross antenna is suggested as one possible way; it would accomplish the imposed task and would still have a remarkably low price for its size.

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2. General Principles

Consideration of the problem of building a very large antenna for a low price leads to the following general principles.

1. One should make the best possible use of the surface of the Earth, with a minimum amount either of digging holes or erecting structures. Whatever kind of antenna we might consider, it needs a regularity of some sort, and the most regular type of geological surface would be a plain (no valley will be as parabolic as a plain may be plane). Therefore, we should look for a solution which basically is plane.

2. Large movable structures are extremely expensive. Thus we should look for a fixed-antenna type.

3. The antenna will probably occupy a relatively large base area. The attenuation in feed lines then would be very high and would necessitate a large number of high quality preamplifiers which have to be tuned in phase over large distances. This is not impossible but still expensive and troublesome. It can be avoided if all surfaces are reflecting and all information finally is reflected to just one point.

4. From the source to this point all waves should have the same path length. Only under this condition can we vary the wavelength as well as observe with a broad bandwidth. We need a variation of the wavelength in order to obtain spectra of at least the brighter sources, and a restriction in bandwidth in case of large dimensions and short wavelengths would cut down the brightness limit considerably.

5. According to formula (I, 47) we need a sky coverage of only 20 minutes of arc to get a total of 3000 sources even under unfavorable conditions. If more sky coverage makes the antenna expensive, we should be satisfied with these 20 minutes.

3. Distribution of the Antenna Surface over the Base Area

The calculations of Paper I are based on the assumption (I, 22) that the effective beam area of the antenna system is the same as that of a single round dish of diameter b, the size of the base area. The following ways in which this can be realized are known at present.

1. The single round dish itself. It has many obvious advantages, but in case of very large dimensions, even for a fixed dish, it would need high structures (or a deep hole) and a very high feed tower.

2. Random distribution of a large number of small elements. With a large enough number of elements the beam pattern might be satisfactory (Deschamps, 1961), but in case of a large base area we do not see an easy way of collecting all the information to one point.

3. Mills cross or similar crosses. The phase switching technique multiplies the two fanshaped beam patterns of the two arms and results in a good pencil beam with relatively small sidelobes. It has the disadvantages of small bandwidth and, at short wavelengths, high attenuation in the feed lines. We also could observe at only one wavelength.

4. Aperture synthesis. (See e.g. Ryle, Hewish and Shakeshaft, 1960). Observations are made with two antennas, at least one of which is movable within the base area. All observations in different positions are reduced together, and a Fourier analysis then yields the brightness distribution of the sky. In order to get the same resolution as in the case of a dish of diameter b, one has the choice of either moving a very large dish into few positions or a small dish into many positions. The first will be expensive, and the second might introduce a high amount of uncertainty. Besides, the disadvantages are

the same as with the Mills cross, and in addition we have to move large parts of the system.

Summarizing, we think that none of the present solutions is the ideal one for our task in mind. The most promising ones seem to be the Mills cross type and the single dish. In the next section we shall try to combine their advantages.

II. PROPOSAL FOR AN INSTRUMENT

1. The Reflecting Cross

Figure 1 shows the general outline of an arrangement which might be called a reflecting cross. The basic idea is to use the phase switching technique of the Mills cross and to fulfill the general principles of section I, 1.

The collecting surface of the antenna forms a cross, S_1, the arms of which stand on circles around two focal points, F. The surface of each arm is tilted outward by 45° so that all rays from the zenith would be reflected to F. Both arms are of equal length, b, and the distance to F is 4b according to (I, 47) of Paper I. At about half of this distance two secondary reflectors, S_2, are introduced. They are plane, vertical surfaces. The secondary reflectors shift both focal points F to nearly the same focal point R, where two separate feeds are connected to a single receiver. One of the feeds is movable horizontally by some meters in order to give the needed coverage in declination (see section II, 6 and III, 3 of Paper I). The surface S_1 is slightly curved vertically and forms a section of a paraboloid which has its focus in F and a vertical axis*). The surface of S_2 is plane. Neither surface is movable.

The outputs of the two feeds are connected by a phase switching device as in a Mills cross, yielding a pencil beam as the product of the two separate beams.

Advantages. All general principles, of section I, 1 are fulfilled. The base lines of all of the reflectors and feeds are on a horizontal plane. The only movable part is one feed. All of the information is finally reflected to just one point. At this point all wavelengths are automatically in phase, allowing a broad bandwidth and observations at various wavelengths. In summary: a low price, no restriction in wavelength, and a relatively simple arrangement.

Disadvantages. The highest number of sources will be reached at short wavelengths, which means we need a high-precision survey of the region and an accurate surface. The tilting of S_1 by 45° and the introduction of S_2 will increase the total surface by about a factor of two. The movability of the beam is restricted to the close neighborhood of the zenith.

A survey of the region could be achieved with the desired precision by modern radar techniques, and the accuracy of a fixed antenna surface is no severe problem. A certain increase of the total surface cannot be avoided, although some other arrangements might give a smaller increase.

The sky coverage could be increased slightly by increasing the focal length or considerably by moving one reflector horizontally in an arrangement somewhat different from the one of Figure 1. But the price would be increased too, even the relative price in dollars per source, and we see no real pressing need for this. The present instruments are not limited in their sky coverage, but they are in fact limited in the average distances they reach: our instrument would reach to much farther distances and we pay for that with

*In practice, the vertical curvature of S_1 is so small that the parabola may be replaced by two straight lines, meeting at half the height of S_1.

