12 METER MILLIMETER WAVE TELESCOPE
MEMO No. 208

12/30/82

Note by John W. Findlay.

This paper has now appeared in print. It was used by John Payne in designing the NS translation stage for the 12-meter.

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Peak Gain of a Cassegrain Antenna with Secondary Position Adjustment

MEHDI S. ZARGHAMEE, MEMBER, IEEE

Abstract—For an enclosed Cassegrain antenna, the loss of peak gain and beam deviation due to structural deformations of the primary reflector and rigid body displacements of the secondary reflector and of the feed are computed from the combined changes in the radio frequency (RF) path length. As the antenna moves in elevation, the position of the secondary reflector may be adjusted mechanically to minimize the loss of peak gain; a general method for the computation of the magnitude of such adjustments and of their effects on the gain and pointing of the system is presented. Numerical results are obtained for a particular case of a 45-ft diameter antenna designed for operation at 95.5 GHz RF for which the computed peak gain of the antenna varies significantly with the elevation angle. The results indicate that the loss of peak gain as the antenna moves in elevation can be substantially reduced by mechanical adjustment of the position of the secondary reflector.

Manuscript received June 29, 1981; revised September 28, 1981. The work reported in this communication is a part of the design review of the ALCOR Millimeter Wavelength Augmentation Radar performed at Simpson Gumpertz & Heger Inc., for Massachusetts Institute of Technology Lincoln Laboratory.

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which is independent of scan angle and depends only on the half-angle θ_a of the illuminated part of the lens. For elements with ideal characteristics, the element spacing can be as large as [6]

$$S = \frac{\lambda}{2 \sin \theta_a}. \tag{10}$$

In practice, the realization of ideal element patterns is difficult and the element spacing should be less than λ (or r_0 should be less than two).

CONCLUSION

A generalized constrained lens with surfaces of unequal radii is discussed. The well-known R-KR lens is shown to be one of the special cases. For a specified radiating aperture and allowable maximum phase error, it is shown that a lens with surfaces of unequal radii can be designed to be more compact compared to an equal radii case in limited coverage applications like that of satellite communications.

ACKNOWLEDGMENT

The author expresses his thanks to Mr. Paul Shelton of the Naval Research Laboratory for his valuable advice.

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***Abstract*—For an enclosed Cassegrain antenna, the loss of peak gain and beam deviation due to structural deformations of the primary reflector and rigid body displacements of the secondary reflector and of the feed are computed from the combined changes in the radio frequency (RF) path length. As the antenna moves in elevation, the position of the secondary reflector may be adjusted mechanically to minimize the loss of peak gain; a general method for the computation of the magnitude of such adjustments and of their effects on the gain and pointing of the system is presented. Numerical results are obtained for a particular case of a 45-ft diameter antenna designed for operation at 95.5 GHz RF for which the computed peak gain of the antenna varies significantly with the elevation angle. The results indicate that the loss of peak gain as the antenna moves in elevation can be substantially reduced by mechanical adjustment of the position of the secondary reflector.**

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I. INTRODUCTION

The structural deformations of a Cassegrain antenna result in surface distortions of the primary reflector and misalignments between the primary and secondary reflector and the feed. The surface distortions of the secondary are usually ignored as they are an order of magnitude smaller than those of the primary reflector. For enclosed antennas, the change in gravity deformations as it moves in elevation is the main source of gain degradation. Both surface distortions of the primary reflector and misalignments betwen the antenna components result in gain degradation and beam deviation. The gain degradation due to the gravity deformations of the primary reflector may be predicted by the tolerance theory of Ruze [1] from the root mean square (rms) of the surface deviations, usually computed with respect to a paraboloidal surface that best fits the deformed geometry of the reflector. The best-fitting is achieved by simultaneously translating and rotating the reflector and changing the focal distance [2]. In many cases, the position of the best-fit paraboloid cannot be determined with accuracy due to ill conditioning of the equations. The ill conditioning is inherent in the best fitting process because rigid-body lateral displacements and rotations of the primary reflector result in similar distributions of the change in the RF path length over the aperture.

