**MULTIPLE MIRROR TELESCOPE OBSERVATORY**

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12 METER MILLIMETER WAVE TELESCOPE
MEMO No. 16

4202

MEMORANDUM TO: M. A. Gordon

FROM: B. L. Ulich

RE: Results of spherical aberration tests on NRAO 11-M telescope

DATE: March 20, 1981

On November 14, 1979, I carried out a series of measurements on the 11-m telescope in an attempt to determine the effect of spherical aberration errors on the telescope gain. I used a cooled mixer receiver at 89.6 GHz ($\lambda$ = 3.35 mm) and observed Jupiter with one beam produced by nutating the subreflector 7 arc minutes at 4.2 Hz. The measured aperture efficiency was 32% and the zenith optical depth was 0.07.

Malacara (1978) suggests using a series of annular diaphragms in the aperture of the optical system to diagnose spherical aberration. Then the best focus for each annular region can be determined. In this way we can measure the longitudinal spherical aberration curve and from that infer the transverse spherical aberration in the nominal focal plane. I did this by covering the subreflector with annular zones of absorbing material, leaving a single annulus uncovered. Table I lists the zones for which observations of Jupiter were made.

The inner and outer radii for each zone were chosen so that (if the surface errors are uniformly distributed over the reflector surface) the signal from each of the four zones would be 25% of the total signal. To make these calculations I used the measured feed pattern at 3.3 mm, which has a subreflector edge taper of -18dB (Gustincic feed only). Then I derived a closed-form analytic expression for the fractional power received from a given zone using the taper efficiency formula:

$$\eta = \frac{\left[ (r^2/2 - ar^4/4) \Big|_{r_1}^{r_2} \right]^2}{(r^2/2 - ar^4/2 + a^2r^6/6)\Big|_{r_0}^{1}} \quad . \tag{1}$$

In equation (1), $r_0$ is the minimum possible normalized radius of the aperture, and the maximum radius is, of course, unity. The limits $r_1$ and $r_2$ are the edges ($r_1 \leq r_2$) of the annular zone being illuminated. The constant a depends on the feed taper according to the following approximate expression for the feed power gain:

$$G = (1 - ar^2)^2. \tag{2}$$

For an edge taper of -18dB, a = 0.874.

Also listed in Table I are the results of focus, beamwidth, and flux density measurements of Jupiter. A comparison of the flux densities measured from each zone shows that three are equal but zone III is significantly, but not drastically, lower. Remembering that I have refocused the subreflector for maximum signal from each zone, I can calculate the total flux density I would expect to measure if all the zones were added in phase. This quantity is simply given by

$$S_{MAX} = (\sqrt{67} + \sqrt{66} + \sqrt{52} + \sqrt{67})^2 = 1005 \text{ Jy}. \quad (3)$$

In fact I measured only 926 Jy which is about 8% less. I believe this is a realistic estimate of the gain loss at 3 mm due to large scale zonal errors in the main reflector (at an ambient temperature of 7°C). It would certainly be interesting to repeat these measurements at an extremely high or low temperature to see if the spherical aberration gain loss is temperature-dependent. The individual zone flux densities show that the region from 55% to 70% of the main reflector is only about 78% as efficient as the remainder of the surface. The outer edge, however, behaves as if it were just as efficient as the inner part. I conclude, therefore, that small-scale surface errors are relatively uniformly distributed over the main reflector, and the simple statement that the outer part of the dish is much worse than the inner part is probably not true, at least as far as machining errors are concerned.

Table I also contains measured half-power beamwidths. These seem consistent except in the case of zone IV, where the azimuth beamwidth is too large. This is probably the result of the feed leg accident, and contributes to the larger azimuth beamwidth relative to the elevation for the whole dish. Figure 1 is a plot of the beamwidth data.

Table I also contains the pointing offsets for maximum gain. If these vary from zone to zone, it means that the annular zones are tilted with respect to each other, resulting in linear phase errors when the signals from each zone combine and therefore in reduced gain. Zones I and III show some tilt in azimuth, and zones I and IV show some elevation tilt. In all cases, the tilts are about one-tenth of the beamwidth or less, and the resulting gain loss is small (a few percent).