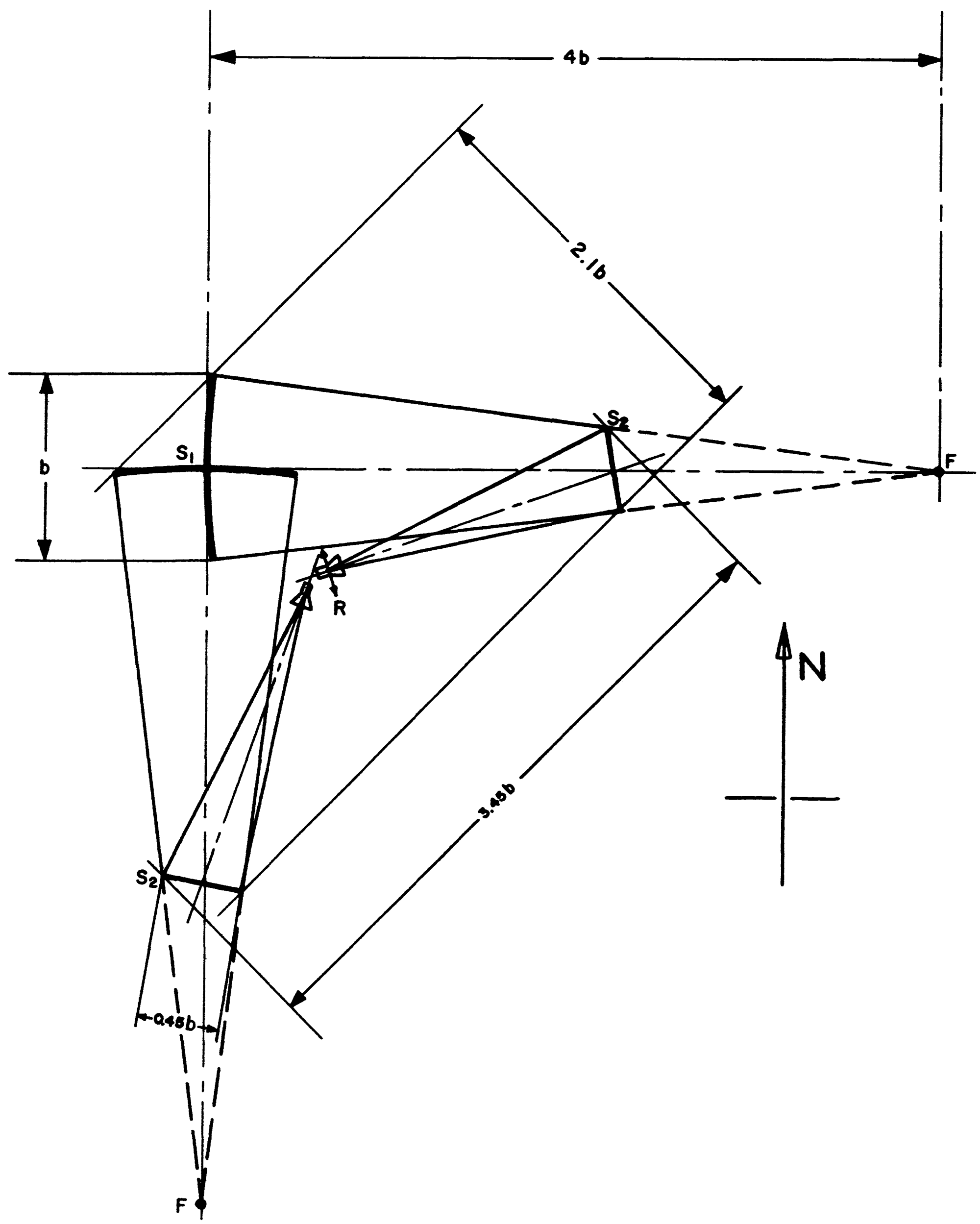

Fig. 1 Radiation from the zenith is reflected horizontally and focussed to F by the two arms of the cross at S_1. Both focal points F are brought to the same receiver station at R by two secondary reflectors at S_2. One of the two feeds can be moved sideways to give the sky coverage in declination.

a limited sky coverage, keeping the total number of sources about constant.

A restriction to selected areas would become necessary anyway, as soon as larger distances are reached. A long, narrow strip around the whole sky will provide a more representative average than would a more concentrated selected area of equal solid angle. The strip will pass the galactic plane at two points and will approach the north galactic pole within about ten degrees (zenith at Green Bank), which might help to separate galactic and isotropic distributions of sources.

The feed movement. We call D the distance over which one of the feeds must be moved in order to give the needed sky coverage s in declination. Measuring D and b in the same units we get, with a focal ratio of 4 according to (I, 47):

$$D = 4\ bs,$$

and with s = 20 minutes of arc,

$$D = 0.023\ b. \tag{1}$$

With an armlength of b = 800 m, for example, the feed must be moved by about 14 m in order to give s = 20'.

2. The Feed Height

The feed must illuminate an arm of height d from a distance vb. The height of the feed, h, is then approximately given by

$$h = \frac{v\ b\ \lambda}{d} \ . \tag{2}$$

The application of (2) to the optimum solutions of Table 7 in Paper 1 gives an increase of h with λ which is so strong that our freedom in the choice of λ gets sharply limited toward longer wavelengths. (For example: λ = 12 cm, with present receivers, gives 4 m height; 18 cm gives 19 m, 27 cm gives 87 m, 40 cm gives 374 m height, and so on). On the other hand, the longer the wavelength, the less surface we need, thus we should choose λ as large as possible. This means we have to find a compromise between the two demands for a convenient feed height and a small surface. Keeping in mind that one of the feeds has to be movable with a good accuracy, we suggest the same height for feed and arm:

$$h = d\ . \tag{3}$$

With v = 4 from (I, 47) we have

$$d^2 = 4\lambda b, \tag{4}$$

and with $d = \pi a^2/8b$ from (I, 40) we have

$$a^4 = \frac{256}{\pi^2}\ b^3\ \lambda. \tag{5}$$

The optimum solutions of Paper I left us with two degrees of freedom: the quality of the receiver and the wavelength. By equation (5) we are now going to sacrifice one degree of freedom in order to achieve the best compromise between feed height and antenna surface according to (3). One degree of freedom still is left: the noise temperature T_n of the receiver. To show this more clearly, we eliminate a and b in (5) to obtain λ as a function of T_n.