The misalignment in the relative position of the best-fit paraboloid and the displaced positions of the secondary and feed results in beam deviation and loss of peak gain. It is possible to break up the misalignment into components of rigid-body displacements of the primary, secondary, and feed and to compute separately the beam deviation and loss of peak gain due to each component of misalignment [3]-[5]. The total beam deviation may be computed by the superposition of the effects of the components of misalignment; however, to compute the loss of peak gain the total misalignment must be considered at one time because, in general, superposition of the effects of components taken one at a time may not hold. Antennas for which the loss of peak gain due to misalignment is significant may demonstrate an acute degradation of peak gain near the limits of their travels in elevation. To minimize the loss of peak gain due to misalignment the position of the secondary reflector may be adjusted by mechanical means. The magnitude of the adjustment depends on the elevation angle of the antenna. Gain degradation may also occur due to astigmatism resulting from gravity deformations of the primary reflector as described by von Hoerner [6], [7] which he suggests correcting by mechanically deforming a flexible subreflector. This communication presents a method for the computation of the adjustment of the position of the secondary reflector that minimizes loss of peak gain.

The beam deviation and the loss of peak gain are calculated directly from the changes in the RF path length resulting from the structural deformation of the primary reflector and the rigid-body displacements and rotations of the secondary reflector and of the feed. The method avoids the best fitting of the primary reflector and the resulting problems from inherent in such calculations. The position of the secondary reflector is then adjusted to minimize the loss of peak gain. The results, obtained for a particular 45-ft antenna, indicate that the loss of peak gain due to deformations caused by gravity can be substantially reduced by adjustment of the position of the secondary.

II. GAIN LOSS AND BEAM DEVIATION

Let us consider an antenna with an axisymmetric illumination function $f(\bar{r})$ where $\bar{r}$ is the aperture position vector defined in polar coordinates by (r, ϕ'), (see Fig. 1). The direction of observation expressed by the angles ϕ and θ may also be expressed by a unit vector $\hat{p}$ where

$$\hat{p} = (\sin\theta\cos\phi, \quad \sin\theta\sin\phi, \quad \cos\theta). \tag{1}$$

Overbars indicate a vector quantity and carets indicates a unit vector.

The gain in the direction of observation $\hat{p}$ of a distorted antenna with a change in RF path length of δ at point $\bar{r}$ on the aperture is expressed by

$$G(\phi, \theta) = \frac{4\pi}{\lambda^2} \frac{\left| \int_A f(\bar{r}) e^{\frac{2\pi j}{\lambda}(\delta - \hat{p}\cdot\bar{r})} \, dS \right|^2}{\int_A f^2(\bar{r})\, dS}. \tag{2}$$

Let G_0 be the no error peak gain obtained from (2) by setting $\delta = \hat{p}\cdot\bar{r} = 0$, then

$$\frac{G}{G_0} = \left| \frac{\int_A f(\bar{r}) e^{\frac{2\pi j}{\lambda}(\delta - \hat{p}\cdot\bar{r})} \, dS}{\int_A f(\bar{r})\, dS} \right|^2. \tag{3}$$

If the variation of $\delta - \hat{p}\cdot\bar{r}$ over the aperature is small as compared to the wavelength λ, we may approximate (3) by expansion as follows:

$$\frac{G}{G_0} = 1 - \left(\frac{2\pi}{\lambda}\right)^2 \frac{\int_A f(\bar{r})(\delta - \hat{p}\cdot\bar{r})^2\, dS}{\int_A f(\bar{r})\, dS} + \left(\frac{2\pi}{\lambda}\right)^2 \left[\frac{\int_A f(\bar{r})(\delta - \hat{p}\cdot\bar{r})\, dS}{\int_A f(\bar{r})\, dS}\right]^2. \tag{4}$$

