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HOLOGRAPHIC ANTENNA MEASUREMENTS: FURTHER TECHNICAL CONSIDERATIONS

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Summary

In a recent report (Moore 1982), Craig Moore discusses determining the surface errors of the new NRAO 12-meter antenna by interferometry of a CW signal from a satellite. The reader is referred to that report for necessary background. The present report provides some analysis of the signal processing for such a scheme. This leads to a formula for the signal-to-noise ratio of the individual measurements which is slightly different from that assumed by Moore (his equation (2)). Furthermore, when the measurements are combined to form am image of the aperture field and the phase of this field image is used to estimate the surface errors, we find an expression for the noise in the latter estimates which agrees with the formula of Scott and Ryle (1977) only in the special case where all measurements have the same noise variance. Use of the on-axis noise in Scott and Ryle's formula (Moore's equation (3)) leads to an overestimate of the SNR requirement.

In addition, we enumerate some considerations other than random noise which affect the accuracy of the measurements. Choice of the type of detector is considered briefly. The dynamic range requirement is analyzed in some detail, and the use of ALC loops is discussed. A digital cross correlator without ALC is recommended, and the number of bits needed to satisfy the dynamic range requirement is calculated.

I. Noise in a Correlation Receiver with Monochromatic Signals

In this section, we determine the effect of receiver noise on a single measurement.

We model the receiver as shown in Fig. 1a, where the signals $s_1(t)$ and $s_2(t)$ are multiplied and low-pass filtered. Let the signals be

$$s_1(t) = V_1 \cos(2\pi f_o t) + n_1(t) \tag{1}$$

$$s_2(t) = V_2 \cos(2\pi f_o t + \phi) + n_2(t) \tag{2}$$

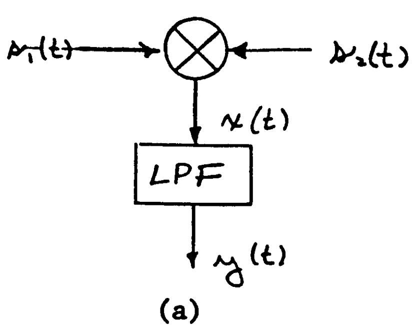
where n_1 and n_2 are independent Gaussian random processes such that

$$\langle n_1 \rangle = \langle n_2 \rangle = 0 \tag{3}$$

$$\langle n_1^2 \rangle = kT_{s1}B \qquad \langle n_2^2 \rangle = kT_{s2}B \tag{4}$$

and with power spectra as illustrated in Fig. 1b. (Here we take $B < f_o$, but the analysis for baseband signals leads to the same final result.) Then the multiplier output is

$$\begin{aligned} x(t) &= s_1(t)\, s_2(t) \\ &= V_1 V_2 \cos(2\pi f_o t) \cos(2\pi f_o t + \phi) + n_1(t)\, n_2(t) \\ &\quad + V_1\, n_2(t) \cos(2\pi f_o t) + V_2\, n_1(t) \cos(2\pi f_o t + \phi). \end{aligned} \tag{5}$$



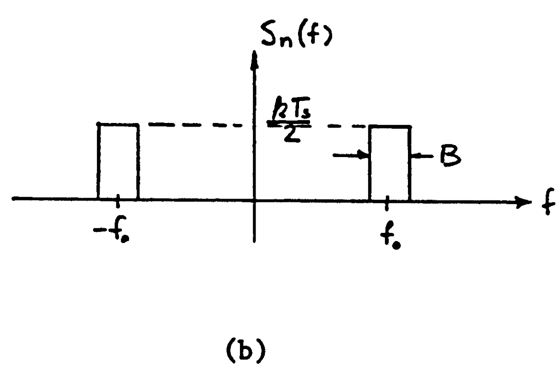


Fig. 1: (a) Correlation receiver model. (b) Power spectral density of receiver noise $n_1(t)$.

