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Confirmation Of S. Srikanth's Pointing Coefficients For GBT Single Subreflector Optics.

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ABSTRACT

This note presents independent ray trace calculations to check the values of GBT pointing coefficients for GBT single subreflector optics given in a note of March 6, 1991 by S. Srikanth. These are pointing offsets caused by translations, rotations, or focal length change of the GBT optical elements (main reflector, subreflector, Gregorian focus feed horn, prime focus feed horn). The ray trace derived error coefficients agree in sign and magnitude with those of Srikanth to a few percent.

1 Introduction

Dated March 6, 1991 a short memorandum was filed by S. Srikanth: "Pointing Coefficients for Single Subreflector Optics." This memorandum, Green Bank archive document GBRN 0006995, presented the results of calculations of pointing offsets caused by perturbations in the optical elements forming the Green Bank Telescope. Included in the memorandum was a comparison with the results of pointing error coefficient calculations by R. Levy of Jet Propulsion Laboratories (dated January 23, 1990). The two sets of calculations agreed in some coefficients but differed by factors of two in others. In neither case were details of the calculations presented.

Recently, John Payne questioned whether those results had been independently validated. It was decided that an optical ray trace program should be run to check the coefficients presented in the above memorandum. This has been carried out using the BEAM 3 optical ray trace codes (Stellar Software, Berkeley, CA). The results are presented in Table 2. The geometric assumptions underlying the present calculations are given in the next section of this document. The setup of the ray tracing files is discussed in a subsequent section. The ray tracing files themselves and a copy of Srikanth's memorandum are appended.

2 The Design Telescope Optical Geometry

The starting point for our discussion is the geometry of the reference optical telescope. The design of the GBT optics is given in GBT Memo 155, "*A Summary of the GBT Optics Design*" by R. Norrod and S. Srikanth, March 1996. A detailed analysis of this design is given in GBT Memo 165, "*GBT Coordinates And Coordinate Transforms*" by M.A. Goldman, February 1997.

For the purposes of the present document, a summary of the relevant features of the reference optical telescope (which we will also refer to as the design telescope) are given below.

The primary reflector surface is an offset portion of a paraboloid of revolution of focal length $f_p = 60$ meters. We call this paraboloid the "design paraboloid" or the "parent paraboloid." The plane parabola which generates this surface has curvature at its vertex point, R_d, which can be shown to be: $C(R_d) = 1 / (2 \cdot f_p) = 0.00833333 \text{ m}^{-1}$. (This curvature is needed to set up the ray tracing files.)

The geometry of the reference optical telescope is described in relation to the telescope's main reflector reference frame coordinate system. This is a right hand Cartesian coordinate system whose origin is located at the vertex point of the parent paraboloid. We assume that it is rigidly embedded in the tipping structure of the telescope. We will call the coordinate axes X_{rd}, Y_{rd}, Z_{rd}. The Z_{rd} axis points from the vertex to the focal point of the parent paraboloid. The plane of the Y_{rd} and Z_{rd} axes defines the plane of symmetry of the design telescope. The Y_{rd} axis points towards the primary reflector which is located in the half-space $Y_{rd} > 0$. The X_{rd} axis is normal to the telescope's plane of symmetry, is parallel to the elevation axis of the telescope, and points towards the right side of the telescope (towards the man lift). The subscript "*rd*" on the axis designates that the coordinates refer specifically to the main reflector coordinate system for the design telescope.

The secondary reflector is a surface patch on a prolate ellipsoid of revolution, the "design ellipsoid" or "parent ellipsoid." Let us call the focal points of this ellipsoid F_0 and F_1. The design focal spacing is $|F_0 F_1| = 11 \text{ meter} \equiv 2 \cdot f_e$. The design eccentricity is $e = 0.528$. These values produce calculated values of the ellipsoid semi-axis lengths: $a = 10.416667$ meter and $b = c = 8.846296$ meter. The ellipsoid focus F_0 is defined to coincide with the parent paraboloid focus, and is defined to be the prime focus of the design telescope. The major axis of the ellipsoid lies in the telescope plane of symmetry and is defined to make an angle $\beta = 5.570°$ with the parent paraboloid axis. The focal point F_1 is defined to be the Gregorian focal point of the design telescope. (Figure 1 and Figure 2).