The best axial focus of each zone from Table I is seen to vary significantly, indicating that the focal length of the best-fit paraboloid to each zone is different. The variation is not as simple as pure spherical aberration, however, since the focal length does not vary monotonically with radius. Zones II and IV have shorter focal lengths than zones I and III by a large distance - about 6 mm. Figure 2 is a plot of relative focal length versus zone radius. Superimposed of these data is a curve calculated from mechanical surface measurements by Payne and Hollis (1975) for the focal length of a zone from the center of the dish out to a given radius as a function of that radius. Qualitatively one can see that these data agree in direction and roughly in magnitude. Thus both direct and indirect measurements show that the focal length of the telescope varies significantly with radius. This is not entirely surprising since the surface was machined by rotating a cutting tool in circles about the central axis.

Quantitatively the relative gain loss due to pure spherical aberration is approximated by (Born and Wolf 1970):

$$G/G_0 \approx 1 - 0.22\ E^2 \qquad (3)$$

where E is the rim phase error in wavelengths. The rim deflection is related to the change in focal length by

$$E = \frac{\Delta f}{8\ (F/D)^2}\ , \qquad (4)$$

where $\Delta f$ is the change in focus in wavelengths and F/D is the focal ratio of the paraboloid. For the 11-m telescope, F/D = 0.8 and E = 0.195 $\Delta f$. Thus the gain reduction as a function of focus change is

$$G/G_0 \approx 1 - 0.0084(\Delta f)^2. \qquad (5)$$

For $\Delta f$ = 2 wavelengths (6.7 mm) the gain reduction would only be about 3%. One can see that for spherical aberration to produce large gain reductions, the focus must change considerably.

We know that the rim of the dish deflects considerably with temperature; according to Figure 10 in Payne and Hollis (1975), the rim moves upward by about 0.43 mm/$^{\circ}$C. At 3.3 mm the gain reduction as a function of temperature difference $\Delta T$ from the best gain temperature is given by

$$G/G_0 \approx 1 - 0.0037\ (\Delta T)^2. \qquad (6)$$

For $\Delta T = 5^{\circ}$C the gain is reduced by 9% and for $\Delta T = 10^{\circ}$C by 37%. This is somewhat larger than the observed values of 8% loss at 10$^{\circ}$C change and 14% loss at 15$^{\circ}$C change (Ulich and King 1977).

In conclusion, I make the following points:

(1) The dish focal length varies with radius in a nonmonotonic fashion.

(2) The small-scale surface errors are rather uniformly distributed over the surface.

(3) The gain loss at 7$^{\circ}$C due to focus errors is rather small ($\leq$ 8%) at 3 mm, but will be much greater at 1 mm wavelength.

(4) At other temperatures spherical aberration is large enough to cause the entire gain reduction which we have observed.