Let $X = \langle y(t) \rangle$ be the "signal" and let $\sigma = (\langle y^2(t)\rangle - X^2)^{1/2}$ be the "rms noise," where y(t) is the output of the LPF. The obtain σ, we calculate the power spectrum of y(t); this is easily obtained from the power spectrum of x(t), which we now compute:

$$S_x(f) = \text{F.T.}\{\langle x(t)\ x(t+\tau)\rangle\} \tag{6}$$

where F.T.{ } denotes the Fourier transform with respect to τ.

$$\begin{aligned} S_x(f) = \text{F.T.}\{ < [&\tfrac{1}{2}V_1V_2 \cos\phi + \tfrac{1}{2}V_1V_2 \cos(4\pi f_o t + \phi) + n_1(t)n_2(t) \\ &+ V_1n_2(t)\cos 2\pi f_o t + V_2n_1(t) \cos(2\pi f_o t + \phi)] \\ &\times [\tfrac{1}{2}V_1V_2\cos\phi + \tfrac{1}{2}V_1V_2\cos(4\pi f_o t + 4\pi f_o\tau + \phi) \\ &+ n_1(t+\tau)\ n_2(t+\tau) + V_1n_2(t+\tau)\ \cos(2\pi f_o t + 2\pi f_o\tau) \\ &+ V_2n_1(t+\tau)\ \cos(2\pi f_o t + \phi)] > \} \\ = \text{F.T.}\ \{ &\tfrac{1}{4}\ V_1{}^2V_2{}^2\cos^2\phi + \tfrac{1}{8}\ V_1{}^2V_2{}^2\cos 4\pi f_o\tau + \rho_1(\tau)\rho_2(\tau) \\ &+ V_1{}^2\rho_2(\tau)\cos(2\pi f_o\tau) + V_2{}^2\rho_1(\tau)\cos 2\pi f_o\tau \} \end{aligned} \tag{7}$$

where $\rho_i(\tau) = \langle n_i(t)n_i(t+\tau)\rangle$, i = 1 or 2. The Fourier transform of each term may be evaluated by inspection, given the simple form of the noise power spectra, and recalling that the transform of a product is the convolution of the transforms. The result, illustrated in Fig. 2, is

$$\begin{aligned} S_x(f) = &\frac{1}{4}V_1{}^2V_2{}^2\cos^2\phi\ \delta(f) + \frac{1}{16}\ V_1{}^2V_2{}^2[\delta(f - 2f_o) + \delta(f + 2f_o)] \\ &+ kT_{s1}\ kT_{s2}B\ \text{tri}(f/B)\ [\frac{1}{2}\text{tri}(f/B) + \frac{1}{4}\ \text{tri}((f - 2f_o)/B)\ ] \\ &+ \frac{1}{2}\ V_1{}^2kT_{s2}\ \text{rect}(f/B) + \frac{1}{2}\ V_2{}^2kT_{s1}\ \text{rect}(f/B) \end{aligned} \tag{8}$$

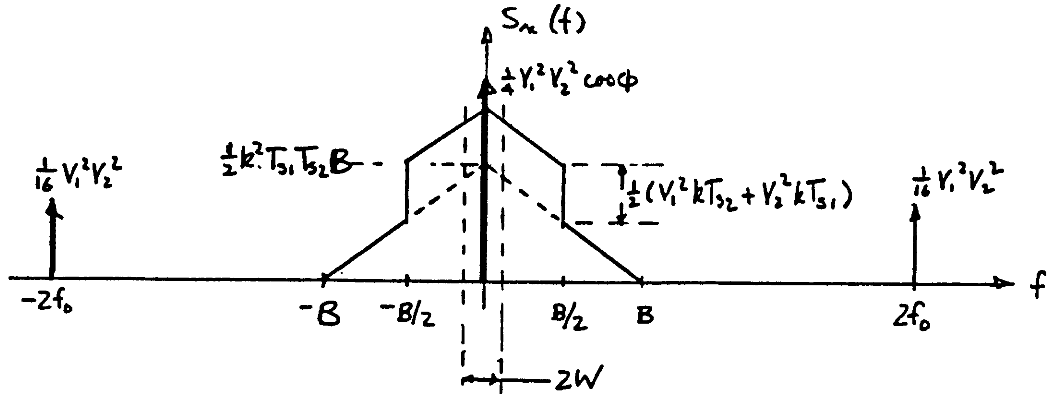


Fig. 2: Power spectral density of multiplier output signal x(t).

