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12 METER MILLIMETER WAVE TELESCOPE
MEMO No. 92

October 30, 1981

Dr. John W. Findlay
National Radio Astronomy Observatory
Edgemond Road
Charlottesville, VA 22901

Dear John:

I am enclosing a new paper by Mehdi Zarghamee of Simpson, Gumpertz and Heger which, I am sure, will be of great interest to you. It provides some fresh insight into the age-old problem of determining the motions of the secondary that are required to maximize peak gain for a Cassegrain antenna as a function of elevation angle. It also covers determination of the beam deviation that results when gain is maximized through proper secondary movement. The antenna that is the subject of the paper is an ESSCO 45-foot radio telescope that was modestly modified by us to serve as the principal component of the ALCOR Millimeter Wavelength Augumentation Radar Project of M.I.T. Lincoln Laboratory. This program is nearing completion with installation scheduled to begin in January, 1982.

The reflector is an improved version of our standard 13.7 meter box beam design that functions well at the required high angular rates and thus would serve as an excellent radio telescope. We did a thorough job of analyzing the structure on the computer using a finite-element approach and the results of our analysis were used by Mehdi as the structural (deflection) inputs for his subreflector motion postprocessor.

Close examination of the paper, especially Figure 3, shows the importance of secondary alignment on gain. In this case, reference alignment is assumed at an elevation angle of 30°. If no further active adjustments are made with changing elevation angle, the impact on gain can be significant as illustrated by the graph. For example, a gain loss of 1 dB or more will accompany an incremental elevation travel of 10° for elevation angles above 55°. Also for small elevation excursions of 10°-15° on either side of the reference position, losses of 0.2-0.4 dB will occur.

If active secondary adjustments are made, the peak gain may be dramatically improved. Figure 3 indicates that lateral adjustment is more sensitive than axial (focusing) adjustment, but the two applied simultaneously are most effective, lowering the gain loss to a maximum of only about 0.2 dB over the entire travel range of 90° in elevation. Figure 5 shows the beam deviation resulting from primary gravity deformations and

(cont'd on page 2)

secondary alignment which must be utilized as a correction term in the computer pointing program. Figure 4 shows the magnitude of lateral and axial adjustment required which amounts to less than 0.5 inches laterally and 0.2 inches axially for this 45-foot dish.

As Mehdi points out in his paper, minimizing the loss of peak gain via active subreflector alignment can be accomplished only when the structural deformations are repeatable. The millimeter wave radar antenna is enclosed in a radome which is equipped with an internal environmental control system. Thus, the structural gravity behavior is well understood and repeatable as opposed to an exposed antenna. Every antenna will exhibit different gravity deflection characteristics, but as long as those deflections are repeatable, the performance should not suffer as a result of improper secondary alignment for a reflector that deforms in an axially symmetric way. Thus it behooves the user to take advantage of these factors so that ultimate antenna performance depends only upon the primary surface deformations and pointing accuracy.

I trust that this information will be helpful to you and we would be pleased to discuss this subject further if you have any comments or questions.

Best Regards,

L. E. Rhoades
Engineering Manager

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PEAK GAIN OF A CASSEGRAIN ANTENNA WITH SECONDARY POSITION ADJUSTMENT

by Mehdi S. Zarghamee*

Abstract

For an enclosed Cassegrain antenna, the loss of peak gain and beam deviation due to structural deformations of the primary reflector and rigid body displacements of the secondary reflector and of the feed are computed from the combined changes in the RF-path length. As the antenna moves in elevation, the position of the secondary reflector may be adjusted mechanically to minimize the loss of peak gain; a general method for the computation of the magnitude of such adjustments and of their effects on the gain and pointing of the system is presented.

Numerical results are obtained for a particular case of a 45-foot diameter antenna designed for operation at 95.5 GHz RF frequency for which the computed peak gain of the antenna varies significantly with the elevation angle. The results indicate that the loss of peak gain as the antenna moves in elevation can be substantially reduced by mechanical adjustment of the position of the secondary reflector.

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I. INTRODUCTION

The structural deformations of a Cassegrain antenna result in surface distortions of the primary reflector and misalignments between the primary and secondary reflector and the feed. The surface distortions of the secondary are usually ignored as they are an order of magnitude smaller than those of the primary reflector. For enclosed antennas, the change in gravity deformations as it moves in elevation is the main source of gain degradation. Both surface distortions of the primary reflector and misalignments between the antenna components result in gain degradation and beam deviation. The gain degradation due to the gravity deformations of the primary reflector may be predicted by the tolerance theory of Ruze[1] from the rms of the surface deviations, usually computed with respect to a paraboloidal surface that best fits the deformed geometry of the reflector. The best-fitting is achieved by simultaneously translating and rotating the reflector and changing the focal distance[2]. In many cases, the position of the best-fit paraboloid cannot be determined with accuracy due to ill-conditioning of the equations. The ill-conditioning is inherent in the best-fitting process because rigid-body lateral displacements and rotations of the primary reflector result in similar distributions of the change in the RF-path length over the aperture.

The misalignment in the relative position of the best-fit paraboloid and the displaced positions of the secondary and feed results in beam deviation and loss of peak gain. It is possible to break up the misalignment into components of rigid-body displacements of the primary, secondary and feed, and to compute separately the beam deviation and loss of peak gain due to each component of misalignment[3]-[5]. The total beam deviation may be computed by the superposition of the effects of the components of misalignment; however, to compute the loss of peak gain, the total misalignment must be considered at one time because, in general, superposition of the effects of components taken one at a time does not hold. Antennas for which the loss of peak gain due to misalignment is significant may demonstrate an acute degradation of peak gain near the limits of their travels in elevation. To minimize the loss of peak gain due to misalignment, the position of the secondary reflector may be adjusted by mechanical means. The magnitude of the adjustment depends on the elevation angle of the antenna. Gain degradation may also occur due to astigmatism resulting from gravity deformations of the primary reflector as described by von Hoerner [6], [7] which he suggests correcting by mechanically deforming a flexible subreflector. This paper presents a method for the computation of the adjustment of the position of the secondary reflector that minimizes loss of peak gain.

