12 METER MILLIMETER WAVE TELESCOPE

MEMO No. 152

NATIONAL RADIO ASTRONOMY OBSERVATORY
Tucson, Arizona

April 15, 1982

12-m Telescope Memorandum

To: H. Hvatum

From: M. A. Gordon

Subject: Weighted Aperture Blockage of the 12-m Telescope

References: Ruze, J. 1968 Microwave Journal, December, pp 76-80
Gordon, M. A. 1981, 12-m Telescope Memo 73
King, L. J. 1982, 12-m Telescope Memo 148

This memorandum supplements Lee King's results by presenting the blockage calculations in detail, the resulting total blockage in percent, and makes a comparison with the present blockage of the 36-ft telescope.

Telescope	Power Efficiency	Power Blockage	Calculator
36-ft	0.8565	14.4 %	Ulich
12-m	0.8785	12.2 %	Gordon

I find the same distribution of aperture blockage as Lee found,

Hub	Strut (Plane Wave)	Strut (Divergent Wave)	Total Strut
25.2 %	40.0 %	34.8 %	74.8 %

Because of the length of the struts, a reduction in width can have a substantial effect upon the percentage blockage. For example, reducing their width by 1 inch would reduce the total blockage from 12.2 to 10.4 %. Also, because of the illumination taper, shrinking the hub assembly to within the cassegrain cut-out circle would reduce the total blockage.

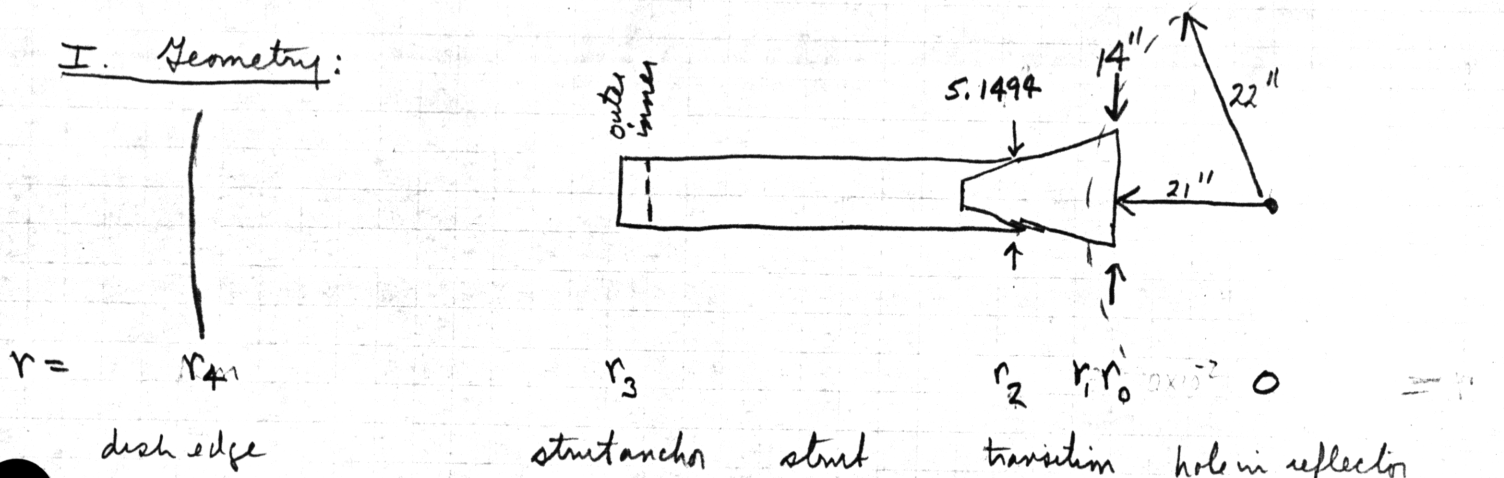
I've attached my worksheets.

Calculations for Aperture Blockage, Revised Quadrupod Structure

M. A. Gordon April 13, 1982

Reference 12-m Memos 73 and 148

I. Geometry:



Radius	Inches	Meters	Normalized
r_0	21	0.5334	8.990×10^{-2}
r_1	22	0.5588	9.313×10^{-2}
r_2	$21 + 10 \sin 36^\circ =$	0.6827	1.138×10^{-1}
r_3 { in	155.5	3.950	0.6583
r_3 { out	176.3	4.479	0.7465
r_4	236.2	6.000	1