Now let the LPF have an ideal rectangular passband of width W. Then

$$\langle y^2 \rangle = \int_{-\infty}^{\infty} S_y(f)\,df = \int_{-W}^{W} S_x(f)\,df$$

$$= \frac{1}{4} V_1{}^2 V_2{}^2 \cos^2\phi + W(kT_{s1}kT_{s2}B + V_1{}^2 kT_{s2} + V_2{}^2 kT_{s1}) \qquad (9)$$

and (from (5))

$$X = \langle x \rangle = \langle y \rangle = \frac{1}{2} V_1 V_2 \cos\phi$$

so

$$\sigma = \sqrt{\langle y^2 \rangle - X^2} = \sqrt{W(kT_{s1}kT_{s2}B + V_1{}^2 kT_{s2} + V_2{}^2 kT_{s1})}\ . \qquad (10)$$

For identical receivers we can let $T_{s1} = T_{s2} = N_o/k$; putting $C_i = \frac{1}{2} V_i^2$ for the carrier powers then gives

$$\sigma_e = \sqrt{W[N_o{}^2B + 2N_o(C_1 + C_2)} = \sqrt{\sigma_n{}^2 + \sigma_1{}^2 + \sigma_2{}^2} \tag{11}$$

and

$$SNR_e = \frac{X}{\sigma_e} = \sqrt{\frac{C_1C_2}{W[N_o{}^2B + 2N_o(C_1 + C_2)]}} \cos\phi \ . \tag{12}$$

If the C_1 term dominates the denominator (as it will for the on-axis measurement if C_1 is the main reflector signal power), then

$$SNR_e \approx \sqrt{\frac{C_2}{2WN_o}} \cos\phi \ . \tag{13}$$

This may be compared with Craig Moore's equation (2) for integrating time $\tau = 1/2W$. His expression gives the square of the SNR. In addition we find that the SNR depends on the carrier to noise density ratio of the weaker signal C_2/N_o, where he has the geometric mean of the two signals. This is significant.

II. Noise Analysis of the Mapping Scheme

Here we calculate, starting from elementary considerations, the noise in a map of the aperture field made from measurements of the type contemplated. It iwll be necessary to make some simplifying assumptions about how the map is computed.

A. Some simple antenna theory.

We will describe the antenna using scalar diffraction theory (ignoring polarization) and monochromatic signals. Extention to wide bandwidths would be straightforward, but is unnecessary here.

Select a reference plane (in principle arbitrary) in which the antenna aperture is defined, and let (ℓ,m) be direction cosines with respect to the normal to this plane. Then a uniform plane wave from direction (ℓ,m) with electric field amplitude E_o in the aperture plane produces an open circuit voltage at the antenna terminals given by

$$V = E_o\, h(\ell,m) \tag{14}$$

where $h(\ell,m)$ is the complex effective length of the antenna (Sinclair 1950).* If the antenna is made to transmit by applying a current source of amplitude I_t to the terminals, then from reciprocity it can be shown that the far field distribution is a superposition of plane waves having amplitudes per unit solid angle

$$e(\ell,m) = \frac{Z_o I_t}{\lambda^2}\, h(\ell,m), \tag{15}$$

where $Z_o = \sqrt{\mu_o/\varepsilon_o}$ is the free space impedance and λ is the wavelength. A fundamental theorem which is the basis of the present project is that the

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*Since the available power received is

$$\frac{1}{8R_a}\,|V|^2 = \frac{1}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\,|E_r|^2\, A_e,$$

where R_a is the real part of the terminal impedance and A_e is the effective area, comparison with (14) shows that A_e and h are related by

$$A_e = \frac{1}{4R_a}\sqrt{\frac{\mu_o}{\varepsilon_o}}\,|h|^2 .$$

Although effective area seems to be more familiar to radio astronomers than effective length, the latter is more appropriate when considering interferometers.