In this paper, the beam deviation and the loss of peak gain are calculated directly from the changes in the RF-path length resulting from the structural deformation of the primary reflector and the rigid body displacements and rotations of the secondary reflector and of the feed. The method avoids the best-fitting of the primary and the resulting ill-conditioning inherent in such calculations. The position of the secondary reflector is then adjusted to minimize the loss of peak gain. The results, obtained for a particular 45-foot antenna, indicate that the loss of peak gain due to deformations caused by gravity can be substantially reduced by adjustment of the position of the secondary.

II. GAIN LOSS AND BEAM DEVIATION

Let us consider an antenna with an axisymmetric illumination function $f(\bar{r})$ where $\bar{r}$ is the aperture position vector defined in polar coordinates by (r, ϕ'), see Fig. 1. The direction of observation expressed by the angles ϕ and θ (see Fig. 1) may also be expressed by a unit vector $\hat{p}$ where:

$$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \tag{1}$$

(Bars indicate a vector quantity and "^" indicates a unit vector.)

The gain in the direction of observation $\hat{p}$ of a distorted antenna with a change in RF-path length of δ at point $\bar{r}$ on the aperture is expressed by:

$$G(\phi, \theta) = \frac{4\pi}{\lambda^2} \frac{\left| \int_A f(\bar{r})\, e^{\frac{2\pi j}{\lambda}(\delta - \hat{p}\cdot\bar{r})}\, dS \right|^2}{\int_A f^2(\bar{r})\, dS} \tag{2}$$

Let G_o be the no error peak gain, obtained from (2) by setting $\delta = \hat{p}\cdot\bar{r} = 0$, then

$$\frac{G}{G_o} = \left| \frac{\int_A f(\bar{r})\, e^{\frac{2\pi j}{\lambda}(\delta - \hat{p}\cdot\bar{r})}\, dS}{\int_A f(\bar{r})\, dS} \right|^2 \tag{3}$$

If the variation of $\delta - \beta \cdot \bar{r}$ over the aperature is small as compared to the wavelength λ, we may approximate Eq. (3) by expansion as follows:

$$\frac{G}{G_o} = 1 - \left(\frac{2\pi}{\lambda}\right)^2 \frac{\int_A f(\bar{r})\,(\delta - \beta\,.\,\bar{r})^2\,dS}{\int_A f(\bar{r})\,dS} + \left(\frac{2\pi}{\lambda}\right)^2 \left[\frac{\int_A f(\bar{r})\,(\delta - \beta\,.\,\bar{r})\,dS}{\int_A f(\bar{r})\,dS}\right]^2 \tag{4}$$

In the direction of peak gain β_o, $\partial\,(G/G_o)/\partial\,\theta = 0$ and $\partial\,(G/G_o)/\partial\,\phi = 0$. Therefore, if we note that

$$\beta\,.\,\bar{r} = r \sin\theta \cos(\phi - \phi') \simeq r\,\theta \cos(\phi - \phi') \tag{5}$$

then

$$\int_A f(\bar{r})\,(\delta - \beta_o\,.\,\bar{r})\;r\;\cos(\phi_o - \phi')\,dS = 0 \tag{6.1}$$

and

$$\int_A f(\bar{r})\,(\delta - \beta_o\,.\,\bar{r})\;r\;\sin(\phi_o - \phi')\,dS = 0 \tag{6.2}$$

From Eqs. (6.1) and (6.2), the direction of peak gain may be expressed by

$$\theta_o = \frac{\int_A f(\bar{r})\;\delta\;r\;\cos(\phi_o - \phi')\,dS}{\int_A f(\bar{r})\;r^2 \cos^2(\phi_o - \phi')\,dS} \tag{7}$$

and

$$\tan\phi_o = \frac{\int_A f(\bar{r})\;\delta\;r\;\sin\phi'\,dS}{\int_A f(\bar{r})\;\delta\;r\;\cos\phi'\,dS} \tag{8}$$

III. CHANGES IN RF-PATH LENGTH

The total change in the RF-path length δ is the sum of the changes in the RF-path length due to the deformation of the primary reflector δ_p, the rigid-body translations and rotations of the secondary reflector δ_s, and the displacements of the feed δ_f; that is

$$\delta = \delta_p + \delta_s + \delta_f \tag{9}$$

Let a point on the surface of the primary reflector undergo a displacement $\bar{u}_p = (u_p, v_p, w_p)$. The resulting change in the RF-path length is twice the axial component of the displacement normal to the paraboloid[8]; that is, $\delta_p = -2\, n_z\, (\bar{u}_p \cdot \hat{n})$ where $\hat{n} = (n_x, n_y, n_z)$ is a unit vector normal to the surface of the primary reflector. In order words

$$\delta_p = \bar{c}_p \cdot \bar{u}_p \tag{10}$$

where the components of the coefficient vector $\bar{c}_p$ for a paraboloid of the form $z = r^2/4f$ may be expressed by

