National Radio Astronomy Observatory
Tucson, Arizona

November 21, 1983

Memorandum

TO: 12-m Memo Series

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SUBJECT: RMS Surface Accuracy and Zero-Frequency Efficiency

I. Introduction

Over the last few weeks, Rick Howard has measured the aperture efficiency of the 12-m telescope, at prime focus, at 2 wavelengths. Because the feeds were scaled versions of each other, the illumination functions were identical. In these circumstances, we can use the 2 measurements to calculate the RMS surface accuracy (weighted by this illumination taper) independent of the zero-frequency efficiency, η_o, of the reflector.

II. Theory

If the surface errors of a parabolic reflector are randomly distributed over the surface, then Ruze (1952) gives the aperture efficiency η at a wavelength λ to be

$$\eta = \eta_o \exp - \left(\frac{4\pi\delta}{\lambda} \right)^2 . \qquad (1)$$

where δ is the RMS surface error.

The measurement of aperture efficiency at 2 different wavelengths allows us to eliminate η_o, giving

$$\frac{\eta_1}{\eta_2} = \exp \left[\left(\frac{4\pi\delta}{\lambda_2} \right)^2 - \left(\frac{4\pi\delta}{\lambda_1} \right)^2 \right] . \qquad (2)$$

The RMS surface error is then

$$\delta = \frac{c}{4\pi} \left(f_2^2 - f_1^2\right)^{-\frac{1}{2}} \left(\ell n \frac{\eta_1}{\eta_2}\right)^{\frac{1}{2}} , \qquad (3)$$

where we've replaced wavelength with frequency for mathematical convenience.

Because measurements of aperture efficiencies η have substantial uncertainties σ, we calculate the indirect error on δ caused by propagation of the measurement uncertainties. This error is given by the standard rule

$$\Delta\delta = \left[\left(\frac{\partial\delta}{\partial\eta_1}\right)^2 \cdot \sigma_1^2 + \left(\frac{\partial\delta}{\partial\eta_2}\right)^2 \cdot \sigma_2^2 \right]^{\frac{1}{2}} , \qquad (4)$$

which gives

$$\Delta\delta = \frac{\delta}{2} \left[\left(\frac{\sigma_1}{\eta_1}\right)^2 + \left(\frac{\sigma_2}{\eta_2}\right)^2 \right]^{\frac{1}{2}} \cdot \left(\ell n \frac{\eta_1}{\eta_2}\right)^{-1} . \qquad (5)$$

Having calculated the RMS error by equations (3) and (5), we can calculate the zero-frequency aperture efficiency from equation (1).

$$\eta_o = \eta \exp \left(\frac{4\pi\delta}{\lambda}\right)^2 , \qquad (6)$$

where the indirect error $\Delta\eta_o$ can be calculated from an equation similar to (4) to be

$$\Delta\eta_o = \frac{\eta_o}{\eta} \Delta\eta \qquad (7)$$

where we consider only the error contribution η in applying equation (4) because it has the largest effect.

III. Data

f	λ	η	σ*
90.0 GHz	3.33mm	0.48	0.03
223.2	1.34	0.19	0.03

* 2σ is maximum uncertainty in η, including errors in calibration, atmospheric extinction, and flux of Venus.

IV. Results

Substitution of the data above into the appropriate equations gives

$$\delta \pm \Delta\delta = 112.4 \pm 10.3\ \mu m$$
$$= \underline{110 \pm 10} ,$$

and

$$\eta_o \pm \Delta\eta_o = 0.575 \pm 0.036$$
$$= \underline{0.57 \pm 0.04} ,$$

where the underlined values reflect the number of significant figures permitted by the measurements.

Ruze, J. 1952, MIT Research Laboratory of Electronics, Tech. Report 248.