The optimum solutions lie at the crossing point of resolution limit, N_{res}, and brightness limit, N_{vis}, where noise and background distortion add up according to section II, 5 of Paper I. To reach N_{lim} sources per steradian we need at this point.

$$N_{res} = N_{vis} = 2N_{lim} = 6 \times 10^5 \text{ sources per steradian.} \tag{6}$$

Equation (I, 13) leads to

$$a^4 = \left(\frac{N_{lim}}{182}\right)^{4/3} \left(\frac{T_o}{g\lambda^{0.3}}\right)^2 \tag{7}$$

for the brightness limit, and equation (I, 24) gives

$$b^3 = 2.21 \times 10^3 \, N_{lim}^{3/2} \, \lambda^3 \tag{8}$$

for the resolution limit. Entering (5) with (7) and (8) we get

$$T_o = 7.68 \times 10^3 \, N_{lim}^{1/12} \, g\lambda^{2.3} , \tag{9}$$

where $g(\lambda)$ is the limiting function defined in Paper I. T_o, the over-all noise temperature, is the sum of receiver noise, T_n, of galactic and extragalactic radiation noise, T_g, and atmospheric noise, T_a. This gives

$$T_n = 7.68 \times 10^3 \, N_{lim}^{1/12} \, g\lambda^{2.3} - (T_a + T_g), \tag{10}$$

and with $N_{lim} = 3 \times 10^5$ sources per steradian finally

$$T_n = 2.20 \times 10^4 \, g\lambda^{2.3} - (T_a + T_g), \tag{11}$$

All quantities on the right hand side of (11) are functions of the wavelength λ, and equation (11) can be solved graphically with the values of Table I, 1 to yield λ as a function of T_n. The values of a, b, and d then are found from equations (4), (5) and (8):

$$\begin{aligned} a &= 15.47 \, N_{lim}^{3/8} \, \lambda = 1.76 \times 10^3 \, \lambda \\ b &= 13.0 \, N_{lim}^{1/2} \, \lambda = 7.13 \times 10^3 \, \lambda \\ d &= 7.22 \, N_{lim}^{1/4} \, \lambda = 1.69 \times 10^2 \, \lambda \end{aligned} \tag{12}$$

Results are shown in Table 1. We have assumed that the noise temperature T_n does not vary with the observing wavelength, with exception of the last line for present vacuum tube receivers where we have solved equation (11) with the values T_n (λ) from Table I, 1.

TABLE 1

Antenna Properties for Equal Height of Feed and Arms

T_n	λ	a	b	d
°K	cm	m	m	m
20	5.3	93	378	9.0
30	6.1	107	433	10.3
50	7.4	129	526	12.5
70	8.4	148	601	14.2
100	9.8	171	696	16.5
200	13.2	232	941	22.3
400	17.7	311	1260	29.9
624	21.3	374	1520	36.0

$\pi a^2/4$ = antenna surface
b = length of arms
d = width of arms = height of feed

3. The actual dimensions

There are four effects which call for dimensions somewhat larger than those calculated in the last section.

1. Tapering. The beamwidth was calculated in Paper I according to $\beta = 1.2\ \lambda/a$ in case of a round dish of diameter a. F. D. Drake (1961) has performed some calculations about the beam pattern of an arrangement as shown in Figure 1. A uniform illumination leads to $\beta = 1.22\ \lambda/b$ with strong sidelobes. A tapering of 10 decibel at the edges leads to $\beta = 1.42\ \lambda/b$ with acceptably small sidelobes: in order to get the same beamwidth as before we thus have to increase the length b by 18%. We adopt 20% in the following.

2. The arms S_1 are tilted by 45° which increases d_1 by $\sqrt{2}$.

3. Feed movement should be possible over a larger distance than the one given by (1), which was based on the demand that the coma be smaller than 1/5 of the beamwidth. It might become desirable, however, to observe a higher number of sources with somewhat less accuracy, and a coma limit of 1/2 of the beamwidth might be enough for this purpose. Instead of (1) we then have

$$D = 0.0575\ b. \qquad (13)$$

The feed should always point to the center of the cross as reflected by S_2, and b_2 must be enlarged by at least 6% to allow for the extreme positions of the feed without cutting off the illumination of S_1.

4. Spillover increases T_0: it can be avoided by increasing d. A certain safety factor was already included in equation (I, 7) where the effective surface was supposed to be 0.7 of the geometrical one. To realize the low noise temperature of a good maser, we now introduce an additional increase of d by 30%. The remaining very small pick up of ground radiation could be made entirely negligible by spreading a scattering (uneven) wire mesh surface below and in front of all reflectors.

In summary we have to apply the following factors to the values of Table 1 in order to get the actual dimensions:

	S_1	S_2
length	tapering 1.2	tapering 1.2 feed mov. 1.06
width	45° tilt $\sqrt{2}$ spillover 1.3	spillover 1.3

TABLE 2

The actual antenna dimensions (safety factors applied)
as a function of receiver quality, for equal height of feed and arms.

b = length of reflectors, see Figure 1
d = width of reflectors
A = total surface

	T_n	λ	b_1	b_2	d_1	d_2	A
	°K	cm	m	m	m	m	10^4 m^2
Maser	20	5.3	454	217	16.5	11.7	2.01
	30	6.1	519	249	18.9	13.3	2.64
	50	7.4	631	302	22.9	16.2	3.89
	70	8.4	721	346	26.2	18.5	5.09
Parametric amplifier	100	9.8	835	401	30.4	21.5	6.81
	200	13.2	1130	542	41.0	29.0	12.5
	400	17.7	1514	726	55.0	38.9	22.4
Vacuum tube receiver	624	21.3	1824	875	66.2	46.8	32.5

which finally gives

$$b_1 = 1.2\,b = 8.55 \times 10^3\,\lambda \qquad b_2 = 0.575\,b = 4.10 \times 10^3\,\lambda$$
$$d_1 = 1.84\,d = 3.11 \times 10^2\,\lambda \qquad d_2 = 1.3\,d = 2.19 \times 10^2\,\lambda\ . \tag{14}$$

The total surface then is given by $A = 2(b_1 d_1 + b_2 d_2)$, and with $2bd = \pi a^2/4$ we have:

$$A = 2.32\,a^2 = 7.19 \times 10^6\,\lambda^2. \tag{15}$$

The results are shown in Table 2, as a function of the receiver quality. This last degree of freedom shall be used in the following sections to find the solution of minimum total cost.

