Interoffice

NATIONAL RADIO ASTRONOMY OBSERVATORY

TUCSON, ARIZONA

June 5, 1981

12 METER MILLIMETER WAVE TELESCOPE

MEMO No. 45

To: 12 M Memo File

From: J. M. Payne

Subject: Errors in Encoders, Addition to Memo # 27

As an addition to my memo #27, I would like to include a note on the Measurement and Correction for Periodic Baldwin Encoder Errors. This note was written by the staff of the MWO at Fort Davis and supports my belief that any errors in the encoder are periodic and may easily be removed by the software.

I think I am correct in saying that Bill Horne now agrees with this and we may now consider the encoder question closed.

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NATIONAL RADIO ASTRONOMY OBSERVATORY

TUCSON, ARIZONA

April 30, 1981

To: Bill Horne

From: John Payne

12 METER MILLIMETER WAVE TELESCOPE
MEMO No. 27

Subject: Errors in Encoders

It seems time to resolve our differences of opinion about the accuracy of the encoder system on the present 11 M telescope.

The encoders used are 20 bits (LSB = 1.24 arc sec) and the manufacturer claims an absolute accuracy of 1 bit when the unit leaves the factory. The encoders are checked using an autocollimator and an optically ground 16 sided reflector. This appears to be similar to the familiar pentaprisim in that it comes from the manufacturer calibrated to better than 1 arc sec. To put the pattern on the disc, a Divided Circle Machine is used and the absolute accuracy is claimed to be better than 0.3 arc secs.

Bits 13 - 20 on the encoder are synthesized from sine and cosine tracks on the encoder and it is my belief that after a time, repeatable periodic errors could appear as a result in the synthesising electronics. John Davis, at Texas has measured such an effect on identical encoders. Note that such an effect is repeatable and is very simply compensated for in the software.

At the telescope at the present time, we have about 5 arc sec RMS random pointing errors after one week of operation and we believe that the encoders contribute very little.

I do not believe that the present encoders will limit the performance of the upgraded telescope.

Measurement and Correction for Periodic Baldwin Encoder Errors

The Baldwin 20-bit encoders are subject to an error which has a period of exactly 128 encoder bits. This stems from the fact that the encoder consists of a code disk with a discrete representation for the most significant 14 bits and an analogue portion for the least significant 7 bits. (There are two representations for bit 14 which are forced to agree.) The analogue portion has two tracks: a sine and a cosine wave which are electronically compared to produce the least significant 6 bits of the encoder reading. If the two circuits are improperly balanced, a periodic error results. This can be remedied by removing the encoder and balancing the two circuits (requires adjustment of six resistors). An easier method, however, is to correct for the error in software: A table can be produced which consists of the encoder error as a function of the last six bits of the actual encoder reading. Thus the computer program need only use the last six bits of the encoder reading as an index in the table, get the corresponding error and subtract it from the encoder reading to produce an error free encoder reading. The remainder of this document describes how one can produce this table so that the error can be removed.

Let the actual encoder reading, r, be written

$$r = e + E_c(2\pi e_6/128) \equiv e + E_c(\theta_c)$$

where e is the true encoder reading and E_c is the encoder error as a function of the least significant six bits, e_6, of the <u>true</u> encoder reading. For small enough E_c, there also exists a single valued function, E, of the least significant 6 bits for r, r_6, such that

$$r = e + E(2\pi\, r_6/128) \doteq e + E(\theta)$$

(If E_c is too large, $E(\theta)$ will be multi-valued and software correction will be, at best, only approximate. In this case, one must adjust the encoder hardware to make E_c smaller.) Since $E(\theta)$ is periodic in θ and has zero mean (any non-zero mean has been absorbed in the software encoder zero-point), it can be represented as a fourier series:

$$E(\theta) = \sum_{i=1}^{\infty} S_i \sin(i\theta) + \sum_{i=1}^{\infty} C_i \cos(i\theta)$$