In the direction of peak gain $\hat{p}_0$, $\partial(G/G_o)/\partial\theta = 0$ and $\partial(G/G_0)/\partial\phi = 0$. Therefore if we note that

$$\hat{p}\cdot\bar{r} = r\sin\theta\cos(\phi - \phi') \cong r\theta\cos(\phi - \phi') \tag{5}$$

then

$$\int_A f(\bar{r})(\delta - \hat{p}_0\cdot\bar{r}) r\cos(\phi_0 - \phi')\, dS = 0 \tag{6a}$$

and

$$\int_A f(\bar{r})(\delta - \hat{p}_0\cdot\bar{r}) r\sin(\phi_0 - \phi')\, dS = 0. \tag{6b}$$

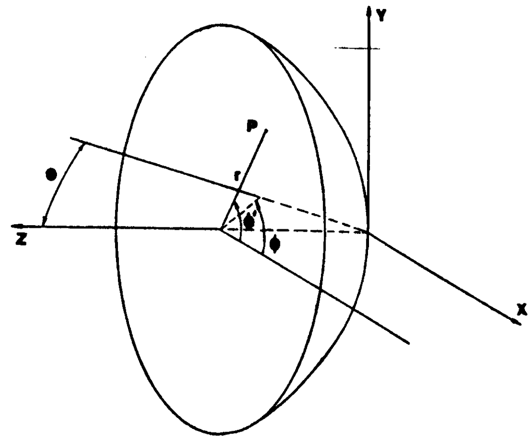

Fig. 1. Coordinates.

From (6a) and (6b), the direction of peak gain may be expressed by

$$\theta_0 = \frac{\int_A f(\bar{r})\delta r\cos(\phi_0 - \phi')\, dS}{\int_A f(\bar{r}) r^2\cos^2(\phi_0 - \phi')\, dS} \tag{7}$$

and

$$\tan\phi_0 = \frac{\int_A f(\bar{r})\delta r\sin\phi' dS}{\int_A f(\bar{r})\delta r\cos\phi' dS}. \tag{8}$$

III. CHANGES IN RF PATH LENGTH

The total change in the RF path length δ is the sum of the changes in the RF path length due to the deformation of the primary reflector δ_p, the rigid-body translations and rotations of the secondary reflector δ_s, and the displacements of the feed δ_f; that is

$$\delta = \delta_p + \delta_s + \delta_f. \tag{9}$$

Let a point on the surface of the primary reflector undergo a displacement $\bar{u}_p = (u_p, v_p, w_p)$. The resulting change in the RF path length is twice the axial component of the displacement normal to the paraboloid [8]; that is, $\delta_p = -2n_z(\bar{u}_p\cdot\hat{n})$ where $\hat{n} = (n_x, n_y, n_z)$ is a unit vector normal to the surface of the primary reflector. In other words

$$\delta_p = \bar{c}_p \cdot \bar{u}_p \tag{10}$$

where the components of the coefficient vector $\bar{c}_p$ for a paraboloid of the form $z = r^2/4f$ may be expressed by

$$c_{p1} = c_{p0}\cos\phi' \tag{11a}$$

$$c_{p2} = c_{p0}\sin\phi' \tag{11b}$$

$$c_{p3} = -\frac{8f^2}{4f^2 + r^2} \tag{11c}$$

where

$$c_{p0} = \frac{4rf}{4f^2 + r^2}. \tag{12}$$

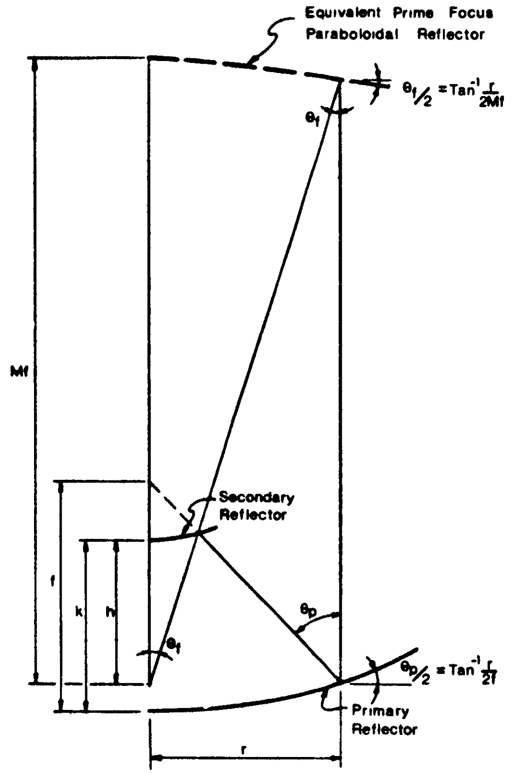

Fig. 2. Geometry of equivalent prime-focus paraboloidal reflector.