$$c_{p1} = c_{po} \cos\phi' \tag{11.1}$$

$$c_{p2} = c_{po} \sin\phi' \tag{11.2}$$

$$c_{p3} = -\frac{8 f^2}{4 f^2 + r^2} \tag{11.3}$$

where

$$c_{po} = \frac{4 r f}{4 f^2 + r^2} \tag{12}$$

The effect of the displacements of the feed can be examined by using the equivalent prime-focus paraboloid concept. In this concept we use the fact that the energy converging on the feed appears to come from an equivalent prime-focus paraboloid (see Fig. 2). Thus the effects of the displacement of the feed in a Cassegrain antenna is equivalent to the displacement of the feed in a prime-focus antenna of focal length Mf. Therefore, if the feed is assumed to undergo a displacement $\bar{u}_f = (u_f, f_f, w_f)$, we can express the corresponding change in the RF-path length δ_f as follows:

$$\delta_f = \bar{c}_f \cdot \bar{u}_f \tag{13}$$

with

$$c_{f1} = c_{fo} \cos\phi' \tag{14.1}$$

$$c_{f2} = c_{fo} \sin\phi' \tag{14.2}$$

$$c_{f3} = \frac{-4(Mf)^2 + r^2}{4(Mf)^2 + r^2} \tag{14.3}$$

$$c_{fo} = -\frac{4 r (Mf)}{4 (Mf)^2 + r^2} \tag{15}$$

The rigid-body displacements of the secondary reflector, translations $\bar{u}_s = (u_s, v_s, w_s)$ and the rotations ψ_{xs} and ψ_{ys} about axes parallel to the x and y axes passing through the vertex of the secondary, may be expressed in terms of equivalent rigid body translations and rotations of the primary reflector and of the feed. The resulting expression for the change in the RF-path length due to displacements of the secondary reflector may be written as follows:

$$\delta_s = \bar{c}_s \cdot \bar{u}_s + c_{s4}\,\psi_{xs} + c_{s5}\,\psi_{ys} \tag{16}$$

where

$$\bar{c}_s = -(\bar{c}_p + \bar{c}_f) \tag{17.1}$$

$$c_{s4} = -c_{p2}\,(k - z) - c_{f2}\,h - r(1 + c_{p3})\sin\phi \tag{17.2}$$

$$c_{s5} = c_{p1}\,(k - z) + c_{f1}\,h + r(1 + c_{p3})\cos\phi \tag{17.3}$$

For the known deformations of a Cassegrain antenna, the changes in the RF-path length are initially computed from Eqs. (9) - (17). The values of δ are then used to compute the direction of peak gain d_o and θ_o from Eqs. (7) and (8) and the corresponding loss of peak gain from Eq. (4).

IV. ADJUSTMENT OF SECONDARY REFLECTOR POSITION

When the structural deformations are repeatable, the secondary position may be adjusted to minimize the loss of peak gain. Let δ_o be the total change in the RF-path length due to the structural deformations of the primary reflector and the rigid-body displacements of the secondary and of the feed, and let ϕ_o and θ_o denote the corresponding direction of peak gain. If as the secondary is adjusted it undergoes additional translations and rotations denoted by $\bar{u}$, and ψ_x and ψ_y, the resulting change in the RF-path length after adjustments δ_a is

$$\delta_a = \delta_o + \bar{c}_s \cdot \bar{u} + \bar{c}_{s4}\,\psi_x + c_{s5}\,\psi_y \tag{18}$$

Without loss of generality, we may assume that $\phi_o = \pi/2$ and $u = \psi_y = 0$. For most enclosed antennas which have a vertical plane of symmetry and are subjected to a linear combination of face-up and face-side gravity loadings, these assumptions are valid.

(If $\phi_o \neq \pi/2$, we may rotate the coordinate axes so that in the rotated coordinates $\phi_o = \pi/2$.) Under these assumptions, Eq. (18) reduces to

$$\delta_a = \delta_o + (k_1 \sin \phi') v + k_2 w + (k_3 \sin \phi') \psi_x \quad (19)$$

where

$$k_1 = -(c_{po} + c_{fo}) \quad (20.1)$$

$$k_2 = c_{s3} \quad (20.2)$$

$$k_3 = -c_{po}(k - z) - c_{fo} h - r(1 + c_{p3}) \quad (20.3)$$

The additional displacement of the secondary reflector changes the direction of peak gain. Let us denote the direction of peak gain after secondary adjustment by $\hat{p}_a$ and the corresponding angles by ϕ_a and θ_a. Since $\phi_o = \pi/2$ and the adjustments do not change $\phi, \phi_a = \phi_o$. If we substitute Eq. (19) into Eq. (7), we obtain an expression for the modified direction of peak gain as follows:

$$\theta_a = \theta_o + \theta_v v + \theta_\psi \psi_x \quad (21)$$

where

$$\theta_v = \frac{\int_R f(r)\, k_1\, r^2\, dr}{\int_R f(r)\, r^3\, dr} \quad (22.1)$$

and

$$\theta_\psi = \frac{\int_R f(r)\, k_3\, r^2\, dr}{\int_R f(r)\, r^3\, dr} \quad (22.2)$$

in which R refers to the interval of radial integration.