III. COST ANALYSIS

It is the task of the instrument to reach $N_{lim} = 3 \times 10^5$ sources per steradian with a minimum of cost. One degree of freedom still is left: the quality (and price) of the receiver. We thus need an estimate of the antenna price as a function of T_n, in order to find the point of minimum total price and also to determine the basic principles of the design. The following estimate necessarily is a very rough one but should serve this purpose. Four parts may be considered separately: surface, supporting structure, anchoring to the ground, and choice of material.

The most important figure is the ratio of the highest expected wind pressure, w, to the maximum allowed stress in slender columns, S. Taking $w = 30\ \text{lb/ft}^2 = 146\ \text{kg/m}^2$ (belonging to a true wind velocity of 106 mph = 170 km/h) and S for a ratio of length to radius of gyration = 50, we have

Material	$\frac{w}{S}$
Steel	1.29×10^{-5}
Aluminum	1.97×10^{-5}
Wood	1.33×10^{-4}

. (16)

If we neglect for a moment all geometrical relations given by the special arrangements of the structure, then w/S is equal to the total cross section of all supporting members divided by the total antenna surface. We multiply w/S by the total surface and by the average length of the members and get the volume of material needed to hold the surface against the wind. We multiply by the specific weight of the material and by the price per unit weight (including erection) and get the price of what we might call the minimum supporting structure. This is done in the first section with some more details.

The values of (16) are very small ones, and this means we would need only a very small amount of material. But (16) also should hold for each individual member, and this means that a member of small cross section should be able to support a large area of the surface. In practice, however, there is something like a minimum member cross section which then will give a minimum distance of any two members at the surface. The surface thus should be self-supporting within this distance, which would need large frames of high cost. It is indeed cheaper to multiply the number of supporting members in order to get smaller frames, and the factor by which to multiply to obtain the minimum cost is determined in the second section.

1. The Minimum Supporting Structure

The task of this structure is to hold the surface of a long, horizontal reflector arm at a certain height above ground against the wind force. This requires at least two basic lines on the ground, and each supported point of the surface must be connected with at least three points of the ground (we do not use the rigidity of the surface, because the surface must be adjustable). We start with a calculation of the safety limit given by the strongest possible wind velocity. Second, we calculate the wind velocity to which useful observations can be made. Third, we insure that the weight of the structure and its bending forces, as well as the weight of snow, may be neglected against the limiting wind force at the surface. The surface may be wire mesh but we assume it to be entirely closed by snow and ice.

a. The Secondary Reflectors

The surface stands vertically and the height H above ground need not be much larger than the width d. One basic line, B_b, therefore should be placed exactly below the surface (see Figure 2). Several supporting members will lead from different heights of the surface to the second basic line, B_a; but for this estimate we regard all members as being of equal length, L, holding the surface at average height, H, under an angle φ. Seen from above, as in Figure 3, each supporting point of the surface should be connected with at least two points of B_a. The longitudinal forces at the surface will be much smaller than the lateral ones, and therefore the angle ψ need not be very large, but we will adopt $\psi = 30°$ to be on the safe side.

We call A_2 the total surface of both secondary reflectors together, w the limiting wind pressure for safety, and K_a and K_b the total forces in all members leading to B_a and B_b. The forces then are given by

$$K_a = \frac{w\ A_2}{\sin\varphi} \quad \text{and} \quad K_b = \frac{w\ A_2}{\tan\varphi} \quad . \tag{17}$$

If we call S the maximum allowed stress in slender columns, the forces then demand a total cross section, q, of the members:

$$q_a = \frac{K_a}{S} = \frac{w\ A_2}{S\sin\varphi} \quad \text{and} \quad q_b = \frac{K_b}{S} = \frac{w\ A_2}{S\tan\varphi} \quad . \tag{18}$$

The length of the member is $L_a = H/(\cos\varphi\cos\psi)$ and $L_b = H/\cos\psi$, and the total volume of material then is given by $V_2 = L_a q_a + L_b q_b$ or

$$V_2 = \frac{w\ A_2\ H}{S\cos\psi}\left(\frac{1}{\tan\varphi} + \frac{1}{\sin\varphi\cos\varphi}\right) = \frac{w\ A_2\ H}{S\cos\psi}\ \frac{3+\cos 2\varphi}{\sin^2\varphi} \ . \tag{19}$$

We ask for the best value of φ and find

$$\left(\frac{3+\cos 2\varphi}{\sin 2\varphi}\right)_{\text{minimum}} = \sqrt{8} \text{ at } \varphi = \frac{1}{2}\ \text{arc}\cos(-1/3) = 54.^\circ 8. \tag{20}$$

With this value and $\psi = 30°$ we obtain $\sqrt{8}/\cos\psi = 3.27$, and the equation (19) then reads

$$V_2 = 3.27\ \frac{w\ A_2 H}{S} \ . \tag{21}$$

b. The Cross Reflectors

Their surfaces are tilted by 45°, which decreases the wind force by $\sqrt{2}$. The direction of the force will be, in the first approximation, perpendicular to the surface, and this

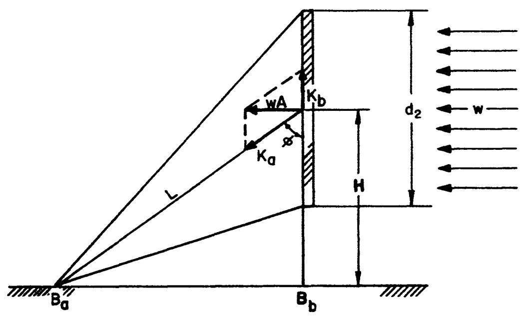

Fig. 2 Secondary reflector, side view.