The effect of the displacements of the feed can be examined by using the equivalent prime-focus paraboloid concept. In this concept we use the fact that the energy converging on the feed appears to come from an equivalent prime-focus paraboloid (see Fig. 2). Thus the effects of the displacement of the feed in a Cassegrain antenna is equivalent to the displacement of the feed in a prime-focus antenna of focal length Mf. Therefore if the feed is assumed to undergo a displacement $\bar{u}_f = (u_f, v_f, w_f)$, we can express the corresponding change in the RF path length δ_f as follows:

$$\delta_f = \bar{c}_f \cdot \bar{u}_f \tag{13}$$

with

$$c_{f1} = c_{f0} \cos \phi' \tag{14a}$$

$$c_{f2} = c_{f0} \sin \phi' \tag{14b}$$

$$c_{f3} = \frac{-4(Mf)^2 + r^2}{4(Mf)^2 + r^2} \tag{14c}$$

$$c_{f0} = -\frac{4r(Mf)}{4(Mf)^2 + r^2}. \tag{15}$$

The rigid-body displacements of the secondary reflector, translations $\bar{u}_s = (u_s, v_s, w_s)$, and the rotations ψ_{xs} and ψ_{ys} about axes parallel to the x and y axes passing through the vertex of the secondary may be expressed in terms of equivalent rigid body translations and rotations of the primary reflector and of the feed. The resulting expression for the change in the RF path length due to displacements of the secondary reflector may be written as follows:

$$\delta_s = \bar{c}_s \cdot \bar{u}_s + c_{s4} \psi_{xs} + c_{s5} \psi_{ys} \tag{16}$$

where

$$\bar{c}_s = -(\bar{c}_p + \bar{c}_f) \tag{17a}$$

$$c_{s4} = -c_{p2}(k - z) - c_{f2} h - r(1 + c_{p3}) \sin \phi' \tag{17b}$$

$$c_{s5} = c_{p1}(k - z) + c_{f1} h + r(1 + c_{p3}) \cos \phi'. \tag{17c}$$

For the known deformations of a Cassegrain antenna the changes in the RF path length are initially computed from (9)-(17). The values of δ are then used to compute the direction of peak gain θ_0 and ϕ_0 from (7) and (8) and the corresponding loss of peak gain from (4).

IV. ADJUSTMENT OF SECONDARY REFLECTOR POSITION

When the structural deformations are repeatable the secondary position may be adjusted to minimize the loss of peak gain. Let δ_0 be the total change in the RF path length due to the structural deformations of the primary reflector and the rigid-body displacements of the secondary and the feed and let ϕ_0 and θ_0 denote the corresponding direction of peak gain. If as the secondary is adjusted it undergoes additional translations and rotations denoted by $\bar{u}$, and ψ_x and ψ_y, the resulting change in the RF path length after adjustments δ_a is

$$\delta_a = \delta_0 + \bar{c}_s \cdot \bar{u} + c_{s4} \psi_x + c_{s5} \psi_y. \tag{18}$$

Without loss of generality we may assume that $\phi_0 = \pi/2$ and $u = \psi_y = 0$. For most enclosed antennas which have a vertical plane of symmetry and are subjected to a linear combination of face-up and face-side gravity loadings, these assumptions are valid. (If $\phi_0 \neq \pi/2$, we may rotate the coordinate axes so that in the rotated coordinates $\phi_0 = \pi/2$.) Under these assumptions, (18) reduces to

$$\delta_a = \delta_0 + (k_1 \sin \phi') v + k_2 w + (k_3 \sin \phi') \psi_x \tag{19}$$