Let us define δ_o' and δ_a' as follows:

$$\delta_o' = \delta_o - r\, \theta_o \cos(\pi/2 - \phi') \quad (23)$$

$$\delta_a' = \delta_a - r\, \theta_a \cos(\pi/2 - \phi') \quad (24)$$

Substitution of Eqs. (19), (21), and (23) into Eq. (24) results in the following:

$$\delta_a' = \delta_o' + k_1' \sin \phi' v + k_2 w + k_3' \sin \phi' \psi_x \quad (25)$$

where

$$k_1' = k_1 - r\ \theta_v \tag{26.1}$$

$$k_3' = k_3 - r\ \theta_\psi \tag{26.2}$$

To select the values of v, w, and ψ_x which will maximize gain, we substitute Eq. (25) into Eq. (4) and set the partial derivatives of G/G_o with respect to v, w, and ψ_x equal to zero. We obtain

$$\int_A f(\bar{r})\, k_i' \sin\phi'\ \delta_a'\, d\,S = 0 \qquad i = 1 \text{ and } 3 \tag{27.1}$$

$$\int_A f(\bar{r})\, k_2'\quad \delta_a'\ d\,S = 0 \tag{27.2}$$

where

$$k_2' = k_2 - \frac{\int_R f(r)\ k_2\, r\, d\, r}{\int_R f(r)\ r\, d\, r} \tag{28}$$

Eqs. (27) are three equations in three unknowns and may be written in the form of $\bar{C}\,\bar{x} = \bar{b}$ where

$$c_{ij} = \pi \int_R f(r)\, k_i'\, k_j'\, rdr \tag{29.1}$$

$$c_{i2} = c_{2j} = 0 \qquad \Big\}\ i, j = 1 \text{ and } 3 \tag{29.2}$$

$$c_{22} = 2\,\pi \int_R f(r)\, k_2'^2\ rdr \tag{29.3}$$

$$b_i = -\int_A f(\bar{r})\, k_i' \sin\phi'\ \delta_o' d\,S \qquad i = 1 \text{ and } 3 \tag{30.1}$$

$$b_2 = -\int_A f(\bar{r})\, k_2'\ \delta_o' d\,S \tag{30.2}$$

Solution of this system yields

$$v = \frac{b_1\, c_{33} - b_3\, c_{31}}{c_{11}\, c_{33} - c_{13}\, c_{31}} \tag{31.1}$$

$$w = \frac{b_2}{c_{22}} \tag{31.2}$$

$$\psi_x = \frac{-b_1\, c_{13} + b_3\, c_{11}}{c_{11}\, c_{33} - c_{13}\, c_{31}} \tag{31.3}$$

Eqs. (31) express the adjustment of the secondary reflector needed to maximize gain.

V. COMPUTED RESULTS

The theoretical development presented above has been coded in a computer program to be used as a postprocessor with a structural analysis package. The program computes initially the beam deviation and loss of peak gain of an enclosed Cassegrain antenna due to gravity deformations of the primary reflector and rigid-body displacements of the secondary and of the feed without adjustments of the position of the secondary reflector. The computed values of δ_o and δ_o' are used to evaluate the surface integrals that define b_i, i = 1, 2, and 3, Eqs. (30). The entries of the coefficient matrix $\overline{C}$, Eqs. (29), which are functions of the geometry of the antenna and of the illumination pattern and are independent of the structural deformations, are computed by evaluating a series of line integrals. The computed values for the adjustment of the position of the secondary reflector to maximize gain are used to modify the changes in the RF-path length and compute the corresponding loss of peak gain after adjustment of the secondary reflector.

The computation has been performed for the gravity deformations of a 45-foot diameter Cassegrain antenna enclosed in a radome. The structural deformations were computed by a finite-element idealization of the structure. It is assumed that the surface panels, the secondary reflector, and the feed are aligned in such a way that when the antenna is at elevation angle α_r, the residual deviations from the ideal antenna configuration are random in nature. (Note that we are ignoring the bias alignment errors.) Then, if δ_o^U and δ_o^S are the changes in the RF-path length due to structural deformation resulting from gravity loads in the face-up and face-side positions, we have

$$\delta_o = \delta_o^U (\sin\alpha - \sin\alpha_r) + \delta_o^S (\cos\alpha - \cos\alpha_r) \tag{32}$$

where α is the elevation angle of the antenna.

The values of δ_o as expressed by Eq. (32) for various elevation angles α have been used in the calculation of the loss of peak gain and beam deviation without or with secondary reflector position adjustment. It was assumed that $\alpha_r = 30^o$, f/D = 0.37, and the magnification factor M = 11. It was further assumed that the illumination function is of the form $f(r) = 1 - 0.75\,(2r/D)^2$ and that the RF frequency is 95.5 GHz.

In the numerical computations performed, we considered the adjustment of the position of the secondary reflector in the lateral and axial directions only. The adjustment for the tilt of the secondary reflector was considered to be counterproductive because the loss of gain corresponding to the nonrepeatable errors in the angular positioning mechanism are expected to be larger than the improvements in gain resulting from the added degree of freedom in the adjustment.

Figure 3 shows the loss of peak gain of the 45-foot diameter antenna considered herein prior to the adjustment of the position of the secondary and after independent adjustments in the lateral and axial directions as well as after a combined axial and lateral adjustment. The results indicate that in a Cassegrain antenna for which the deformations of the primary reflector structure are almost homologous (i.e. the primary reflector deforms into an almost parabolic shape), a significant loss of peak gain may occur as a result of secondary position misalignment. Furthermore, the loss of peak gain of a Cassegrain antenna may be substantially reduced by suitable adjustments of the position of the secondary reflector in the lateral and axial directions. The magnitude of the lateral adjustment v and of the axial adjustment w for various elevation angles are shown in Figure 4.

As we adjust the position of the secondary reflector, the loss of peak gain is reduced and the beam deviation, i.e. the value of θ_o corresponding to the peak axial gain, increases significantly (see Fig. 5). Note that if the error in the position of the secondary were primarily due to subreflector droop, an adjustment of the secondary position would have reduced the gain loss and the beam deviation simultaneously; whereas, if the error were for example primarily due to a rotation of the secondary reflector, an adjustment of the secondary position in the lateral direction made to maximize gain is expected to increase beam deviation. Note that the the beam deviation due to gravity deformations is repeatable and can be calibrated out.