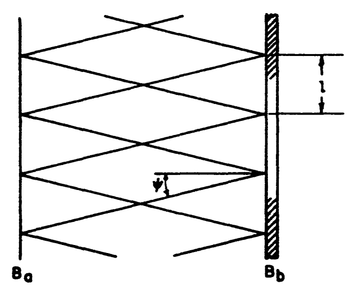

Fig. 3 Secondary reflector, seen from above.

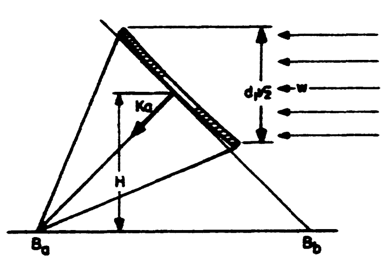

Fig. 4 Primary reflector, side view.

calls for an arrangement as shown in Figure 4. In first approximation the whole wind force is in K_a, while K_b just holds the weight of the structure. This is not a good approximation; but it should be safe enough to give K_b a limiting value of 1/3 of K_a. We call A_1 the total surface of both cross arms and have

$$K_a = w\ A_1/\sqrt{2} \text{ and } K_b = K_a/3, \tag{22}$$

and in a similar way as before we arrive at

$$V_1 = \frac{4}{3}\ \frac{wA_1H}{S}\ . \tag{23}$$

c. In Summation

Cross arms and secondary reflectors together need for the members of their supporting structure a total volume of material

$$V = V_1 + V_2 = \frac{w\ H}{S}\ (1.33\ A_1 + 3.27\ A_2), \tag{24}$$

and with help of equations (14) and (15) we finally have

$$V = 1.82\ \frac{wHA}{S}\ , \tag{25}$$

where A is the total surface given in Table 2.

Next, we should replace the average height, H, by the quantities given in Table 2. If the reflectors would sit flat on the ground, H would equal d/2. In addition we need a certain clearance off the ground. It might be necessary, also, to reduce the remaining small spillover of the illumination by scattering wire mesh screens below and in front of the reflectors, in which case the clearance should increase proportional to d: considering d/4 as enough and keeping in mind that d_1 is larger than d_2 just because of the tilt, without being higher above ground, we have

$$H = d/2 + d/4 = 0.75\ d_2\ . \tag{26}$$

No geological plane is exactly plane, thus we ought to include the influence of the length, b, on the average height. It turns out, though, that the additional increase of the height can be neglected against (26) as long as the slope of the plane is less than 1/100. Under this assumption we get from (25) and (26)

$$V = 1.36\ \frac{w}{S}\ Ad_2\ . \tag{27}$$

We may express A and d_2 in terms of λ from (14) and (15) and find for the equal-height solutions of Table 2:

$$V = 2.15 \times 10^9\ \frac{w}{S}\ \lambda^3\ . \tag{28}$$

d. Antenna Movement in Strong Wind

The previous calculation was based on two safety limits: the strongest expected wind (pressure w) and the maximum allowed stress in slender members, S. If we divide S by E, the modulus of elasticity, we get the maximum allowable relative elongation or compression, $\Delta H/H$, of a member of length H, in case of the strongest expected wind. The actual movement of the antenna will be about two times Δ H depending on the geometrical arrangement. This movement will be proportional to the square of the wind velocity, and it should be not larger than about 1/10 of the wavelength, λ. If we call v_{lim} the velocity of the strongest expected wind, and v_o the velocity up to which useful observations are possible, we have the condition:

$$\frac{S}{E} \left(\frac{v_o}{v_{lim}}\right)^2 \leq 0.1 \frac{\lambda}{2H} , \tag{29}$$

or

$$v_o = v_{lim} \left(0.05 \frac{\lambda E}{H S}\right)^{1/2} . \tag{30}$$

The quotient λ/H is the same for all solutions of Table 2, and from equations (26), (14) and (12) we find $\lambda/H = 6.07 \times 10^{-3}$. The quotients E/S are about the same for steel, aluminum and wood, and condition (30) then reads

$$v_o = 0.50\, v_{lim} , \tag{31}$$

which means we can observe all the way up to one half of the strongest possible wind, or practically all the time.

e. Weight of Structure and Snow, Compared with Wind Force

The weight of the supporting structure is given by (27) as about $wd_2A\,\rho/S$, where ρ is the specific weight of the material, and the wind force is about wA. The ratio of weight to force then is

$$\frac{\text{weight}}{\text{force}} = d_2\,\rho/S = \begin{cases} 0.026 \text{ for steel} \\ 0.014 \text{ for aluminum} \\ 0.021 \text{ for wood} \end{cases} \tag{32}$$

with the maximum value of d_2 = 46.8 m of Table 2. Thus the weight may be neglected as long as the weight of surface plus frames, divided by the weight of the supporting structure, is small compared to 40.

A maximum snow load of 10 lb/foot^2 on the 45° reflector, as compared to a safety limit of 30 lb/foot^2 for the wind pressure, would call for an increase of 30% for the cross sections of the structure. We neglect this contribution, however, under the assumption that a wind of this strength will be able to blow away a snow layer of this thickness.