The subreflector surface patch is symmetric with respect to the design telescope's plane of symmetry. It has an approximate center point, I_1, which lies in the plane of symmetry. The point I_1 together with the point F_1 define an optical axis for the design telescope. The design telescope receiver room is located so that the phase center of each (non-offset) receiver room feed horn will be at the Gregorian focus point F_1 and the horn's electromagnetic axis will lie along the line $F_1 I_1$. The plane through F_1 perpendicular to line $F_1 I_1$ is the Gregorian focal plane of the design telescope. For the design telescope, an optical ray which starts at F_1 towards I_1 will reflect specularly through F_0, reflect specularly from the parent paraboloid and emerge parallel to the paraboloid axis.

Specifically, the location of the point I_1 is defined by:

(2.1) $\angle F_0 F_1 I_1 = \alpha = 17.899°$, and it follows by simple trigonometry that:

(2.2) $\angle F_0\, I_1\, F_1 = \gamma = 36.127028°$,

and the ray $F_0\, I_1$ makes an angle with the $+Z_{rd}$ axis of

(2.3) $\alpha + \gamma - \beta = 48.456028°$,

(2.4) $|F_0\, I_1| = r_2 = 5.7341748\,\mathrm{m}$.

To set up the ray trace files we will need to reference the parent ellipsoid to one of its mathematical vertices and compute the curvature of its generating ellipse at this vertex. We will reference the parent ellipsoid to the upper vertex point on its major axis, which we will call V_{el}. It can be shown that the curvature at a vertex of the generating ellipse is $C(V_{el}) = \pm(a\,/\,b^2)$. We also need to compute a "shape factor" for the ellipsoid, defined by:

(2.5) Ellipsoid shape factor $= 1 - e^2$ $(= 0.721216$ for the parent ellipsoid$)$.

In Table 1 we list the coordinates of the design telescope reference points, in the main reflector coordinate system.

Table 1. Design Telescope Reference Point Coordinates (Meters).

Point	X_{rd}	Y_{rd}	Z_{rd}
R_d	0	0	0
F_0	0	0	$f_p = 60$
F_1	0	$-2f_e \cdot \sin\beta = -1.0676797$	$f_p - 2f_e \cdot \cos\beta = 63.802874$
I_1	0	$-r_2 \cdot \sin(\alpha+\gamma-\beta) = -4.2917258$	$f_p + r_2 \cdot \cos(\alpha+\gamma-\beta) = 63.802874$
V_{el}	0	$(a - f_e) \cdot \sin\beta = 0.4772204$	$f_p + (a - f_e) \cdot \cos\beta = 64.893452$

3 Ray Trace Computations Of Error Pointing Coefficients

The Beam 3 ray trace codes are set up to compute an optical system in the following way. An optics (.OPT) file is generated which lists sequentially the properties of the optical surfaces and optical indexes to be traversed sequentially. A ray (.RAY) file is also generated, which lists an initial point and direction for each ray which is to traverse the optical system. Rays are characterized by X, Y, Z Cartesian coordinates and direction cosines U, V, W to the respective X, Y, Z coordinate axes. The optical code files are configured so that code X, Y, Z axes correspond to telescope X_{rd}, Y_{rd}, Z_{rd} axes respectively. Aspheric surfaces of revolution generated by rotation of a plane conic curve, are described by: a vertex point location, the curvature of the generating conic at its vertex, a shape factor if the surface is ellipsoidal, and local tilt and pitch of the surface's axis of symmetry to the coordinate axes. The ray trace calculation generates a vertex list (.VXL file) giving the coordinates of the intersection point of each ray with the optical surfaces, sequentially, and the direction cosines of the ray on exit from the surface.