aperture plane field when transmitting is the Fourier transform of $e(\ell,m)$, which from (15) is proportional to $h(\ell,m)$, which in turn can be determined in the receiving mode via (14). Combining these gives

$$E_a(x,y) = \frac{Z_o I_t}{\lambda^2} \iint_{\ell^2+m^2<1} \frac{h(\ell,m)}{\sqrt{1-\ell^2-m^2}} e^{-j2\pi(\ell x + my)/\lambda} \, d\ell \, dm \qquad (16)$$

for the aperture field that would be produced by current I_t. Since I_t is arbitrary, we consider from now on the normalized quantity

$$F(x,y) = \frac{\lambda^2}{Z_o I_t} E_a(x,y). \qquad (17)$$

B. The Measurements

We have available a reference antenna whose effective length function $h_r(\ell,m)$ is known. Its response to a plane wave is

$$V_r = E_o h_r(\ell,m) \qquad (18)$$

and the cross-correlation of the reference and test antennas' outputs is

$$VV_r^* = |E_o|^2 \, h(\ell,m) \, h_r^*(\ell,m). \qquad (19)$$

Let M_k be a measurement of the cross correlation (19) in the presence of noise:

$$M_k = |E_o|^2 \, h(\ell_k,m_k) \, h_r^*(\ell_k,m_k) + \varepsilon_k \qquad (20)$$

where ε_k is a zero-mean error. Let there be K measurements $(k=1,\ldots,K)$ at various (ℓ,m), preferably on a rectangular grid. Our task is to estimate $F(x,y)$ from these measurements.

The best linear estimator must be of the form

$$\hat{F}(x,y) = \sum_{k=1}^{K} \frac{M_k}{|E_o|^2 h_r^*(\ell_k,m_k)} e^{j2\pi(\ell_k x + m_k y)/\lambda} W_k \tag{21}$$

where $\{W_k\}$ are weights which may be adjusted to control the errors and the resolution. Using (16) and (17), it is easily shown that

$$\hat{F}(x,y) = F(x,y)**b(x,y) + \delta(x,y) \tag{22}$$

where

$$b(x,y) = \sum_{k=1}^{K} W_k e^{j2\pi(\ell_k x + m_k y)/\lambda} \tag{23}$$

and

$$\langle\delta(x,y)\rangle = 0. \tag{24}$$

The latter follows from $\langle\varepsilon_k\rangle = 0$. If, in addition, $\langle\varepsilon_k\varepsilon_\ell\rangle = 0$ for all $k \neq \ell$ (uncorrelated errors), we find that

$$\langle\delta^2\rangle = \sum_{k=1}^{K} \left| \frac{W_k}{|E_o|^2 h_r(\ell_k,m_k)} \right|^2 \left\langle\varepsilon_k^2\right\rangle . \tag{25}$$

Note that (25) shows that the variance of the error in the estimated aperture field is independent of aperture position (x,y).

C. Noise Analysis

One way to assign the weights in (21) is to consider (21) to be a discrete approximation to (16):

$$\hat{F}(x,y) = \sum_{k=1}^{K} \frac{\hat{h}(\ell_k, m_k)}{\sqrt{1 - \ell_k^2 - m_k^2}} e^{j2\pi(\ell_k x + m_k y)/\lambda} \Delta\ell\Delta m \qquad (26)$$

so that $W_k = \Delta\ell\Delta m/\sqrt{1 - \ell_k^2 - m_k^2}$ and $\hat{h} = M_k/|E_o|^2 h_r^*$. Here we assume a rectangular grid of spacing $\Delta\ell, \Delta m$. For small ℓ_k, m_k the square root factor may be dropped. Then, from (25), the variance of $\hat{F}$ is

$$<\delta^2> = \left(\frac{\Delta\ell\Delta m}{|E_o|^2}\right)^2 \sum_{k=1}^{K} \frac{\left\langle \varepsilon_k^2 \right\rangle}{|h_r(\ell_k, m_k)|^2} \quad . \qquad (27)$$