2. The Surface Conditions

The minimum supporting structure is determined by the values (16) which define the ratio of the total structural cross section to the total antenna surface. This ratio should be the same for each individual member too. In order to avoid a large frame system at the surface, we ought to take a high number of thin supporting members, but there is a practical limit. The price per pound of material will stay about constant as we go to parts of smaller size, but beyond a certain limit we reach unusually small sizes entailing a higher amount of labor in their connection, and the price per pound would increase rapidly.

If we call q_o the cross section of the material in the smallest possible member and l_o the distance of any two supporting points at the surface, and if two members join at each such point, we have for example from (18) with $A = l_o^2$:

$$l_o = (2 \sin \varphi \frac{S}{W} q_o)^{1/2} \quad . \tag{33}$$

If we would regard a steel pipe of 3 cm diameter and 1.5 mm wall thickness as the smallest size, we get $q_o = 1.4\ cm^2$ and $l_o = 4.2$ m.

There are two extreme possibilities. First, to accept a distance of about 4 m and to supply the surface with frames self-supporting within this distance. This might be recommended for a very large height above ground. Second, to introduce so many more members that no frame at all is needed, which would be best for a very small height. As we are interested in medium heights, the best solution will lie somewhere in between.

We consider a frame made from thin sheets perpendicular to the surface, (see Figure 5) with free ends at each supporting point. The maximum bending stress then is given by $S = Mc/I$, where the bending moment is given by $M = Wl/6$ with $W = wl^2/2$ (the supported surface is $l^2/2$). For the distance of the maximum stress from the neutral axis we have $c = h/2$, and the moment of inertia is given by $I = \tau h^3/12$, which gives

$$S = \frac{Mc}{I} = \frac{w\, l^3}{2\tau h^2} \quad . \tag{34}$$

The maximum allowable stress is given by

$$S = \frac{S_o}{1 + \frac{1}{2000}(h/\tau)^2} \quad , \tag{35}$$

and from both formulae we can derive the following one:

$$(\tau h)^2 = \frac{\tau^4}{2(p/l)^3\, S_o/w \; - \; 1/2000} \quad . \tag{36}$$

The volume of material in one frame is phl, and there are $2A^2/l^2$ frames in the total surface A. The total volume of material in all frames then is

$$V = 2A\tau h/l \quad . \tag{37}$$

The comparison of (37) and (36) shows that the thickness, τ, can be adjusted so as to give a minimum volume of material. By minimizing (36) we obtain:

$$\left.\begin{array}{l}\tau = 0.10(w/S_o)^{1/3} l \\ h = 2.58(w/S_o)^{1/3} l\end{array}\right\} \quad h/\tau = 25.8\ . \tag{38}$$

It should be noted that the value for h/τ does not depend on material constants. If we enter formula (37) with these values we get the volume of material in all frames connecting neighboring supporting points. Next, we need a secondary system of smaller frames, and so on, until we reach a distance small enough for self-support of wire mesh. A rough estimate showed that we have to multiply (37) by about 2 in order to get the volume of all kinds of frames together:

$$V_{fr} = 4 A\tau h/l = 1.04\,(w/S_o)^{2/3}\ Al\ . \tag{39}$$

In order to find the best value for l, the distance of supporting points, we start with the minimum supporting structure defined by (33). Now, we divide the distance l_o into m parts of equal length, which means we multiply the number of members by m^2, all members having the same cross section, q_o. The total volume of material then is found from (27) and (39):

$$V_{tot} = 1.36\,\frac{w}{S}\ Ad_2m^2 + 0.897\ \left(\frac{w}{S}\right)^{2/3} Al_o/m\ , \tag{40}$$

where we have replaced S_o by 5S/4 for slender columns. By minimizing (40) with respect to m we obtain:

$$m = 0.690\left(\frac{S}{W}\right)^{1/9}\left(\frac{l_o}{d_2}\right)^{1/3} = 0.124\left(\frac{S}{W}\right)^{5/18}\left(\frac{\sqrt{q_o}}{\lambda}\right)^{1/3}, \tag{41}$$

where we have used (33) for l_o and (14) for d_2. Finally, we insert (41) into (40) and obtain:

$$V_{tot} = 1.95\left(\frac{w}{S}\right)^{7/9} A\, d_2^{1/3}\ l_o^{2/3} = 9.95 \times 10^7 \left(\frac{w}{S}\right)^{4/9} \lambda^{7/3}\, q_o^{1/3}. \tag{42}$$

In this minimized solution, the frames have exactly twice as much volume as the supporting members have. The distance of neighboring members in this solution then becomes

$$l = l_o/m = 10.3\ \left(\frac{S}{W}\right)^{2/9} \lambda^{1/3}\ q_o^{1/3} \tag{43}$$

and the number of members within the height d_2 is

$$d_2/l = 21.3\left(\frac{w}{S}\right)^{2/9}\left(\frac{\lambda}{\sqrt{q_o}}\right)^{2/3}\ . \tag{44}$$

We summarize the last results. For any receiver of a given quality, represented by

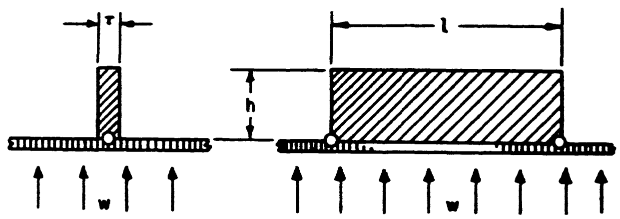

Fig. 5 Frame to hold antenna surface.
supporting points

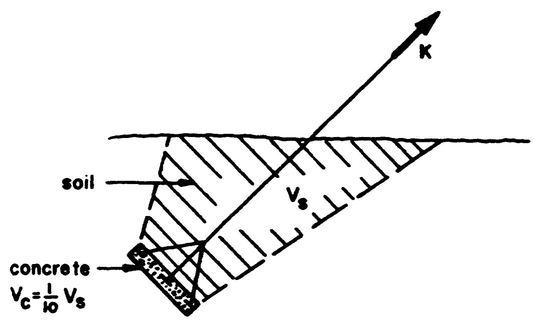

Fig. 6 Anchoring at ground.