where

$$k_1 = -(c_{p0} + c_{f0}) \tag{20a}$$

$$k_2 = c_{s3} \tag{20b}$$

$$k_3 = -c_{p0}(k - z) - c_{f0} h - r(1 + c_{p3}). \tag{20c}$$

The additional displacement of the secondary reflector changes the direction of peak gain. Let us denote the direction of peak gain after secondary adjustment by $\hat{p}_a$ and the corresponding angles by θ_a and ϕ_a. Since $\phi_0 = \pi/2$ and the adjustments do not change ϕ, $\phi_a = \phi_0$. If we substitute (19) into (7), we obtain an expression for the modified direction of peak gain as follows:

$$\theta_a = \theta_0 + \theta_v v + \theta_\psi \psi_x \tag{21}$$

where

$$\theta_v = \frac{\int_R f(r) k_1 r^2 \, dr}{\int_R f(r) r^3 \, dr} \tag{22a}$$

and

$$\theta_\psi = \frac{\int_R f(r) k_3 r^2 \, dr}{\int_R f(r) r^3 \, dr} \tag{22b}$$

in which R refers to the interval of radial integration.

Let us define δ_0' and δ_a' as follows:

$$\delta_0' = \delta_0 - r\theta_0 \cos(\pi/2 - \phi') \tag{23}$$

$$\delta_a' = \delta_a - r\theta_a \cos(\pi/2 - \phi'). \tag{24}$$

Substitution of (19), (21), and (23) into (24) results in the following:

$$\delta_a' = \delta_0' + k_1' \sin\phi' v + k_2 w + k_3' \sin\phi' \psi_x \tag{25}$$

where

$$k_1' = k_1 - r\theta_v \tag{26a}$$

$$k_3' = k_3 - r\theta_\psi. \tag{26b}$$

To select the values of v, w, and ψ_x which will maximize gain we substitute (25) into (4) and set the partial derivatives of G/G_0 with respect to v, w, and ψ_x equal to zero. We obtain

$$\int_A f(\bar{r}) k_i' \sin\phi' \delta_a' \, dS = 0, \qquad i = 1 \text{ and } 3 \tag{27a}$$

$$\int_A f(\bar{r}) k_2' \, \delta_a' \, dS = 0 \tag{27b}$$

where

$$k_2' = k_2 - \frac{\int_R f(r) k_2 r \, dr}{\int_R f(r) r \, dr}. \tag{28}$$

Equation (27) is three equations in three unknowns and may be written in the form of $\bar{C}\bar{x} = \bar{b}$ where

$$\left.\begin{aligned} c_{ij} &= \pi \int_R f(r) k_i' k_j' r \, dr \qquad (29a) \\ c_{i2} &= c_{2j} = 0, \qquad (29b) \end{aligned}\right\} i, j = 1 \text{ and } 3$$

$$c_{22} = 2\pi \int_R f(r) k_2'^2 r \, dr \tag{29c}$$

$$b_i = -\int_A f(\bar{r}) k_i' \sin\phi' \delta_0' \, dS, \qquad i = 1 \text{ and } 3 \tag{30a}$$

$$b_2 = -\int_A f(\bar{r}) k_2' \delta_0' \, dS. \tag{30b}$$

Solution of this system yields

$$v = \frac{b_1 c_{33} - b_3 c_{31}}{c_{11} c_{33} - c_{13} c_{31}} \tag{31a}$$

$$w = \frac{b_2}{c_{22}} \tag{31b}$$

$$\psi_x = \frac{-b_1 c_{13} + b_3 c_{11}}{c_{11} c_{33} - c_{13} c_{31}} \tag{31c}$$

Equation (31) expresses the adjustment of the secondary reflector needed to maximize gain.

V. COMPUTED RESULTS

The theoretical development presented above has been coded in a computer program to be used as a postprocessor with a structural analysis package. The program computes initially the beam deviation and loss of peak gain of an enclosed Cassegrain antenna due to gravity deformations of the primary reflector and rigid-body displacements of the secondary and of the feed without adjustments of the position of the secondary reflector. The computed values of δ_0 and δ_0' are used to evaluate the surface integrals that define b_i, $i = 1$, 2, and 3, in (30). The entries of the coefficient matrix $\bar{C}$, in (29), which are functions of the geometry of the antenna and of the illumination pattern and are independent of the structural deformations are computed by evaluating a series of line integrals. The computed values for the adjustment of the position of the secondary reflector to maximize gain are used to modify the changes in the RF path length and compute the corresponding loss of peak gain after adjustment of the secondary reflector.