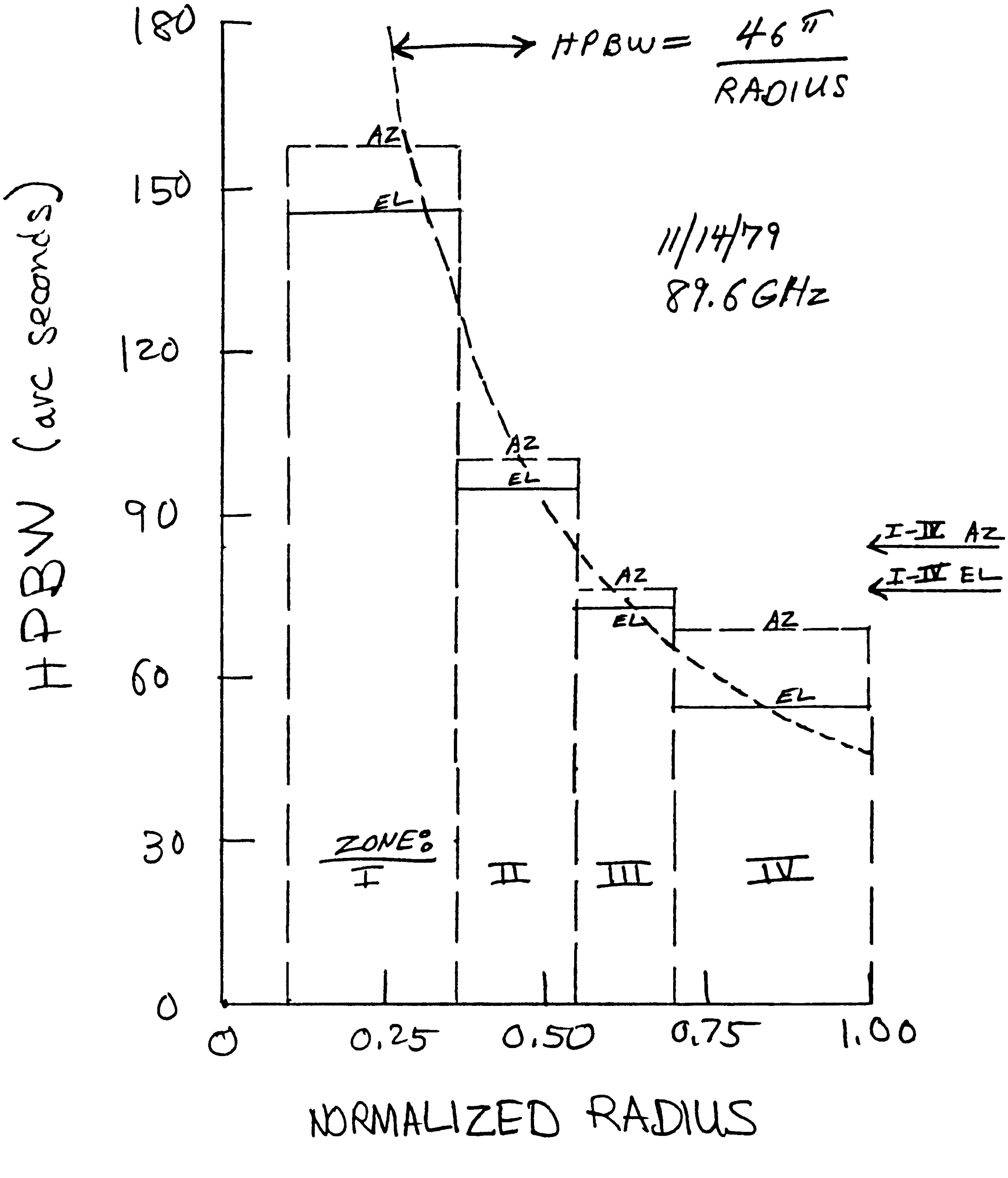
cc: Findlay
Hollis
Howard
Napier
Payne

TABLE I

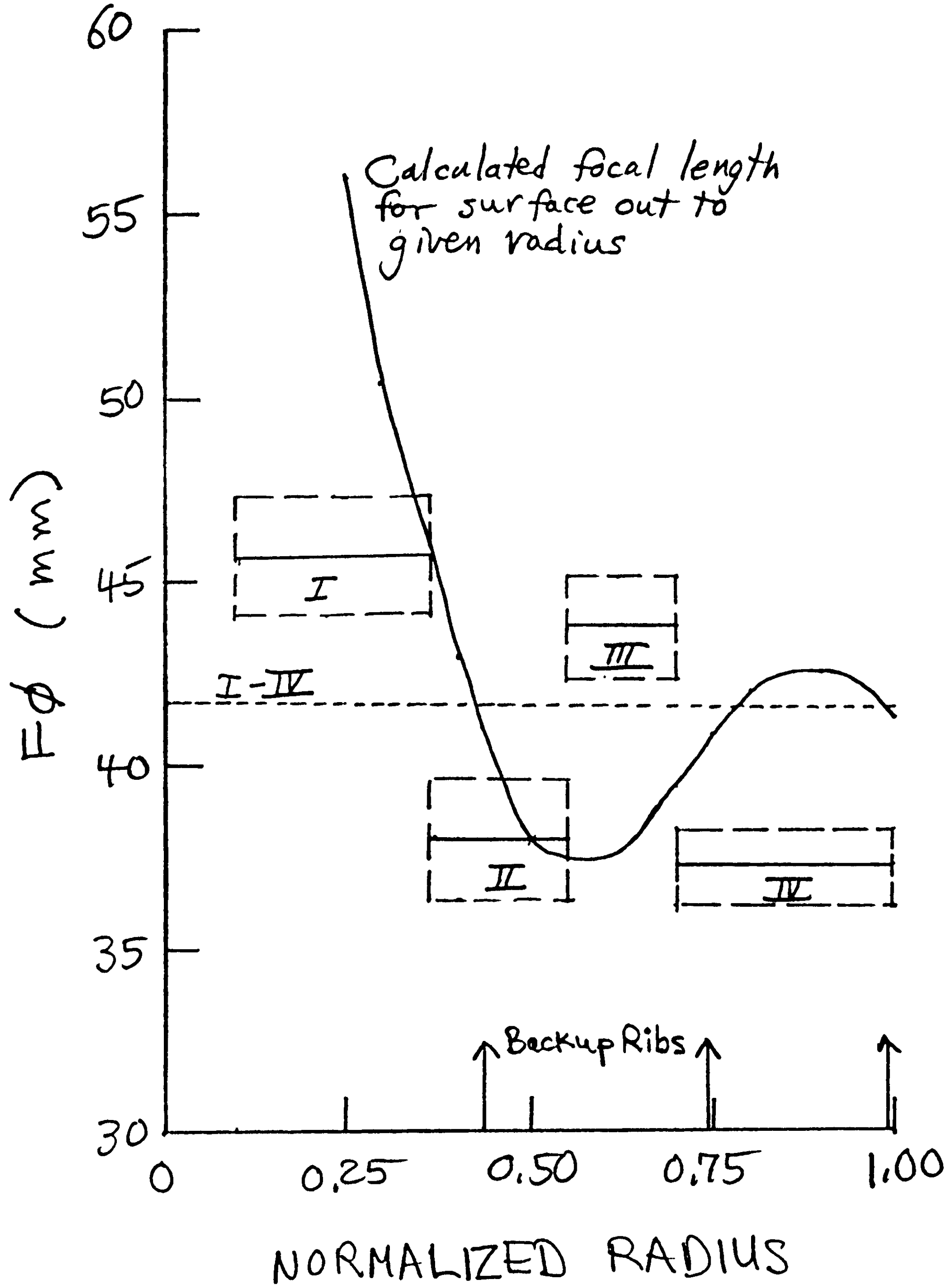
SUMMARY OF OBSERVATIONS

| Zone | I | II | III | IV | I-IV |
|---|---|---|---|---|---|
| Normalized Inner Radius | 0.10 | 0.37 | 0.55 | 0.70 | 0.10 |
| Normalized Outer Radius | 0.37 | 0.55 | 0.70 | 1.00 | 1.00 |
| FØ (mm) | 45.7 ± 1.6 | 38.0 ± 1.6 | 43.8 ± 1.3 | 37.3 ± 1.0 | 41.7± 0.7 |
| Tϕ (°C) | 4.2 | 2.8 | 1.6 | 2.2 | 3.1 |
| T1 (°C) | 3.9 | 2.3 | 1.5 | 2.1 | 2.9 |
| T2 (°C) | 6.8 | 6.4 | 6.3 | 6.5 | 6.8 |
| AZ HPBW (") | 158 | 100 | 76 | 69 | 84 |
| EL HPBW (") | 146 | 95 | 73 | 55 | 76 |
| <HPBW> (") | 152 | 97 | 74 | 62 | 80 |
| Flux Density of Jupiter (Jy) | 67 | 66 | 52 | 67 | 926 |
| ΔAZ (") | -6 | 1 | 8 | -2 | 0 |
| ΔEL (") | -14 | 0 | 0 | 6 | 0 |

# FIGURE 1


HPBW = 46" / RADIUS
11/14/79
89.6 GHz
HPBW (arc seconds)
180
150
120
90
60
30
0
AZ
EL
I-IV AZ
I-IV EL
ZONE:
I
II
III
IV
0
0.25
0.50
0.75
1.00
NORMALIZED RADIUS

# FIGURE 2



Calculated focal length for surface out to given radius
60
55
50
45
40
35
30
Fφ (mm)
I
II
III
IV
I-IV
Backup Ribs
0
0.25
0.50
0.75
1.00
NORMALIZED RADIUS

# REFERENCES

Born, M. and Wolf, E. (1970). Principles of Optics, Pergamon: Oxford, Chapter 9.

Malacara, D. (1978). Optical Shop Testing, Wiley: New York, 375.

Payne, J. and Hollis, J. (1975). "Further Measurements on the 36-ft. Surface." NRAO Memorandum of May 14, 1975.

Ulich, B.L. and King, L. (1977). "Correction of Astigmatism in the 36-Foot Telescope," NRAO Tucson Operations Internal Report No. 4.