To evaluate the effect of this noise, consider measurement of an ideal circular aperture with uniform illumination, so that

$$F(x,y) = \begin{cases} \dfrac{h(0,0)}{\pi(D/2\lambda)^2} , & r \leq D/2\lambda \\ 0, & r > D/2\lambda \end{cases} \qquad (28)$$

where $r = (x^2 + y^2)^{\frac{1}{2}}$ and D is the aperture diameter. Although the phase of F is zero, the phase of $\hat{F}$ has rms value

$$\Delta\phi = \frac{\sqrt{<\delta^2>/2}}{|F|} \quad , \qquad (29)$$

provided $\langle\delta^2\rangle$ is small. The inferred error in reflector surface position is then

$$\Delta z = \lambda \frac{\Delta\phi}{4\pi} = \frac{\lambda}{4\pi\sqrt{2}} \frac{\Delta\ell\Delta m}{|E_o|^2\ |h_r|} \frac{\pi(D/2\lambda)^2}{|h(0,0)|} \left[\sum_{k=1}^{K} \langle\varepsilon_k{}^2\rangle\right]^{\frac{1}{2}}$$

$$= \frac{\lambda}{4\pi\sqrt{2}} \Delta\ell\Delta m \frac{\pi(D/2\lambda)^2}{\langle M_o\rangle} \left[\sum_{k=1}^{K} \langle\varepsilon_k{}^2\rangle\right]^{\frac{1}{2}}$$

$$= \frac{1}{16\sqrt{2}} \frac{\Delta\ell\Delta m\, D^2\, K^{\frac{1}{2}}\sigma_{AV}}{\lambda\langle M_o\rangle} \tag{30}$$

where σ_{AV} is the rms of the measurement errors, $\langle M_o\rangle$ is the expected value of the on-axis measurement (even if this point is not actually measured) and we have taken the reference antenna pattern h_r to be constant. If we desire a resolution of Δ in the aperture plane, then $K \geq (D/\Delta)^2$ points must be sampled. The sampling theorem requires $\Delta\ell, \Delta m \leq \lambda/D$. Taking equality in both cases (although this would allow serious aliasing) gives

$$\Delta z = \frac{1}{16\sqrt{2}} \frac{\lambda D}{\Delta\cdot SNR} \tag{31}$$

where $SNR = \langle M_o\rangle/\sigma_{AV}$. Apart from the numerical factor, this agrees with the formula of Scott and Ryle (1977). But in their case, the source is assumed weak, so that $\langle\varepsilon_k{}^2\rangle$ is the same for all measurements and SNR can be interpreted as the on-axis signal to noise ratio; whereas in the general case, we see that SNR should be interpreted as the ratio of on-axis signal to rms average noise over all measurements. Scott and Ryle have $4\pi = 12.57$ where we have $16\sqrt{2} = 22.62$.*

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*They dropped a factor of $4/\pi$ in the ratio of resolution element area to dish area, and they missed a factor of $\sqrt{2}$ because only the phase component of the noise is significant.