Ray trace computations of the design telescope were carried out initially to confirm the setup of the optics and ray files. A ray from the Gregorian focus point F_1 to the parent ellipsoid subreflector's reference point I_1 should pass through the prime focus point F_0 of the parent paraboloid and, after reflection from the paraboloid, be directed parallel to the paraboloid's axis. This was confirmed for the files GBT.OPT and GBT.RAY which were configured to model the design telescope.

Optics files are generated to study the behavior of a reference ray from the design telescope Gregorian focus towards the design subreflector reference point when the telescope is perturbed from its design configuration by motions of single optical elements. The deviation of the exit ray direction, after reflection from the perturbed telescope's paraboloid, is examined when the subreflector is translated (by changing a coordinate of the ellipsoid vertex) or rotated (by changing the tilt or pitch of the ellipsoid). It is examined when the paraboloid is tilted or pitched or translated, by similar means. For these perturbations the original file GBT.RAY is used together with a separate optics file GBT_n.OPT which configures the perturbed optical telescope. To study the effect of feed horn translation (or feed room translation when the active feed horn is fixed relative to the receiver room) the original GBT.OPT optics file is used, together with a separate ray file GBT02.RAY which translates the origin of the reference ray but preserves its initial direction. The exit ray direction angle shift is computed per unit optical element shift in coordinate or orientation.

A right hand sign convention is used for rays rotating about the positive coordinate axes. A positive rotation of an exit ray about the $+X_{rd}$ axis corresponds to the ray leaving with an increased elevation.

The perturbations of the telescope configuration are small, and the small angle approximation: sin(angle) =angle, cos(angle) = 1 holds for deviations of the exit reference ray angle to the $+Z_{rd}$ axis. If either the U or the V direction cosine of the exit ray vanishes, and the other of these two cosines is small, one may read the output file directly to find the ray deviation angle. For such cases the rotation, in radians, of the exit ray about the $+X_{rd}$ axis is equal to $-V_{final}$ when U_{final} vanishes; the rotation of the exit ray to the $+Y_{rd}$ axis is equal to $+U_{final}$ when V_{final} vanishes. Here $U_{final}, V_{final}, W_{final}$ are the direction cosines of the ray after leaving the final optical surface.

In our study of perturbed telescope configurations we have restricted perturbations of optical component orientation to the following cases. When the paraboloid surface alone is perturbed, by a change in orientation, we leave the paraboloid vertex fixed at the point R_d. That is, the perturbed telescope's paraboloid vertex point remains unmoved with respect to the subreflector and Gregorian feed, although the paraboloid becomes tilted about the X_{rd} or the Y_{rd} axis. When the subreflector surface alone is perturbed, by a change of tilt or pitch, we leave the subreflector reference point I_1 fixed with respect to the paraboloid and feed. The Srikanth and Levy results did not explicitly cite the invariant point of these perturbations. It is not clear, for example, whether the ellipsoid reference point or the ellipsoid vertex was held fixed in perturbing the telescope configuration in those results.

As an example of how the optics file is reconfigured for a perturbed telescope, let us give the example of the configuration where the subreflector is tilted by a small angle, θ radians, about the $+X_{rd}$ axis, while leaving the subreflector reference point I_1 fixed. The ellipsoid vertex point transforms from point V_{el} to a new point, V'_{el}. Under this perturbation of the telescope configuration: $R_d \rightarrow R_d$, $F_0 \rightarrow F_0$, $F_1 \rightarrow F_1$, $I_1 \rightarrow I_1$. Coordinates of the latter points remain fixed. The coordinates of the ellipsoid vertex of the perturbed telescope are given by:

$$(3.1) \qquad \begin{bmatrix} X_{rd}(V'_{el}) \\ Y_{rd}(V'_{el}) - Y_{rd}(I_1) \\ Z_{rd}(V'_{el}) - Z_{rd}(I_1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X_{rd}(V_{el}) \\ Y_{rd}(V_{el}) - Y_{rd}(I_1) \\ Z_{rd}(V_{el}) - Z_{rd}(I_1) \end{bmatrix}$$