The computation has been performed for the gravity deformations of a 45-ft diameter Cassagrain antenna enclosed in a radome. The structural deformations were computed by a finite-element idealization of the structure. It is assumed that the surface panels, the secondary reflector, and the feed are aligned in such a way that when the antenna is at elevation angle α_r, the residual deviations from the ideal antenna configuration are random in nature. (Note that we are ignoring the bias alignment errors.) Then if δ_0^u and δ_0^s are the changes in the RF path length due to structural deformation resulting from gravity loads in the face-up and face-side positions, we have

$$\delta_0 = \delta_0^u (\sin\alpha - \sin\alpha_r) + \delta_0^s (\cos\alpha - \cos\alpha_r) \tag{32}$$

where α is the elevation angle of the antnna.

The values of δ_0 are expressed by (32) for vaious elevation angles α have been used in the calculation of the loss of peak gain and beam deviation without or with secondary reflector position adjustment. It was assumed that $\alpha_r = 30°$, $f/D = 0.37$, and the magnification factor $M = 11$. It was further assumed that the illumination function is of the form $f(\bar{r}) = 1 - 0.75(2r/D)^2$ and that the RF is 95.5 GHz.

In the numerical computations performed we considered the adjustment of the position of the secondary reflector in the lateral and axial directions only. The adjustment for the tilt of the secondary reflector was considered to be counterproductive because the loss of gain corresponding to the nonrepeatable errors in the angular positioning mechanism are expected to be larger than the improvements in gain resulting from the added degree of freedom in the adjustment.

Fig. 3 shows the loss of peak gain of the 45-ft diameter antenna considered herein prior to the adjustment of the position of the secondary and after independent adjustments in the lateral and axial directions as well as after a combined axial and lateral adjustment. The results indicate that in a Cassagrain antenna for which the deformations of the primary reflector structure are almost homologous (i.e., the primary reflector deforms into an almost parabolic shape), a significant loss of peak gain may occur as a result of secondary position misalignment. Furthermore the loss of peak gain of a Cassegrain antenna may be substantially reduced by suitable adjustments of the position of the secondary reflector in the lateral and axial directions. The magnitude of the lateral adjustment v and of the axial adjustment w for various elevation angles are shown in Fig. 4.

As we adjust the position of the secondary reflector the loss of peak gain is reduced and the beam deviation, i.e., the value of θ_0 corresponding to the peak axial gain, increases signif-

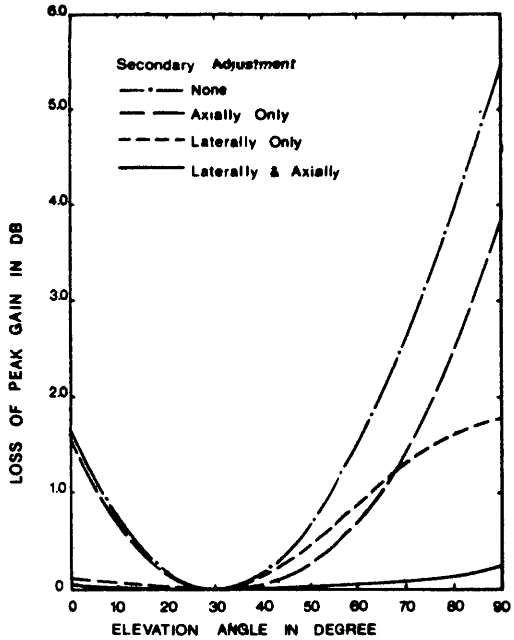

Fig. 3. Loss of peak gain with and without adjustment of position of secondary reflector.