Suppose we now drive the antenna at a constant velocity, v_c, and sample encoder readings at a constant rate. Then the apparent antenna velocity, v, as judged by differencing encoder reading is

$$v = e_{new} - e_{old} + E(\theta_{new}) - E(\theta_{old})$$

We have $e_{new} - e_{old} = v_c$ by definition

and $$\theta_{new} \approx \theta_{old} + 2\pi\, v_c/128 \equiv \theta_{old} + \varphi$$

We can now write $v(\theta, \varphi) \approx \frac{128\varphi}{2\pi} + E(\theta) - E(\theta - \varphi)$

where we have substituted $\theta = \theta_{new}$

If we choose a velocity, v_c, such that $\varphi = \pi$, and substitute the fourier series for $E(\theta)$ we find that the even harmonics of θ cancel and we have

$$E(\theta) - E(\theta - \varphi) = \sum_{j=1}^{\infty} \Big\{ S_{2j-1} \sin([2j-1]\theta) - S_{2j-1} \sin([2j-1][\theta - \pi])$$

$$+ C_{2j-1} \cos([2j-1]\theta) - C_{2j-1} \cos([2j-1][\theta - \pi]\Big\}$$

$$E(\theta) - E(\theta - \pi) = \sum_{j=1}^{\infty} \Big\{ 2 S_{2j-1} \sin[(2j-1)\theta] + 2C_{2j-1} \cos[2j-1)\theta] \Big\}$$

Since $E(\theta) - E(\theta - \pi)$ has zero mean, we can calculate $E(\theta) - E(\theta - \pi)$ by simply removing the mean from $v(\theta, \pi)$. (In practice we choose φ to be slightly different from π so that we slowly drift through an entire period in θ as we sample $v(\theta, \pi)$.)

Thus, we now have a measurement of the odd harmonics in θ of $E(\theta)$ and can use this to construct a correction table, $T(\theta)$, to remove this part of $E(\theta)$. $T(\theta) = \frac{1}{2}(v(\theta, \pi) - \langle v(\theta, \pi)\rangle)$. Once this is done, we can remeasure $v(\theta, \pi)$ to verify it is now flat. If not, the new $v(\theta, \pi)$ can be used to construct a more accurate $T(\theta)$ by adding the new corrections to the old. If necessary, the process can be iterated until a $T(\theta)$ is found which produces a flat $v(\theta, \pi)$.

Having removed the odd harmonics of $E(\theta)$ with the correction table, we can now remove the even harmonics by a similar process: we choose a v_c such that $\varphi \sim \frac{\pi}{2}$ or $\frac{3\pi}{2}$. (We usually choose $\frac{3\pi}{2}$ because our antenna slews more smoothly at this velocity.) Since we have removed the odd harmonics in θ with $T(\theta)$, $v(\theta, \frac{3\pi}{2})$ can be written

$$v(\theta, \frac{3\pi}{2}) \approx \frac{128 \frac{3\pi}{2}}{2\pi} + \sum_{j=1}^{\infty} \{S_{2j} \sin (2j\theta) - S_{2j} \sin (2j[\theta - \frac{3\pi}{2}]) +$$

$$+ C_{2j} \cos (2j\theta) - C_{2j} \cos (2j [\theta - \frac{3\pi}{2}])\}$$

$$v(\theta, \frac{3\pi}{2}) \approx 96 + \sum_{j=1}^{\infty} \{2 S_{2j} \sin (2j\theta) + 2 C_{2j} \cos (2j\theta)\}$$

Thus, we now have a measurement of the even harmonics in θ of $E(\theta) \approx \frac{1}{2}(v(\theta, \frac{3\pi}{2}) - \langle v(\theta, \frac{3\pi}{2})\rangle)$. This new correction can be added to $T(\theta)$ to produce the complete correction table for the encoder. As above, the process may be iterated should the new $v(\theta, \frac{3\pi}{2})$ not be quite flat. Using this procedure, one should be able to reduce the periodic encoder errors to $\lesssim 1$ encoder unit.