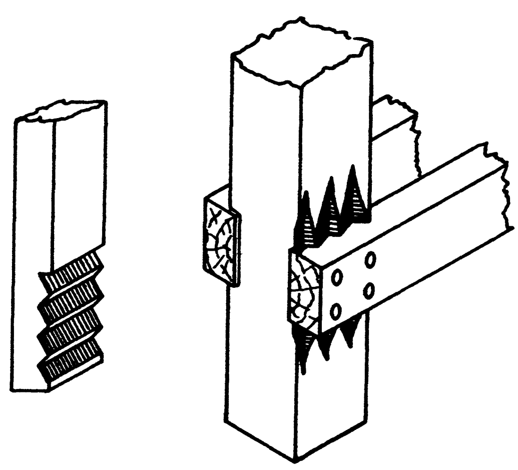

Fig. 7 Joints in wooden structure.

the noise temperature T_n, the optimum solution where feed and arms are of equal height is given in Table 2. The wavelength λ is a function of T_n and all antenna properties can easily be expressed in terms of λ by equations (14) and (15). The task of holding the antenna surface against the wind force is accomplished by a composite arrangement of longitudinal supporting members and lateral frames. By minimizing the total amount of material we arrive at a distance between members given by (43) and a total volume of material given by (42). The size of the frame then is given by (38). All of these equations contain one free parameter, the cross section of the supporting members, q_o, which should be chosen as small as possible. The total volume of material increases with $q_o^{1/3}$; if we had the choice between various members of equal proportions but different size, and if we introduce τ as the wall thickness, we get

$$V_{tot} \sim \tau^{2/3} \quad . \tag{45}$$

This means that we should choose the point where the price per pound of the material, rising with decreasing wall thickness, just rises proportional to $\tau^{-2/3}$. This defines the point of minimum cost.

3. <u>The Anchoring at the Ground</u>

The forces in the supporting members are given by (17) and (22). We add up all forces and obtain with help of (14) and (15)

$$K = \text{total force} = 1.19 \text{ wA} . \tag{46}$$

We suggest anchoring the members as indicated in Figure 6, and we impose the condition that the weight of soil be two times the total force:

$$\rho_s V_s = 2 \text{ K}$$

or

$$V_s = 2.38 \ \frac{W}{\rho_s} \ A \tag{47}$$

where V_s is the volume of soil needed to anchor the members and ρ_s is the density of soil. We adopt \$ 1/yard^3 = \$ 1.4/m^3 for excavations, and we multiply by two for the more complicated refilling, and have \$ 2.8/m^3. The members must be anchored to concrete or to wooden logs. As an example, we take concrete at a price of \$ 40/yard^3 = \$ 56/m^3 and suggest filling 1/10 of the volume with it. Excavation plus concrete then add up to \$ 8.4/m^3, and the price will be $P_a = 8.4 \ V_s$. With w = 146 kg/m^2 for the maximum wind force and ρ_s = 2000 kg/m^3 for the density we arrive at

$$P_a = 1.47 \text{ A} \qquad \begin{cases} P_a \text{ in Dollars} \\ A \text{ in m}^2 \end{cases} \tag{48}$$

There is an additional contribution to the price because the supporting members must be somewhat longer than in the previous calculations in order to reach down under ground. An estimate gives about 10% for this elongation. In the minimized solution for members and frames the price of the members is 1/2 of that of the frames or 1/3 of the total price of the structure. Thus, an elongation of the members by 10% will increase this total price by 3%, but in order to be on the safe side we will adopt 5%.

The price for excavation plus concrete in (48) is proportional to the surface, and so is the price of the wire mesh surface itself. Therefore, we include this contribution now, adopting

$$\$\, 1/m^2 \text{ for galvanized wire mesh} \tag{49}$$

with a distance between wires of about $\lambda/16$.

The price of excavation, concrete, wire mesh and elongation of members then is:

$$P_a = 2.47\, A + P_{str}/20 \tag{50}$$

or, replacing A by λ with (15) and measuring λ in m:

$$P_a = 1.78 \times 10^7 + P_{str}/20 \;\; \lambda^2 \, . \tag{51}$$

4. The Choice of the Material

In the above calculations no material constant is replaced by its numerical value; thus all of the formulae are valid for any construction material. We could not choose the material before equation (42) had been derived, but now we have to choose the material to make the decision as to the best receiver to use.

The price of the structure will be determined by

$$P_{str} = V \rho \, p, \tag{52}$$

where V is the volume of the material, ρ its density, and p the price per pound (raw material + fabrication + erection). If only the minimum supporting structure were needed, then the price would be proportional to the combination $\rho\, p/S$ of material constants, see equation (27). This is different for the minimized arrangement of longitudinal members and lateral frames where the combined volume is given by equation (42). We increase the constant of (42) by 5% according to (50) for the parts which are under ground, and the price of the combined structure then is

$$P_{str} = 1.045 \times 10^8 \, \lambda^{7/3} \;\; (w/S)^{4/9} \; q_o^{1/3} \; \rho \, p, \tag{53}$$

where q_o again is the cross section of a single supporting member. Separating the material constants we get a price factor, F:

$$P_{str} \sim \frac{\rho\, p}{S^{4/9}} \; q_o^{1/3} = F. \tag{54}$$

Values for ρ and S are obtained easily, but not so for p and q_o. The price per pound, p, will stay about constant for larger q_o but will increase for very small q_o, and the minimum of the product $p\, q_o^{1/3}$ will occur at the value of q_o where p just increases proportional to $q_o^{-1/3}$. From some examples we tried to estimate this point in a rough way. The results are shown in Table 3 and might be wrong by about 30%. The price per unit weight in Table 3 is seven times the price of the raw material; this is about twice as much as usually is adopted for larger q_0 and should be safe enough. (The values for S are taken for slender columns with a ratio of length to radius of gyration equal to 50).