The small angle approximation is valid and (3.1) simplifies to:

$$(3.1) \qquad \begin{bmatrix} X_{rd}(V'_{el}) \\ Y_{rd}(V'_{el}) \\ Z_{rd}(V'_{el}) \end{bmatrix} = \begin{bmatrix} X_{rd}(V_{el}) \\ Y_{rd}(V_{el}) - \theta \cdot (Z_{rd}(V_{el}) - Z_{rd}(I_1)) \\ Z_{rd}(V_{el}) + \theta \cdot (Y_{rd}(V_{el}) - Y_{rd}(I_1)) \end{bmatrix}$$

The optics file parameter TILT is modified from β degrees to $\beta + \theta \cdot (180/\pi)$ degrees. The optics file data entries are modified to indicate the new ellipsoid vertex coordinates and ellipsoid tilt.

The optics files GBT_n.OPT each describe the changes made to the design telescope GBT.OPT file to reconfigure the file to generate the individual perturbations of the telescope configuration. The final direction cosine entries for each ray in the GBT_n.VXL ray trace output files provide the exit ray deviation angles when evaluated as described above.

4 Results

The ray trace results are summarized and listed in Table 2, together with a comparison of Levy's 1990 JPL results and Srikanth's 1991 results. The coefficients have also been converted to units of arcseconds per millimeter, using the conversion 1 radian/meter = 206.265 arcsec/mm.

$\Delta\phi = \frac{\partial\phi}{\partial\Theta}\Delta\Theta$

Table 2. Differential Error Pointing Coefficients

Element Shift	Pointing Coefficient				arcsec / mm	
	BEAM3, 1999	Srikanth 1991	JPL	unit	1999	1991
Subrefl. Rot.about an						
axis parallel to $+X_{rd}$ axis	+0.159 (EL)	+0.1504	+0.0657	mrad/mrad		
Subreflector $+Y_{rd}$ Shift	+0.0143 (EL)	+0.0141	+0.0120	mrad/mm	+2.950	+2.908
Subreflector $+Z_{rd}$ Shift	+0.0115 (EL)	+0.0103	+0.0089	mrad/mm	+2.372	+2.125
Feed $+Y_{rd}$ Shift	-0.0051 (EL)	-0.0051	-0.0031	mrad/mm	-1.052	-1.052
Feed $+Z_{rd}$ Shift	-0.0011 (EL)	-0.0012	-0.0005	mrad/mm	-0.227	-0.248
Parab. Rot. about $+X_{rd}$ axis	+1.552 (EL)	+1.5490	+1.5453	mrad/mrad		
Paraboloid $+Y_{rd}$ Shift	-0.0092 (EL)	-0.0091	-0.0090	mrad/mm	-1.898	-1.877
Paraboloid $+Z_{rd}$ Shift	-0.0104 (EL)	-0.0092	-0.0087	mrad/mm	-2.145	-1.898
Prime Focus Beam Deviation	(EL)	+0.9280		mrad/mrad		
Factor in (Y_{rd}, Z_{rd})-plane						
Subrefl. Rot. about an	+0.131 (AZ)	+0.1336		mrad/mrad		
axis parallel to $+Y_{rd}$ axis						
Subreflector $+X_{rd}$ Shift	-0.0191 (AZ)	-0.0183		mrad/mm	-3.940	-3.775
Feed $+X_{rd}$ Shift	+0.0053 (AZ)	+0.0051		mrad/mm	+1.093	+1.052
Parab. Rot. about $+Y_{rd}$ axis	+1.832 (AZ)	+1.7710		mrad/mrad		
Paraboloid $+X_{rd}$ Shift	+0.0139 (AZ)	+0.0130		mrad/mm	+2.867	+2.681
f_p change (+ → increase)	-0.0104 (EL)	-0.00924		mrad/mm	-2.145	-1.906
Prime Focus Beam Deviation						
Factor in (X_{rd}, Z_{rd})-plane	(AZ)	+0.9400		mrad/mrad		