II. Illumination Taper:

Assume that electric field density varies as

$$f(r) = 1 - a r^2$$

John Payne tells me most feeds have an 11db taper; so

$$\frac{\text{edge power}}{\text{center power}} = f^2(r=1) \doteq (1-a)^2 = 10^{-1.1} = 0.0793$$

$$1 - a = 0.2818, \quad a = 0.7182$$

$$\boxed{f(r) = 1 - 0.7182\, r^2}$$

III. Maximum Achievable Aperture

$$A_0 = \underbrace{\int_0^{2\pi}\int_0^1 f(r)\, r\, dr\, d\theta}_{\text{dish}} - \underbrace{\int_0^{2\pi}\int_0^{r_0} f(r)\, r\, dr\, d\theta}_{\text{central hole}}$$

$$= \pi\left(1 - \frac{a}{2}\right) \quad - \pi r_0^2\left(1 - \frac{a}{2} r_0^{2\cdot}\right)$$

$$= 2.013 \quad - \quad 2.476 \times 10^{-2}$$

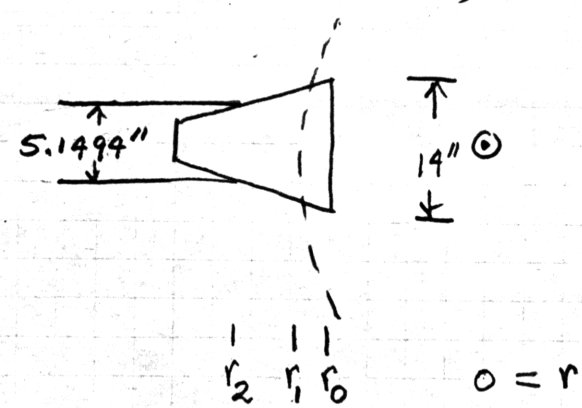
$$\boxed{A_0 = 1.988}$$

(normalized units of $r_4 = 1$)

IV. Hub Blockage

The only part of the hub which contributes to the blockage is the part outside of the central hole of the dish; i.e., the sections between r_1 and r_2.

Let the width of the taper section be described by



$$w(r) = b + kr$$

$$w(r_0) = b + k r_0 \equiv w_0$$

$$w(r_2) = b + k r_2 \equiv w_2$$

$$k = \frac{w_2 - w_0}{r_2 - r_0}$$

$w_0 = 14'' = 0.356\text{m} = 5.927 \times 10^{-2}\ r$

$w_2 = 5.1494'' = 0.1308\text{m} = 2.180 \times 10^{-2}\ r$

$$b = \frac{w_0 r_2 - r_0 w_2}{r_2 - r_0}$$

By substitution,

$$k = \frac{(2.180 - 5.927)\times10^{-2}}{(11.38 - 8.890)\times10^{-2}} = -1.505$$

2.49

6.745 $\times10^{-3}$ 1.938 $\times10^{-3}$

$$b = \frac{(5.927 \times 1.138)\times10^{-3} - (8.890 \times 2.180)\times10^{-4}}{2.490\times10^{-2}} = 1.931\times10^{-1}$$

$$\boxed{W(r) = 0.1931 - 1.505\,r}$$

The blockage of the 4 hub sections extending past the cassegrain circle is

$$A_1 = 4\int_{r_1}^{r_2} w(r)\,f(r)\,dr$$

$$\frac{A_1}{4} = \int_{r_1}^{r_2} (b + kr)(1 - ar^2)\,dr$$

$$= \int_{r_1}^{r_2} \left(b + kr - abr^2 - akr^3\right) dr$$

$$= \left[br + \frac{k}{2}r^2 - \frac{ab}{3}r^3 - \frac{ak}{4}r^4\right]\Bigg|_{r_1}^{r_2}$$

$$= b(\Delta r) + \frac{k}{2}(\Delta r)^2 - \frac{ab}{3}(\Delta r)^3 - \frac{ak}{4}(\Delta r)^4$$

$$= T_1 + T_2 - T_3 - T_4$$

T: $\Delta r = r_2 - r_1 = 1.138\times10^{-1} - 0.9313\times10^{-1}$

$= \underline{2.067\times10^{-1}}$

$T_1 = b(\Delta r) = 0.1931\,(2.067\times10^{-1}) = 3.991\times10^{-2}$

$T_2 = \frac{k}{2}(\Delta r)^2 = \frac{-1.505}{2}(\Delta r)^2 = -3.215\times10^{-2}$

$T_3 = \frac{ab}{3}(\Delta r)^3 = \frac{(0.7182)(0.1931)}{3}(\Delta r)^3 = 4.083\times10^{-4}$

$T_4 = \frac{ak}{4}(\Delta r)^4 = \frac{(0.7182)(-1.505)}{4}(\Delta r)^4 = -4.933\times10^{-4}$

$\frac{A_1}{4} = (3.991\times10^{-2}) + (-3.215\times10^{-2}) - (4.083\times10^{-4}) - (-4.933\times10^{-4})$

$= 7.845\times10^{-3}$

$$\boxed{A_1 = 3.138\times10^{-2}}$$

V. Strut Blockage (Plane Wave)

The width of the strut is

$W = 5.1494'' = 0.1308\,m = 2.180\times10^{-2}$

The blockage for an incoming plane wave is

$$A_2 = 4\int_{r_2}^{r_3} W\, f(r)\,dr$$

$$= 4W\int_{r_2}^{r_3}(1 - ar^2)\,dr$$

$$= 4W\left(r - \frac{a}{3}r^3\right)\Big|_{r_2}^{r_3}$$

$$= 4W\cdot\Delta r\left[1 - \frac{a}{3}(\Delta r)^2\right]$$, $\Delta r \equiv r_3 - r_2 = 6.327\times10^{-1}$ (OUTER)