ACKNOWLEDGEMENT

The author wishes to thank Dr. John Ruze and Dr. Joseph Antebi for their valuable comments.

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NOTATIONS

A	=	Domain of aperture
$\bar{c}$	=	Coefficient vector, see Eqs. (11, 14, 17)
$\bar{C}$	=	Coefficient matrix, see Eqs. (29)
f	=	Focal length of primary reflector
$f(\bar{r})$	=	Aperture illumination function
G	=	Antenna gain
G_o	=	No error peak gain
h	=	Distance between feed and the vertex of secondary, see Fig. 2
k	=	Distance between the vertex of primary and the vertex of secondary, see Fig. 2
k_1, k_2, k_3	=	Coefficients, see Eqs. (20)
k_1', k_2', k_3'	=	Coefficients, see Eqs. (26) and (28)
$\hat{p}$	=	Unit vector in the direction of observation
$\hat{p}_o$	=	Unit vector in the direction of peak gain
$\bar{r}$	=	Aperture position vector = (r, ϕ') in polar coordinates, see Fig. 1
$\bar{u}_p$	=	displacement vector of a point on the primary reflector = (u_p, v_p, w_p)
$\bar{u}_s$	=	Rigid body translation of secondary reflector
$\bar{u}_f$	=	Rigid body translation of feed
u, v, w, ψ_x, ψ_y	=	Magnitude of adjustment of the position of the secondary reflector
x, y, z	=	Coordinates of points on the primary reflector
α	=	Elevation angle of antenna
α_r	=	Elevation angle at which the deviations from the ideal antenna configuration are assumed to be random
δ	=	change in the RF-path length, a function of aperture position
ϕ, θ	=	Angles defining the direction of observation of the deformed Cassegrain system with respect to the undeformed system

ϕ_o, θ_o	=	Direction of observation corresponding to the peak gain (θ_o is also referred to as beam deviation)
θ_V, θ_ψ	=	Beam deviation due to a unit lateral displacement and a unit rotation of the secondary.
ψ_{xs}, ψ_{ys}	=	Components of rotation of secondary about axes parallel to x and y axes through the vertex of the secondary
λ	=	Wavelength

Subscripts

p	=	primary reflector
s	=	secondary reflector
f	=	feed
a	=	pertaining to adjusted position of secondary reflector