5 Pointing Error Coefficients From Exit Ray Deviations

The ray trace program results provide exit ray deviation angles per unit displacement or rotation of an optical element of a conceptual telescope, the design telescope. We assume that these results will be used to provide error estimates of pointing errors of the as-built GBT telescope, given the errors of placement of the real optical elements. These error estimates would be used to provide *a-priori* correction terms in the telescope pointing series. To do this, calculated rotations of the reference exit ray from the $+Z_{rd}$ axis must be converted to error correction terms of the form ΔEL_{corr} and $(\cos EL)\cdot\Delta AZ_{corr}$. Exit ray rotations are sufficiently small, for realistic optical element perturbations, that error terms generated by individual optical element perturbations are separately additive. We next derive pointing error and correction terms generated by small exit ray rotations about the $+X_{rd}$ and $+Y_{rd}$ axes.

We start with the reference exit ray from the design telescope. This ray starts at the design Gregorian focus F_1 and initially travels to the parent ellipsoid reference point I_1. After reflections from the parent ellipsoid and parent paraboloid it exits the telescope parallel to the $+Z_{rd}$ axis. We denote a unit vector in the direction of this design telescope exit ray by $\widehat{r}_d$.

When the reference telescope is oriented at azimuth angle AZ and elevation angle EL, the main reflector frame unit basis vectors can be represented with respect to the ground coordinate frame of the GBT by unit vectors:

$$
\begin{aligned}
&\widehat{X}_{rd} = (\cos AZ)\cdot\widehat{E} + (-\sin AZ)\cdot\widehat{N}\ , \\
(4.1)\qquad &\widehat{Y}_{rd} = (\sin EL\cdot\sin AZ)\cdot\widehat{E} + (\sin EL\cdot\cos AZ)\cdot\widehat{N} + (-\cos EL)\cdot\widehat{Zen}\ , \\
&\widehat{Z}_{rd} = (\cos EL\cdot\sin AZ)\cdot\widehat{E} + (\cos EL\cdot\cos AZ)\cdot\widehat{N} + (\sin EL)\cdot\widehat{Zen} = \widehat{r}_d
\end{aligned}
$$

We use the notation $\widehat{N}$ (North), $\widehat{E}$ (East), $\widehat{Zen}$ (Zenith), for the ground frame base vectors because we have previously assigned X, Y, Z as base vectors for the ray trace codes.

Let us assume that the telescope configuration is perturbed by a single optical element shift or rotation, desribed by a telescope design parameter p so that the exit ray is rotated by a small angle $\theta(p)$ about the $+X_{rd}$ axis. The exit ray then rotates to the direction of the vector

$$(4.2)\qquad \overrightarrow{r} = \widehat{r}_d + \Delta\overrightarrow{r}(p) = \widehat{r}_d + \theta(p).[\widehat{X}_{rd}\times\widehat{Z}_{rd}] \quad \text{which gives}$$

$$(4.3)\qquad \Delta\overrightarrow{r} = -\theta(p)\cdot\widehat{Y}_{rd}\ .$$

The rotated ray remains in the median plane of the design telescope; it has only rotated in elevation. The $+X_{rd}$ axis is the elevation axis of the design telescope; the perturbation rotates the exit ray by an angle $(\Delta EL(p))_{exit\_ray} = +\theta(p)$ in elevation. This may be considered to be an error $EL_{err}(p)$ in the pointing elevation angle, which requires a pointing correction $EL_{corr}(p) = -EL_{err}(p)$. That is,

$$(4.4)\qquad EL_{corr}(p) = -EL_{err}(p) = -(\Delta EL(p))_{exit\_ray} = -\theta(p)\ .$$

If θ is a smooth function of p (when other telescope parameters are held fixed), then for small changes in p we have

$$(4.5)\quad (\Delta EL)_{exit\_ray} = (\frac{d\theta}{dp}) \cdot \Delta p \ .$$

The value of $(\frac{d\theta}{dp})$ found by the ray tracing then can be considered as a differential elevation pointing error rate coefficient when a small change in parameter p produces a small exit ray rotation about the $+X_{rd}$ axis.