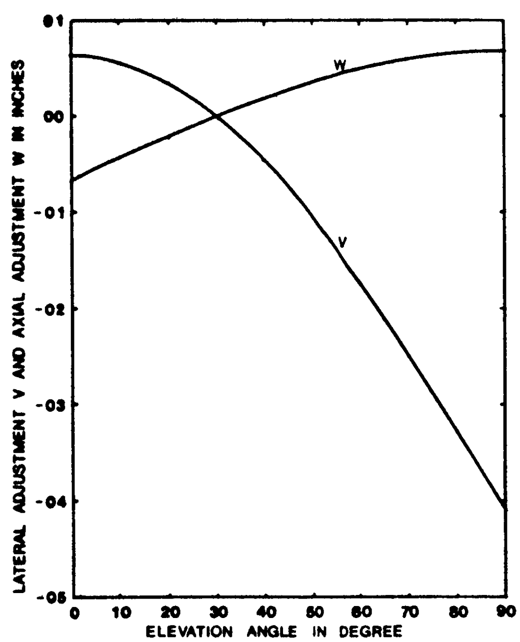

Fig. 4. Variation of lateral adjustment v and axial adjustment w of the position of secondary reflector with elevation angle; no rotational adjustment of position of secondary reflector allowed.

icantly (see Fig. 5). Note that if the error in the position of the secondary were primarily due to subreflector droop on adjustment of the secondary position would have reduced the gain loss and the beam deviation simultaneously whereas if the error were for example due to distortions of the primary reflector, an adjustment of the secondary position in the lateral direction made to maximize gain could increase beam deviation. Note that the beam deviation due to gravity deformations is repeatable and can be calibrated out.

NOMENCLATURE

$\bar{A}$	Domain of aperture.
$\bar{c}$	Coefficient vector, see (11), (14), (17).
$\bar{C}$	Coefficient matrix, see (29).

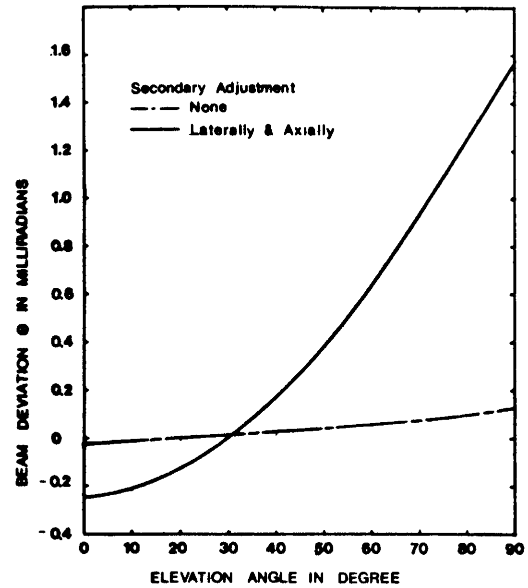

Fig. 5. Beam deviation θ due to gravity deformation with and without lateral adjustment of position of secondary reflector; no rotational adjustment of position of secondary reflector allowed.

f	Focal length of primary reflector.
$f(\bar{r})$	Aperture illumination function.
G	Antenna gain.
G_0	No error peak gain.
h	Distance between feed and the vertex of secondary, see Fig. 2.
k	Distance between the vertex of primary and the vertex of secondary, see Fig. 2.
k_1, k_2, k_3	Coefficients, see (20).
k_1', k_2', k_3'	Coefficients, see (26) and (28).
$\hat{p}$	Unit vector in the direction of observation.
$\hat{p}_0$	Unit vector in the direction of peak gain.
$\bar{r}$	Aperture position vector = (r, ϕ') in polar coordinates, see Fig. 1.
$\bar{u}_p$	Displacement vector of a point on the primary reflector = (u_p, v_p, w_p).
$\bar{u}_s$	Rigid body translation of secondary reflector.
$\bar{u}_f$	Rigid body translation of feed.
u, v, w, ψ_x, ψ_y	Magnitude of adjustment of the position of the secondary reflector.
x, y, z	Coordinates of points on the primary reflector.
α	Elevation angle of antenna.
α_r	Elevation angle at which the deviations from the ideal antenna configuration are assumed to be random.
δ	Change in the RF path length, a function of aperture position.
ϕ, θ	Angles defining the direction of observation of the deformed Cassegrain system with respect to the undeformed system.
ϕ_0, θ_0	Direction of observation corresponding to the peak gain (θ_0 is also referred to as beam deviation).
θ_v, θ_ψ	Beam deviation due to a unit lateral displacement and a unit rotation of the secondary.
ψ_{xs}, ψ_{ys}	Components of rotation of secondary about axes parallel to x and y axes through the vertex of the secondary.
λ	Wavelength.