TABLE 3
Adopted Material Constants and Resulting Price Factor

Material	ρ	S	p	q_0	F
	g/cm^3	kg/cm^2	$/kg	cm^2	rel. to steel
Steel	7.8	1130	1.54	3	1.00
Aluminum	2.7	740	6.30	1	1.18
Wood	0.6	110	0.70	100	0.32

Regarding the uncertainty of the adopted values for p and q_0, the difference in the price factor, F, between steel and aluminum is not very significant, but the difference between metal and wood certainly is. One disadvantage of wood might be that the lifetime of the structure gets decreased unless a high maintenance price is paid for regular painting. But this is not serious if we save more than half a million dollars in the structure. Another argument could be the difficulty of wooden joints as compared with welding or bolting. A suggestion is made in Figure 7, where all joining surfaces are supplied with ripples perpendicular to the line of force. This could be done relatively fast with a portable grinding stone of rippled surface. -- There might be some more arguments, but the difference in price is so striking that we definitely recommend taking wood as the structural material.

5. The Choice of T_n and the Total Price

The last decision we have to make is with respect to the quality of the receiver which determines the size of the antenna. The total price of receiver plus antenna will have a minimum somewhere and this then would be the best combination.

An estimate of this kind is presented in Table 4. The first column gives the noise temperature of the receiver (including feed lines, switches, etc.) and the next column is a rough guess as to the price of such a receiver. Column 3 is the observing wavelength. In column 4 we have the price of concrete, excavation and wire mesh according to (51), in column 5 the price of the structure according to (53) with the adopted values for wood of Table 3, and in column 6 the combined price of the antenna. This, plus the price of the receiver, then gives the total price of column 7. For comparison, we have added in the last column the total price calculated in the same way for steel. We see again the great supremacy of wood, especially for larger sizes. For the lowest total price we find:

$$\begin{aligned} \text{Material} &= \text{Wood} \\ \text{Receiver} &= \text{Maser with } 30^\circ \text{ noise temperature} \\ \text{Total Price} &= \$438{,}000 \end{aligned} \qquad (55)$$

These values might be altered by time considerations. Masers with the specified qualities are supposed to be available for the adopted price within the near future, but at present there is no way of telling exactly how long it will take until the better ones are developed and could be produced in an economical way. If one waits with the antenna until this question is settled, the values (55) are all right. But if the design of an antenna of this kind should start right now, it would be better to design it for a maser of about 50° or more.

TABLE 4

Estimated Prices

				Wood				Steel
Receiver	T_n	P_{rec}	λ	P_{gr}	P_{str}	P_{ant}	P_{tot}	P_{tot}
	°K	$ 10^3	cm	$ 10^3	$ 10^3	$ 10^3	$ 10^6	$ 10^6
Maser	20	200	5.3	50	190	240	0.440	0.85
	30	110	6.1	66	262	382	0.438	1.01
	50	80	7.4	97	413	510	0.590	1.49
	70	70	8.4	126	561	687	0.757	1.98
Parametric amplifier	100	62	9.8	171	793	964	1.03	2.74
	200	52	13.2	310	1600	1910	1.96	6.33
	400	46	17.7	557	3170	3730	3.78	10.7
Vacuum tube receiver	624	30	21.3	808	4880	5690	5.72	16.3

TABLE 5

Some structural details
l = distance at surface between supporting members
τ = width of frames
h = height of frames
V = total volume of wood
D = feed movement for 50 min. of arc.
B = largest distance between any two antenna parts

T_n	l	τ	h	V	D	B
°K	m	cm	cm	m^3	m	km
30	6.34	3.00	77.4	624	24.9	1.52
50	6.76	3.20	82.6	984	30.2	1.84
70	7.07	3.34	86.3	1336	34.6	2.10

The values for antenna sizes are already given in Table 2. In Table 5 we give some additional structural details. The second column shows the distance between supporting members according to (43) which then is the length of the frames. Width and height of frames are given in column 3 and 4, according to (38). Column 5 gives the total volume of wood in members and frames together according to (42). The feed movement for the enlarged sky coverage of equation (13) is shown in column 6, and the largest distance of any two antenna parts in column 7 according to Figure 1.

6. Additional costs and the final price	Estimated Price (Thousands of dollars)
a. Many smaller items have been neglected in the structure, but they might add up considerably (adjustment of surface, screws, bolts...). On the other side, a solid sheet of 3 cm x 80 cm for the frame is quite a waste of material which calls for a better solution. This should pay for the smaller items.	0
b. Accurate measurements. The first thing needed is a very dependable survey of the whole region, and the adjustment of the surface should be made with high precision radar techniques.	70
c. Shielding wire mesh against spillover (area equal to antenna surface)	30
d. Feeds. One feed is fixed to the ground, the second one is movable sideways by 30 m. Adopting aluminum sheets of 3 mm wall thickness with wooden frames and structures, the price of a horn-type feed is estimated at $30,000 if fixed to the ground, and $70,000 additional if movable:	130
e. Electronic equipment in addition to receiver price	20
f. Buildings, roads, power lines, etc.	50
Total	300

The above total is nearly independent of the choice of receiver. The highest entry is the feeds; their dimensions increase with the wavelength but the needed accuracy decreases, which should give about a constant price. Thus the final prices are the following:

T_n	Final price
30°	$ 740,000
50°	890,000
70°	1,060,000

These prices already contain many safety factors and are supposed to be realistic. Because of the uncertain availability of extremely low-noise masers in the near future, we would most strongly recommend the antenna with the 50° maser.

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