Let us now consider the case that a small change in a general telescope parameter q rotates the exit ray by a small angle $\phi(q)$ about the $+Y_{rd}$ axis, where ϕ is a smooth function of q. In this case the perturbation rotates the exit ray from the direction of $\hat{r}_d$ to the direction of the vector

$$(4.6)\quad \vec{\rho} = \hat{r}_d + \Delta\vec{\rho} = \hat{r}_d + (\phi \cdot \hat{Y}_{rd}) \times \hat{Z}_{rd} = \hat{Z}_{rd} + (\phi \cdot \hat{X}_{rd}) \ .$$

The vector increment $\Delta\vec{\rho}$ is always horizontal since $\hat{X}_{rd}$ has no component in zenith; the exit beam is thus rotated in azimuth, but not in elevation. The exit beam's azimuth shift, $(\Delta AZ(q))_{exit\_ray} = \Delta AZ$ is found as follows. When $\hat{r}_d$ is rotated in azimuth from AZ to $AZ + \Delta AZ$ its increment, $\Delta\vec{\rho}$, can be calculated using the last equation of (4.1). We have,

$$(4.7)\quad \phi \cdot \hat{X}_{rd} = (\cos EL \cdot \cos AZ \cdot \Delta AZ) \cdot \hat{E} + (-\cos EL \cdot \sin AZ \cdot \Delta AZ) \cdot \hat{N} = (\cos EL) \cdot (\Delta AZ) \cdot \hat{X}_{rd}$$

This gives, by arguments similar to those presented above,

$$(4.8)\quad (\cos EL) \cdot AZ_{corr}(q) = -(\cos EL) \cdot AZ_{err}(q) = -(\cos EL) \cdot (\Delta AZ(q))_{exit\_ray} = -\phi(q) \ ,$$

$$(4.5)\quad (\Delta AZ)_{exit\_ray} = (\frac{d\phi}{dq}) \cdot \Delta q \ .$$

We see that the exit ray deviation angle rate $(\frac{d\phi}{dq})$ provides a differential azimuth pointing error rate for the case of exit ray rotations about the $+Y_{rd}$ axis.

6 Summary

The ray trace computations carried out using the BEAM 3 codes agree well with the 1991 results of Srikanth. Agreement is typically within 3%, at worst 8%. The differences may depend on the precise values of telescope parameters used in the 1991 and 1999 computations, and the precise location of the axis about which the subreflector rotates. In all cases, the signs of the coefficients were in agreement. The results confirm those of Srikanth, rather than those of Levy, where they disagree.

The coefficients also directly give error pointing coefficent rates. Some care should be taken in interpretation of the coefficients'signs to be used in applying the results. Here, the viewpoint taken is that when an optical component is moved from its design telescope configuration, the exit ray (which can also be considered as the reverse of incoming sky ray) is shifted to a wrong position on the sky. The shift rates in elevation angle, and azimuth angle multiplied by cosine of elevation angle have the same signs as the respective coefficients computed by Srikanth. If one wishes correct the as-built telescope in pointing by making use of statically measured optical component shifts from their design values, or measured under dynamical observing conditions, one should examine precisely how the available information on component shifts is to be used. The physical significance of the ray shifts should be examined, before entering calculated component error pointing correction terms into the telescope pointing equations.