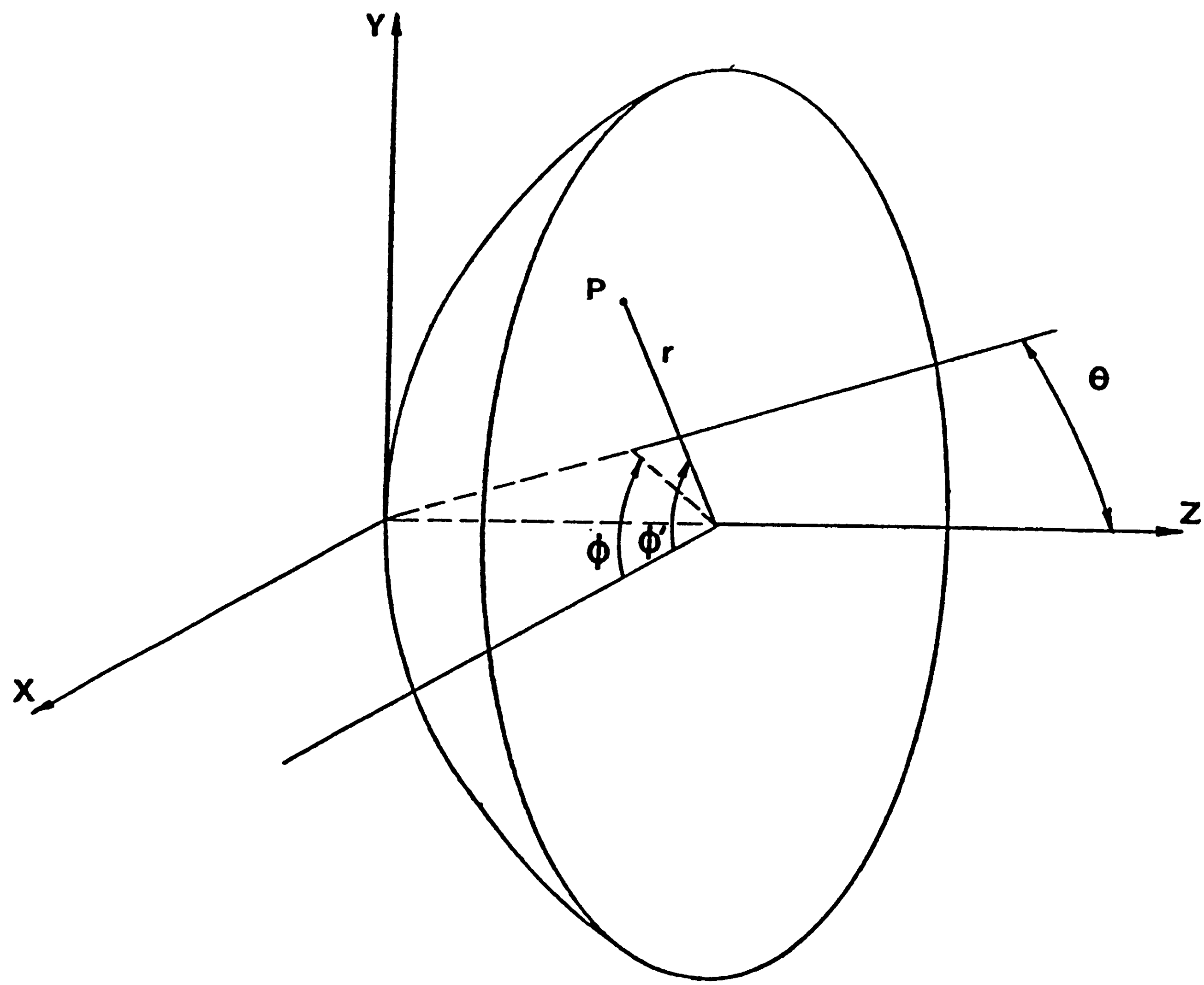

Figure 1 Coordinate Systems

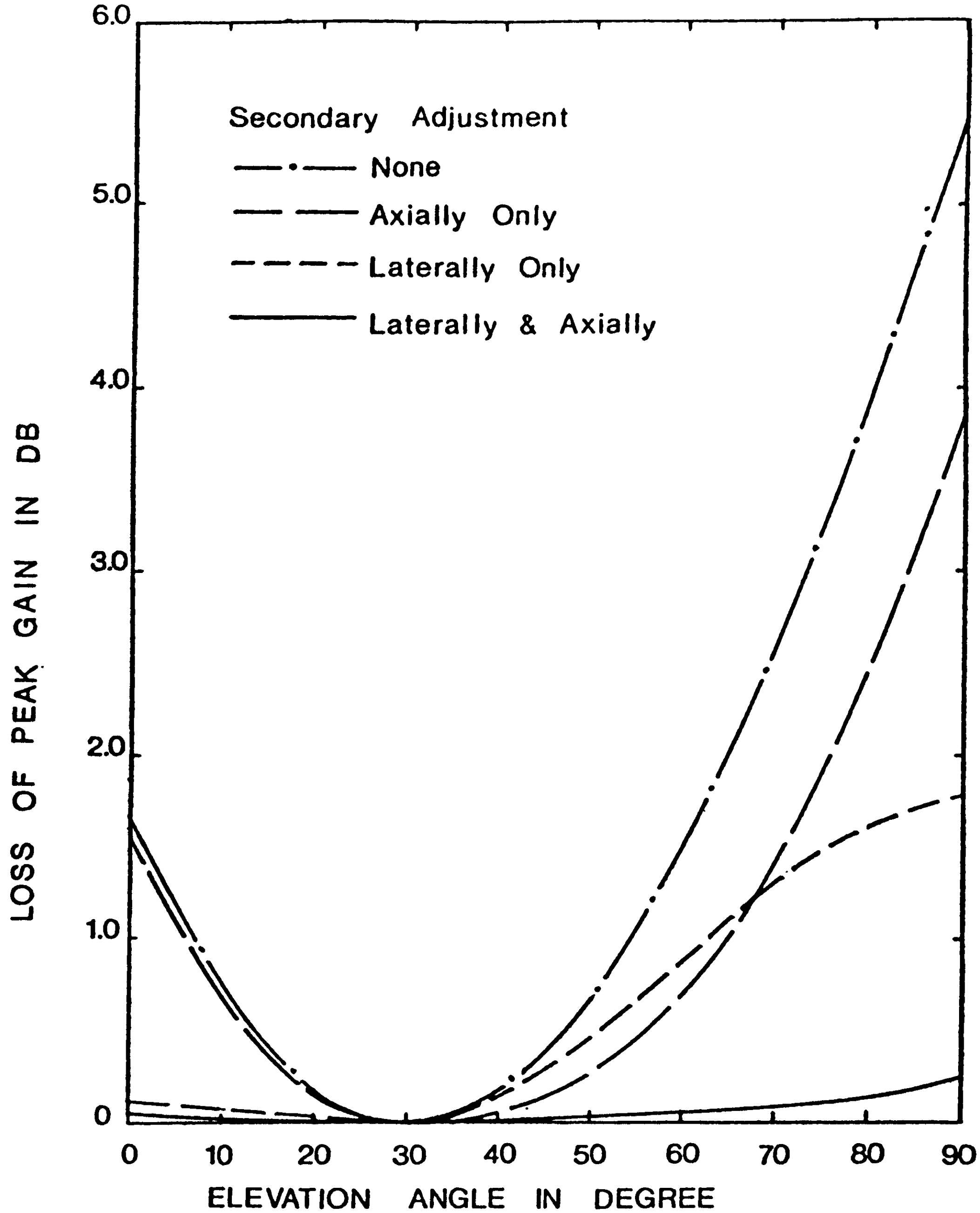

Figure 3 Loss Of Peak Gain With And Without Adjustment Of The Position Of Secondary Reflector.