By substitution

$$A_2 = 4\cdot(2.180\times10^{-2})(6.327\times10^{-1})\left[1-\frac{0.7182}{3}(6.327\times10^{-1})^2\right]$$

$$\boxed{A_2 = 4.988\times10^{-2}}$$

VI. Strut Blockage (Spherical Wave)

This is blockage of the reflected wave from the surface to the prime focus. Ruze (1952) derives the equation,

$$A_3 = 4\cdot\frac{W}{2r_2}\Big[\{(1-r_3^2)-\frac{a}{2}(1-r_3^4)\}$$

$$-2f\tan\alpha\{(1-r_3)-\frac{a}{3}(1-r_3^3)\}$$

$$+\frac{\tan\alpha}{2f}\{\frac{(1-r_3^3)}{3}-\frac{a}{5}(1-r_3^5)\}\Big]$$

$f \equiv$ focal length $= 200'' = 5.080\text{m} = 0.8467$

$\alpha =$ angle between strut and focal axis $= 36°$

$\tan\alpha = 0.7265$

Term 1: $\{(1-r_3^2)-\frac{a}{2}(1-r_3^4)\}$ (INNER)

$$= \{(1-0.6583^2)-\frac{0.7182}{2}(1-0.6583^4)\} = 2.750\times10^{-1}$$

Term 2: $2f\tan\alpha\{(1-r_3)-\frac{a}{3}(1-r_3^3)\}$

$$= 2\cdot(0.8467)(0.7265)\underbrace{\{(1-r_3)-\frac{0.7182}{3}(1-r_3^3)\}}_{1.706\times10^{-2}} = 2.099\times10^{-1}$$

Term 3: $\frac{\tan\alpha}{2f}\{\frac{(1-r_3^3)}{3}-\frac{a}{5}(1-r_3^5)\}$

$$\frac{0.7265}{2(0.8467)}\underbrace{\{\frac{(1-r_3^3)}{3}-\frac{0.7182}{5}(1-r_3^5)\}}_{1.124\times10^{-1}} = 4.820\times10^{-2}$$

$$A_3 = 4 \frac{W}{2 r_2} \left[\text{Term 1} - \text{Term 2} + \text{Term 3} \right]$$

$$= 4 \frac{2.180 \times 10^{-2}}{2 \cdot (0.1138)} \left[\underbrace{0.2750 - 0.2099 + 0.04820}_{0.1133} \right]$$

$$\boxed{A_3 = 4.341 \times 10^{-2}}$$

VII. Scattering Efficiency of Struts

At 150 GHz, or $\lambda = 2.0$ mm, the width of the strut is

$$\frac{W}{\lambda} = \frac{5''}{2\text{mm}} = \frac{127\text{mm}}{2\text{mm}} \approx 64 \text{ wavelengths}$$

From Ruze's Figure 3, the induced current ratio (ICR) is

$$ICR \approx 1$$

which means that the strut blockage is its optical shadow.

VIII. The Percentage Blockage

$$\text{Power factor} = \left[1 - \frac{A_1}{A_0} - \left(\frac{A_2 + A_3}{A_0} \right) ICR \right]^2$$

$$= \left[1 - \frac{A_1}{A_0} - \frac{A_2}{A_0} - \frac{A_3}{A_0} \right]^2$$

$$= \left[1 - \text{Hub} - \text{Strut} - \text{Strut Divergence} \right]^2$$

$$= \left[1 - \frac{3.138 \times 10^{-2}}{1.988} - \frac{4.988 \times 10^{-2}}{1.988} - \frac{4.341 \times 10^{-2}}{1.988} \right]^2$$

$$= \left[1 - 1.578 \times 10^{-2} - 2.509 \times 10^{-2} - 2.184 \times 10^{-2} \right]^2$$

$$= [0.9373]^2 \qquad 6.27\% \text{ field blockage}$$

$$= 0.8785 \qquad \underline{\underline{12.15\%}} \text{ power blockage}$$

IX. Analysis of Components

	Hub	Strut	Strut Divergence
Percentage of the Blockage =	25.2 %	40.0 %	34.8 %
		74.8% (Strut + Strut Divergence)	

X. Where Can Improvements be made?

Hub: Shrinking the hub to within the cut-out circle could be very helpful. It now extends 4.878 inches beyond the cut-out circle. Here the f(r) illumination taper is greatest. A small shrinkage would help.

Strut: Reducing the width by 1", from 5.1494 → 4.1494 Changes the blockage

Field	6.27%	→ 5.33%
Power	12.15%	→ 10.38%

Running Strut to Rim: This typically reduces the power blockage to 0.75 of the conventional design.