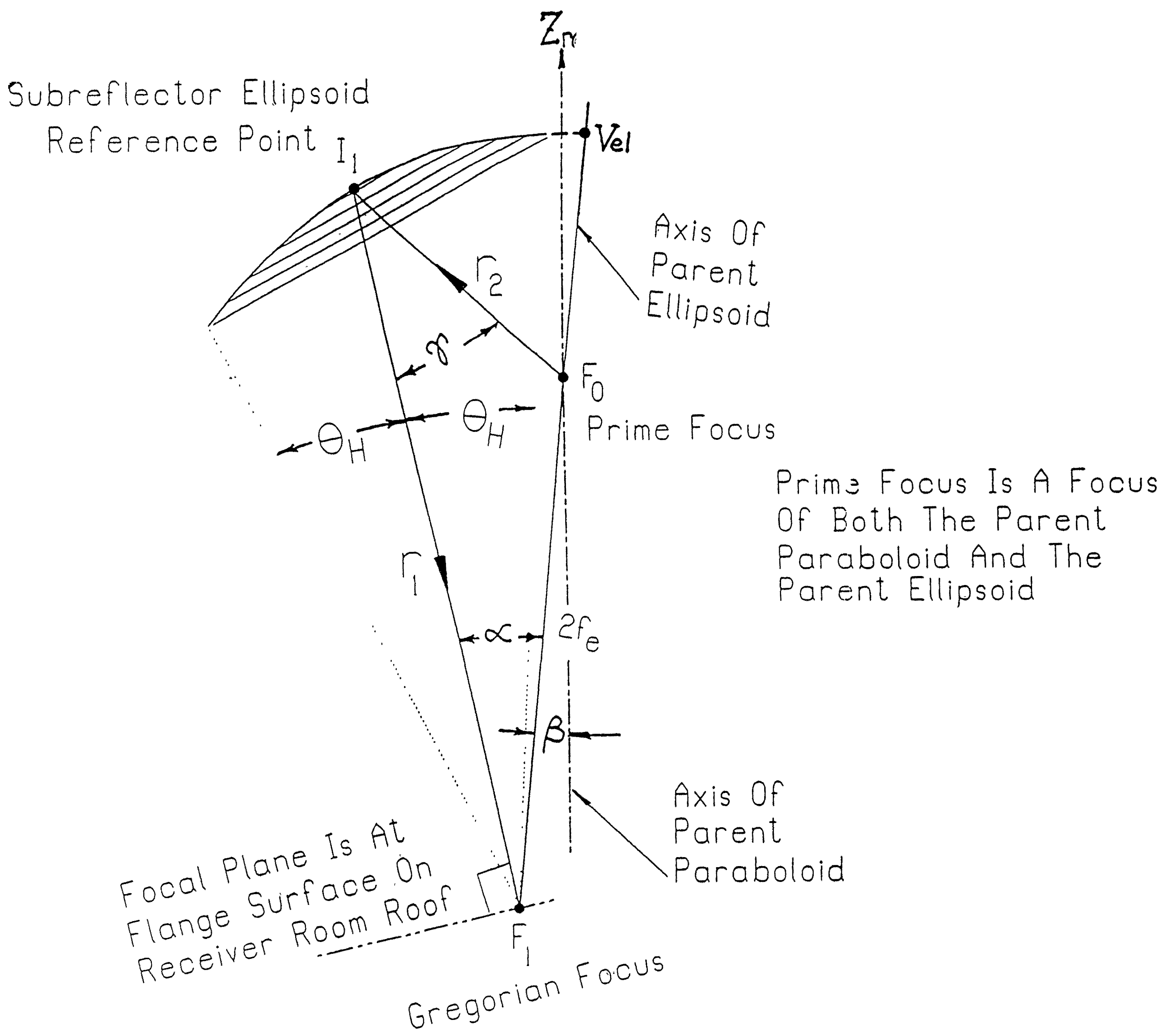

Figure 1. The Subreflector Ellipsoid Reference Geometry