Subscripts

p	Primary reflector.
s	Secondary reflector.
f	Feed.
a	Pertaining to adjusted position of secondary reflector.

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The author wishes to thank Dr. John Ruze and Dr. Joseph Antebi for their valuable comments.

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Effects of Phase and Amplitude Quantization Errors on Hybrid Phased-Array Reflector Antennas

VINCENT MRSTIK

***Abstract*—The tolerance of hybrid array/reflector antennas to feed element phase and amplitude quantization errors is examined. The effects of quantization errors on the peak and root mean square (rms) sidelobes are derived for an example parabolic reflector.**

I. INTRODUCTION

A recent paper reported on an investigation [1] of a hybrid array/reflector antenna in which a large reflector is fed by a relatively small phased array. The paper showed that small array feeds can correct off-axis aberrations and significantly extend the scanning capability of reflector antennas beyond the few tens of beamwidths possible with conventional feeds [2].

Although this recent study, as well as prior studies [3]–[6] of hybrid array reflector antennas, considered many factors including aperture blockage, aperture tapering (to achieve low sidelobes), feed element position errors, as well as variations in the number of feed elements utilized, the impact of feed element phase and amplitude quantization errors has not been reported. The analyses to date have generally assumed ideal feed elements capable of being set to any desired phase and amplitude.

This communication reports on an investigation which was undertaken to address the following issues. 1) What is the impact of truncating the array feed by eliminating elements whose power is below some specified threshold? 2) What is the impact of quantizing the phase and amplitude of the feed element excitations?

These two issues are important in assessing the feasibility of implementing array feeds.

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II. APPROACH

Radiation patterns were computed for scanning in a plane orthogonal to the longitudinal axis of a parabolic cylinder reflector which is fed by an array of line sources having controllable phase and amplitude (Fig. 1). A parabolic cylinder was selected for analysis since it provides a means of examining the scanning properties of an array-fed reflector, yet avoids the computational load of performing a two-dimensional integration over the reflector—as would be necessary for a circular aperture. Although the behavior of a circular aperture can be expected to differ in detail from the results given here, these results provide some qualitative insight into the behavior of circular apertures as well.

The radiation pattern computations were carried out following the procedures described by Mrstik and Smith [1]. The induced current method was used to compute the radiation patterns of a single feed element. The principle of superposition was then applied to obtain the composite pattern for an array of feed elements. The element excitations to generate the desired radiation patterns were computed as the complex conjugate of the currents induced by an incident field from the desired direction. An illumination weighting factor was included to synthesize different radiation patterns. The computations performed here differ from those in the earlier paper by only three aspects, as follows.

1) *Feed Element Quantization Errors Included:* After the phase and amplitude of each feed element for generating the desired radiation pattern is computed, the phase and/or amplitude of each element is quantized and the resulting radiation then computed. (See Fig. 2 for example of element truncation and power quantization.)
2) *Truncated Gaussian Illumination Used with−30.7 dB Edge Illumination:* In order to investigate the effects of amplitude and phase quantization on low-level sidelobes, a highly tapered Gaussian illumination function was used which (in the absence of other error sources) yields very low sidelobes. The first sidelobe is down −47.5 dB and the root mean square (rms) sidelobes in a "sample region" as described later are down 67 dB.
3) *Aperture Blockage Effects Excluded:* In order to focus on the effects of quantization errors, and hence to derive a lower limit on antenna sidelobe levels due to quantization errors alone, the effects of aperture blockage by the feed and/or support structure were not included.