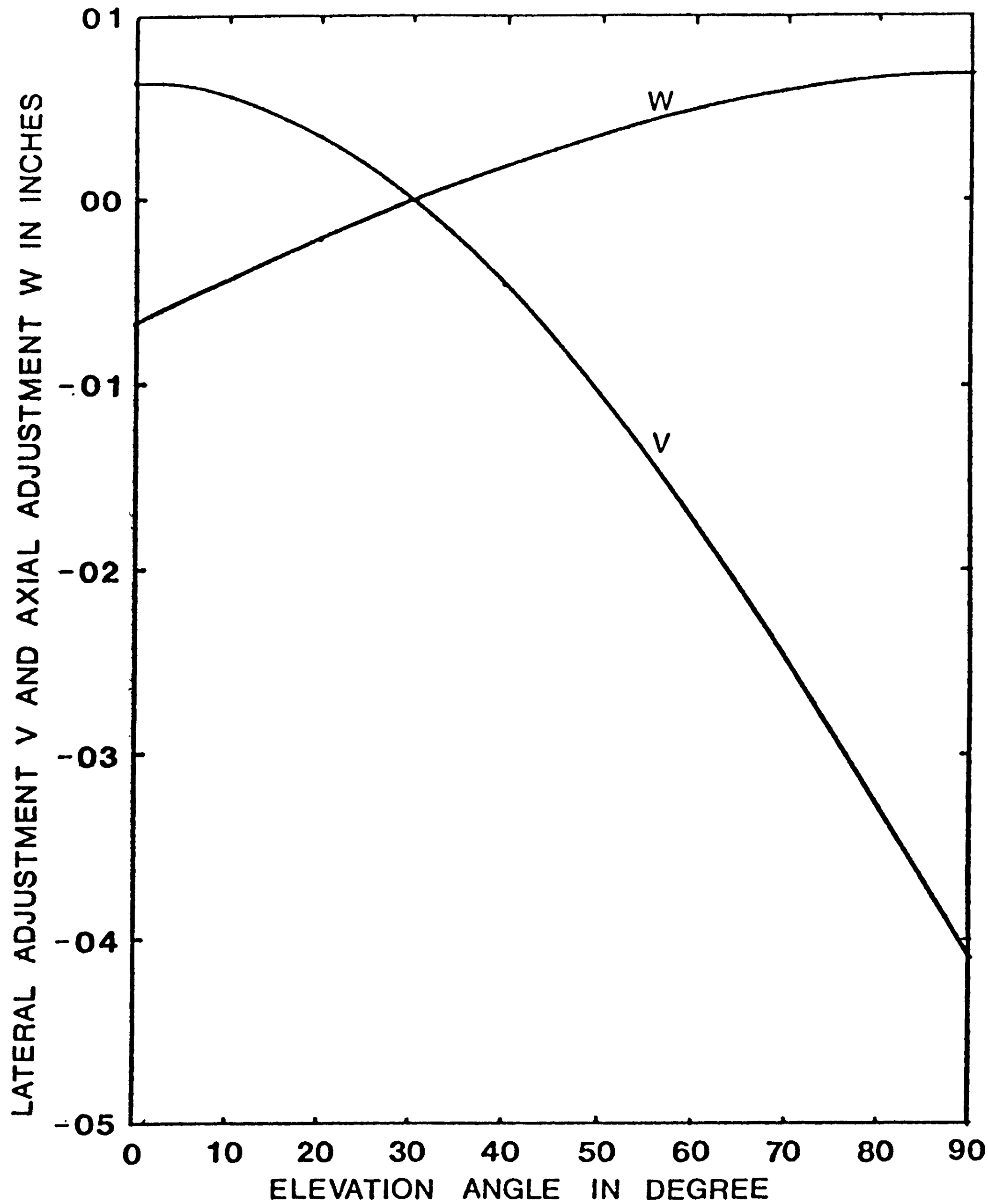

Figure 4 Magnitude Of Lateral Adjustment V And Axial Adjustment W Of Secondary Reflector.