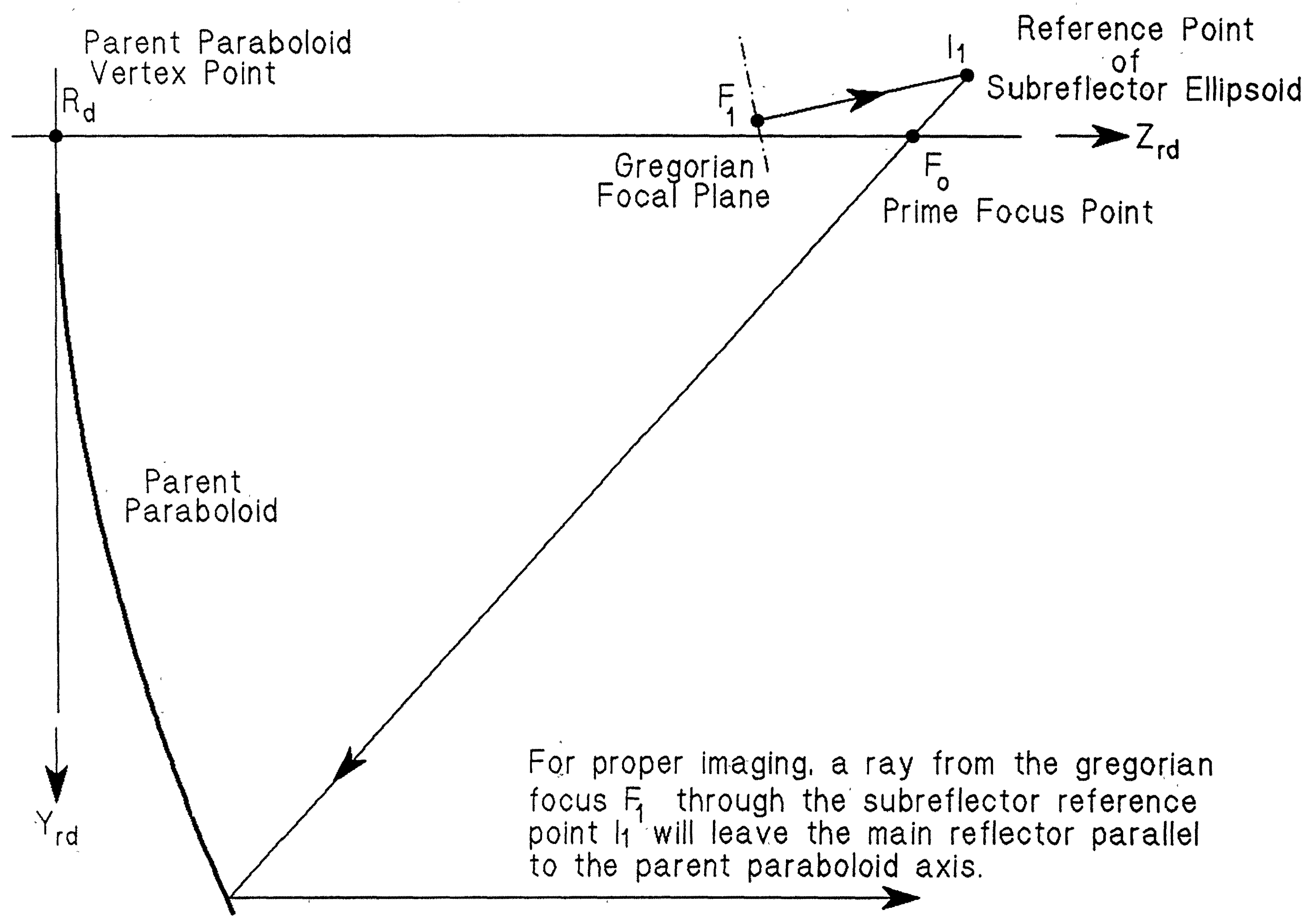

Figure 2. Optical Geometry of the Design Telescope.