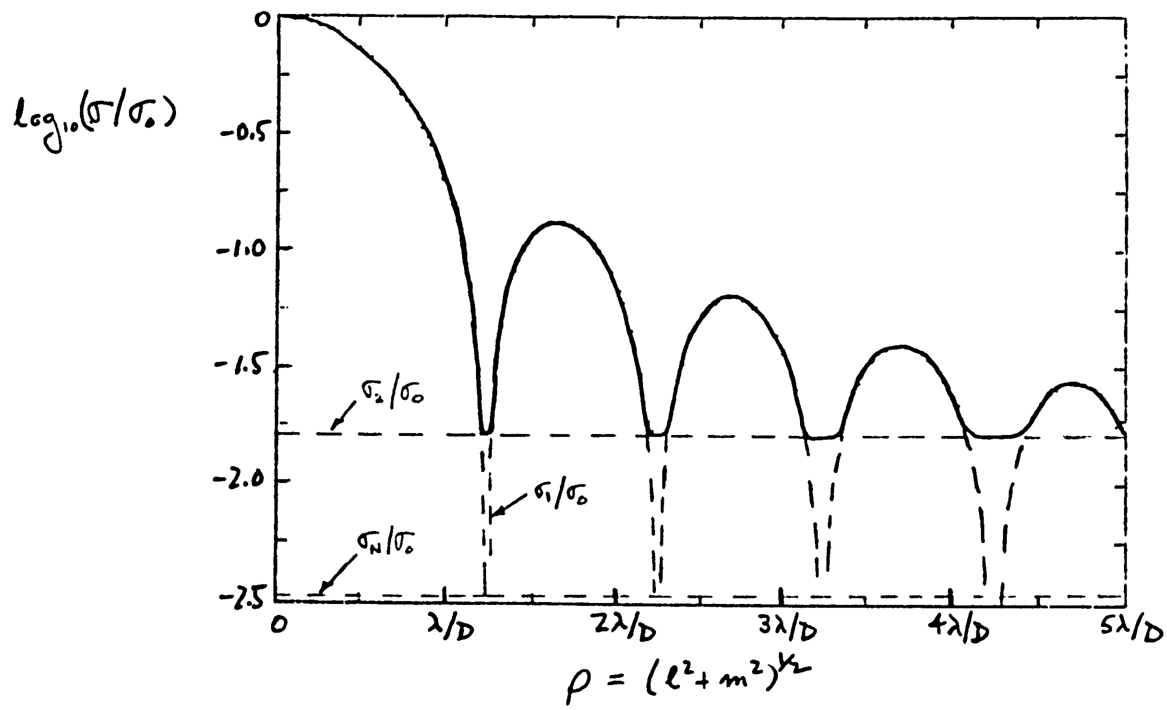

Fig. 3: Noise in a single measurement vs. distance off axis, for a uniformly illuminated main reflector and broad-beamwidth reference antenna (see eqn. (11)). Relative signal and noise powers from Moore (1982).

Figure 3 is a plot of the single-measurement noise as a function of distance off axis, based on equation (11). The dashed lines σ_N, σ_1, and σ_2 show the contributions of the first, second, and third terms, respectively. A uniform circular aperture is assumed, so that

$$h(\ell,m) = \left(\frac{\pi R_a}{Z_o}\right)^{\frac{1}{2}} D \; \frac{J_1(2\pi a\rho)}{\pi a\rho} \quad , \qquad (32)$$

where R_a is the radiation resistance, $a = D/2\lambda$, $\rho = (\ell^2 + m^2)^{\frac{1}{2}}$, and J_1 is the usual Bessel junction. In the plot, the relative strengths of the three terms are based on Table I of Moore (1982) with 10 kHz bandwidth. Although no allowance has been made for the variation of reference antenna gain with ρ, this variation has also been dropped from (30) and (31), so there should be little effect on the final result.

I have evaluated σ_{AV} for various sampling parameters, assuming uniform illumination of the main reflector; the results are given in Table A. Also given are the corresponding SNR and integrating time needed to achieve $\Delta z = 20\ \mu m$, again using parameters from Moore's Table I.

TABLE A: Noise and Integrating Time Requirements

SAMPLING INTERVAL	RESOLUTION	TOTAL NUMBER OF SAMPLES	For 12-m Telescope Parameters*				
						Δz=20 μm	
$\Delta \ell D/\lambda$	D/Δ	$K=(\Delta\ell \cdot \Delta/\lambda)^{-2}$	σ_{AV}/σ_o	M_o/σ_{AV}	M_o/σ_o	τ,ms	$K\tau$,s
1.0	25	625	.046	442	20.3	3.7	2.31
	50	2,500	.027	884	23.9	5.1	12.8
	100	10,000	.020	1,768	35.4	11.2	112.
0.75	25	1,000	.047	331	15.6	2.2	2.2
	50	4,000	.028	663	18.6	3.1	12.4
	100	16,000	.020	1,326	26.5	6.3	101.
0.5	25	2,500	.047	221	10.4	1.0	2.5
	50	10,000	.028	442	12.4	1.4	14.0
	100	40,000	.020	884	17.7	2.8	112.