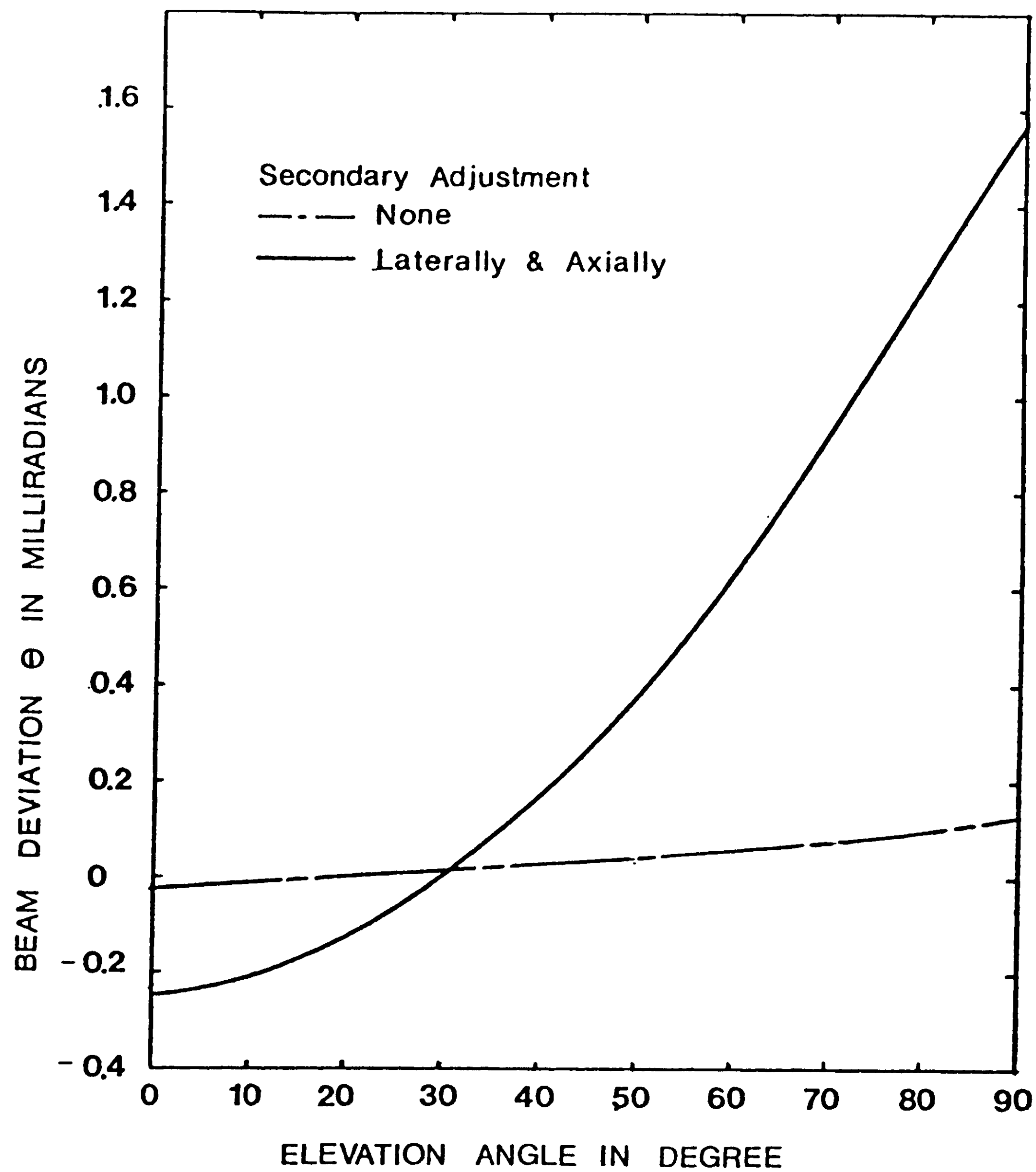

Figure 5 *Beam Deviation Θ Due To Gravity Deformation With And Without Lateral Adjustment Of The Position Of Secondary Reflector, $\alpha_r = 30°$*

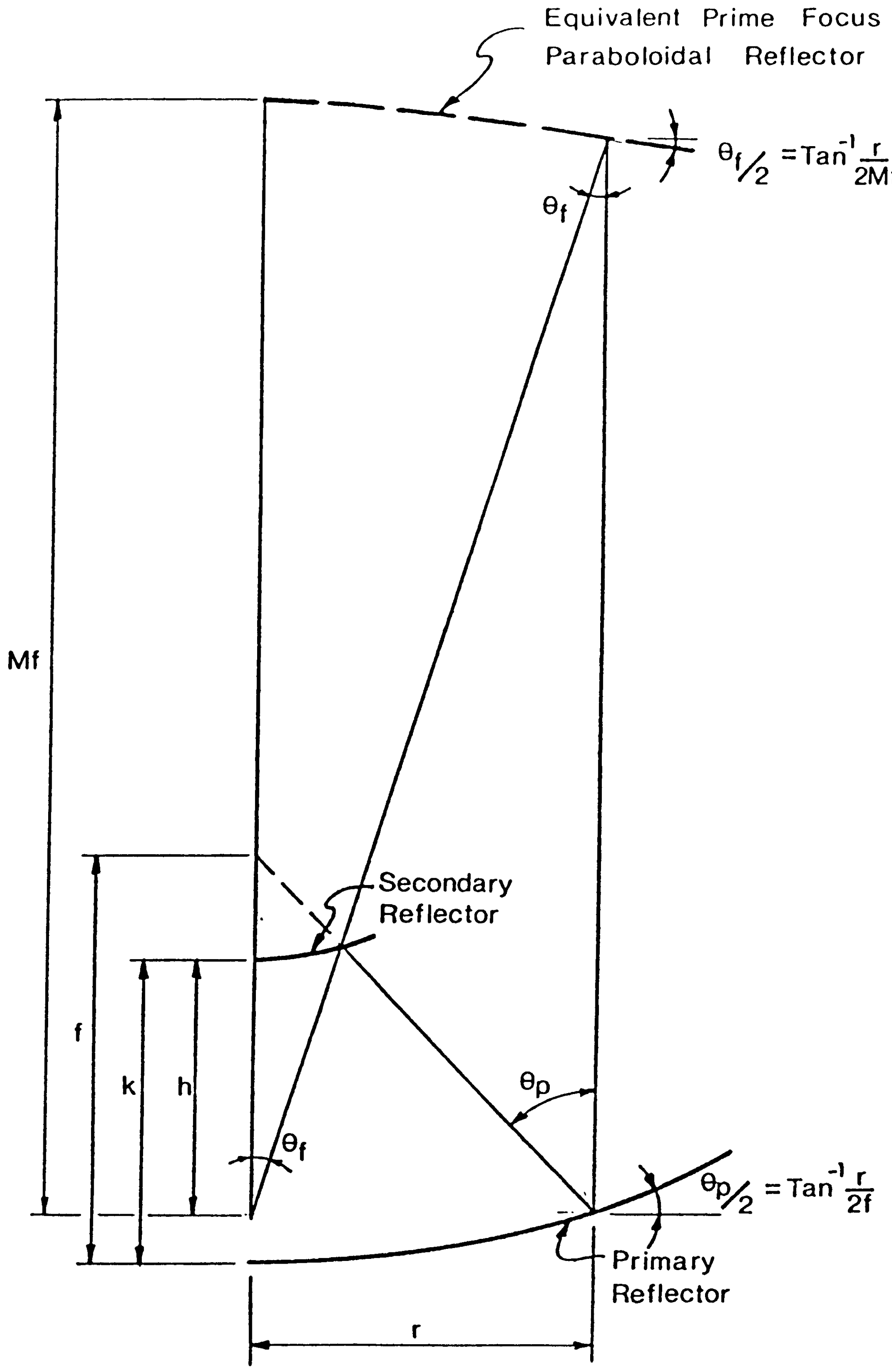

Figure 2 Geometry Of The Equivalent Prime Focus Paraboloidal Reflector.