*λ = 8 mm, D = 12 m, signal and noise powers from Table I of Moore (1982).

III. Accuracy Considerations Other Than Random Noise

Without discussing them in detail, I list here some factors which could affect the accuracy of the final result and which will at least have to be considered in the data processing.

1. Variation of received power due to satellite transmitter changes, satellite pointing errors, atmospheric effects, and changes in receiver gains.

2. Inaccuracy in our knowledge of the reference antenna's beam.

3. Errors in the cross-correlator, such as saturation.

4. Pointing errors in the antenna under test.

5. Variations in the aperture of the antenna under test during the measurements (for example, due to motion of the satellite over a significant elevation range during the measurements).

One aspect of item 3 will be considered in the next section.

By using careful observing strategies, it should be possible to minimize the effects of items 1, 2, and 5 to whatever extent is required. Item 4 is potentially a problem; obviously, the pointing accuracy must be much better than λ/D, but we need to measure the surface to much better than λ. A careful analysis is needed but has not yet been done.

IV. Dynamic Range Requirement

It is a general property of Fourier transforms that large dynamic range in one domain corresponds to high precision in the other domain. Thus, in Fourier synthesis telescopes, large dynamic range in the map requires the absence of systematic errors in the measurements; but, for objects having high contrast (where large dynamic range is desired), the required accuracy need only be achieved over a small range of measured values.

In the holography problem, the situation is reversed; the aperture field distribution shows only small variations, leading to a large range of values in the measurement domain. The dynamic range of the measurements then determines the precision with which the aperture field can be estimated.

Suppose we wish to be able to detect variations in aperture field phase of $2\pi\Delta z/\lambda$, corresponding to surface position error $\Delta z/2$. If α is the amplitude of a sinusoidal variation at spatial frequency (ℓ,m), then the corresponding measurement in the (ℓ,m) direction must have an error less than $(2\pi\alpha/\lambda)M_o$, where M_o is the on-axis measurement. Note that the measurement is of the <u>voltage</u> pattern of the antenna, so the required dynamic range in decibles is $20 \log(2\pi\alpha/\lambda)$.*

However, the dynamic range requirement is somewhat mitigated by the following considerations. For a circular reflector antenna, the aperture field can be taken to be zero outside a circle of the antenna's diameter, D. The field is then determined by samples of its Fourier transform on a square grid of spacing λ/D (see Figure 4). The ideal antenna has a uniform aperture field (or one smoothly tapered in radius), and it may be reasonable to assume that the actual antenna differs only slightly from this. The transform of the ideal aperture is then small at all sample points except $(\ell,m) = (0,0)$. If no effort is made to measure the central point accurately (or if it is not measured at all), the dynamic range required for the remaining points is greatly reduced. In practice, some oversampling is desirable and some margin must be allowed for the antenna being further from ideal than expected.

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*For the 12-m telescope, let $\Delta z = 20\ \mu m$, $\lambda = 8$ mm; then dynamic range $\geq$ 36 dB is needed.

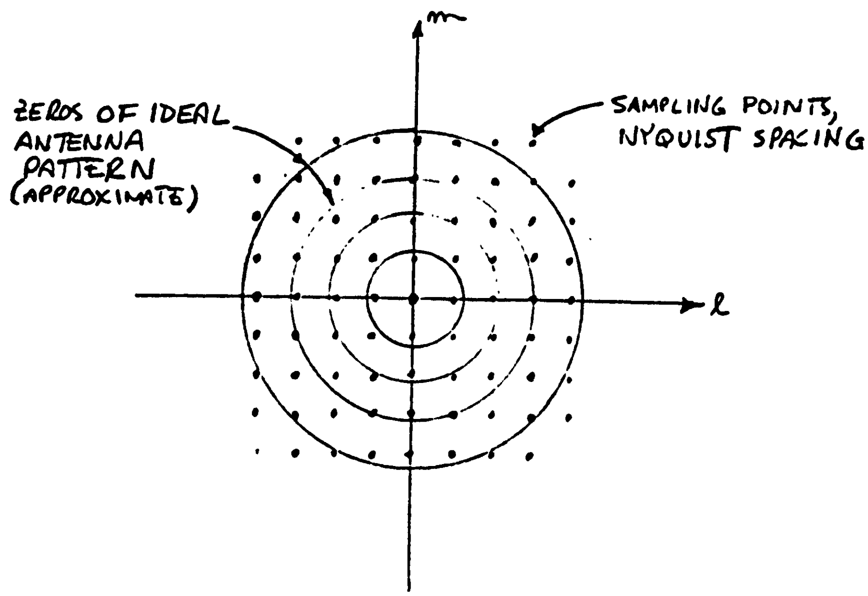

Fig. 4: Illustration of sampling grid relative to antenna pattern. Since many samples fall near zeros of the pattern, σ_{AV} can be much smaller than σ_o.

For the ideal, uniform aperture the antenna pattern is given by (32). With Nyquist sampling, ignoring the central point reduces the dynamic range requirement by a factor of

$$\frac{h(\lambda/D)}{h(0)} = \frac{J_1(\pi)}{\pi/2} = .181 \ (-14.9 \text{ dB}).$$

With a more conservative sampling internal of 0.8 λ/D, the reduction is

$$\frac{h(.8\lambda/D)}{h(0)} = \frac{J \ (0.8\pi)}{0.4\pi} = .393 \ (-8.1 \text{ dB}).$$

If the central point is measured inaccurately or not at all, there are two effects on the aperture field map: a d.c. offset due to the error in the zero-spatial-frequency term itself; and a complex scale error due to the unknown scale of the other terms relative to the zero frequency one. Neither of these has any effect on determination of the quantity of primary interest; namely the variation of phase across the aperture.

However, there could be some difficulty if the satellite power varies significantly during the measurements and it is desired to return to the central point periodically to check this. To ensure sufficient dynamic range in that case, it would be desirable to include a 6 to 10 dB attenuator in the signal channel which could be switched in or out under computer control. Although not essential, it would be good to know accurately the complex gain change when this switch is thrown.

V. ALC or Not?

The advantage of ALC in one or both channels is that it reduces the dynamic range required of the cross-multiplier. The dynamic range requirement is then shifted to square law detectors ahead of the ALC loop, or to measurements of the settings of the ALC loop attenuators. There seems to be no advantage to the use of ALC unless the latter measurements can be made more accurately than the cross-product measurement without ALC. In addition, it is necessary that the ALC loop attenuator not introduce any significant phase shift.

If the required dynamic range is > 30 dB, as seems to be the case, it will be difficult to achieve with analog circuitry. ALC would probably be used on both channels, and careful calibration of the ALC attenuators might succeed in determining the signal powers to sufficient accuracy. On the other hand, at the bandwidth contemplated for this receiver ($\leq$ 10 kHz), it is easily feasible to achieve the required dynamic range with digital circuitry. Quantization to

8 bits (256 levels) should be sufficient, and devices for 8-bit A to D conversion and 8 x 8 bit multiplication are readily available for operation at the required rates. Finer quantization is also possible (see next section).

A digital cross correlator without ALC is therefore recommended.

VI. Quantization

The voltage dynamic range of a signed, K-bit quantizer is 2^{K-1} if 3-level quantization of the weakest signals is adequate; or 2^{K-L} if $2^{L+1}-1$ levels are kept for the weakest signals. Taking L = 2 (7 levels), we have

$$2^{K-2} \geq \left(\frac{2\pi\Delta z}{\lambda}\right)^{-1}$$

or

$$K \geq 2-\log_2(2\pi\Delta z/\lambda),$$

where no special allowance has been made for the central point. In view of the argument of the last section, a safety factor of 2 to 3 is therefore included.*

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*Again using Δz = 20 μm and λ = 8 mm gives K ≥ 7.99 bits. Allowing saturation of the on-axis point reduces the requirement to 6 or 7 bits.

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