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MEMORANDUM

From: B.L. Ulich

Subject: Absolute calibration of millimeter continuum observations

The aperture efficiency of the NRAO 36 foot antenna at Kitt Peak is known to vary significantly with frequency, ambient temperature, and differential solar heating of the reflector surface. These gain degradations are not predictable with precision, and absolute calibration cannot be maintained to better than about 10 - 15% for extended periods of time. Thus suitable calibration sources must be monitored to correct for changes in the overall system gain. The following is a discussion of an absolute flux density scale at millimeter wavelengths, which leads to significant changes at 3.5 MM and shorter wavelengths. Table I is a list of standard calibration sources at the three wavelengths currently being used for continuum observations on the 36 foot antenna. At each frequency a suggested value for the brightness temperature or flux density is also given. The Appendix describes the calculation of the observed flux density and antenna temperature of a calibration source.

At 9.55 MM wavelength the absolute flux density scale is well determined. Relative measurements of DR21 by Dent and absolute measurements at several observatories are consistent with the values in Table I. Errors in the relative and absolute calibration of the standard sources are probably less than 5%. On this scale the aperture efficiency of the 36 foot antenna at this wavelength is 51% and varies only slightly with thermal effects.

Relative planetary measurements near 3 MM wavelength at the NRAO, the Aerospace Corporation, and The University of Texas Millimeter Wave Observatory (MWO) are all consistent with the 85 GHz temperatures in Table I. The absolute temperatures reported by Aerospace are about 13% lower than the values suggested here, and the absolute temperatures measured at Texas are about 3% higher. Precise measurements of DR21 relative to Jupiter and Saturn rule out the possibility of temperatures as low as 140 and 130 K, respectively. The spectrum of DR21 extrapolated from longer wavelengths predicts a flux density of 16.2 f.u. at 85 GHz. This value leads to brightness temperatures of 167 and 144 K for Jupiter and Saturn, respectively. However, the spectrum of DR21 turns up at short millimeter wavelengths, as shown in Figure 1, and the correct value is probably slightly above the extrapolated spectrum even at 3 MM wavelength. The flux density scale at 85 GHz in Table I was chosen to best fit the expected planetary spectra, shown in Figures 2 - 5, with particular emphasis paid to Mars, Venus, and to a lesser extent, Saturn. Mars and Venus were chosen as primary standards because of their predictable spectra.

This scale leads to a flux density of 17.0 ± 0.7 f.u. for DR21, which is above the extrapolated spectrum by a marginally significant amount. This new flux scale results in about a 22% upward revision of many previous 85 GHz continuum measurements on the NRAO 36 foot antenna. The relative and absolute intensities of the calibration sources are accurate to within about 5%. At this frequency the aperture efficiency varies from about 24 to 40%, depending on the ambient temperature and solar heating of the reflector.

The brightness temperatures at 1.2 MM wavelength are the result of relative measurements by Rather and others with a cooled bolometer. The absolute flux density scale was derived by assuming a brightness temperature of 150 K for Jupiter. The Mars and Venus temperatures agree well with expected values. The anomolously high brightness temperature of Saturn, which is referred to the disk only, indicates that Saturn's rings contribute significantly to the observed intensity at 1 MM. Absolute 2.13 MM measurements at the MWO also indicate an increase in temperature with frequency, as shown in Figure 5. The DR21 observations, while somewhat noisy, definitely indicate an upturn in the spectrum. Measurements at 2.13 MM at the MWO and at 350 μM by Harper further confirm this departure from the extrapolated spectrum. The relative temperatures at 1.2 MM are accurate to within about 5%, and the absolute flux density scale for sources with black body spectra is probably accurate to within about 10%. The aperture efficiency of the 36 foot antenna at 1.2 MM is on the order of 5% under good conditions but varies greatly with thermal distortion of the reflector. Ruze's antenna tolerance theory predicts an efficiency of about 6% at 250 GHz and about 2% at 300 GHz for the RMS surface tolerance of 0.15 MM.

TABLE I

MM CALIBRATION SOURCES

Frequency (GHz)	31.4	85.0	250
Wavelength (MM)	9.55	3.53	1.2
HPBW (')	3.67	1.30	1.0
Aperture Efficiency (%)	51	24-40	~5
Sun (K)	8800	7000	5800
Average Moon (K)	238	225	220
New Moon (K)	200	168	120
Venus (K)	430	350	290
Mars (K)	206	210	215
Jupiter (K)	155	174	150
Saturn (K)	139	150	194[a]
DR21 (f.u.)[b]	18.7	17.0	42±10
(S/C_S)DR21 (f.u.)	18.5	15.6	37± 9

[a]This applies only to a ring inclination of 26^o as seen from the Earth.

[b]The true flux density is the observed flux density corrected for source resolution by Dent's factor $C_S = 1.0 + 0.15/B^2$, where B is the HPBW in minutes of arc.

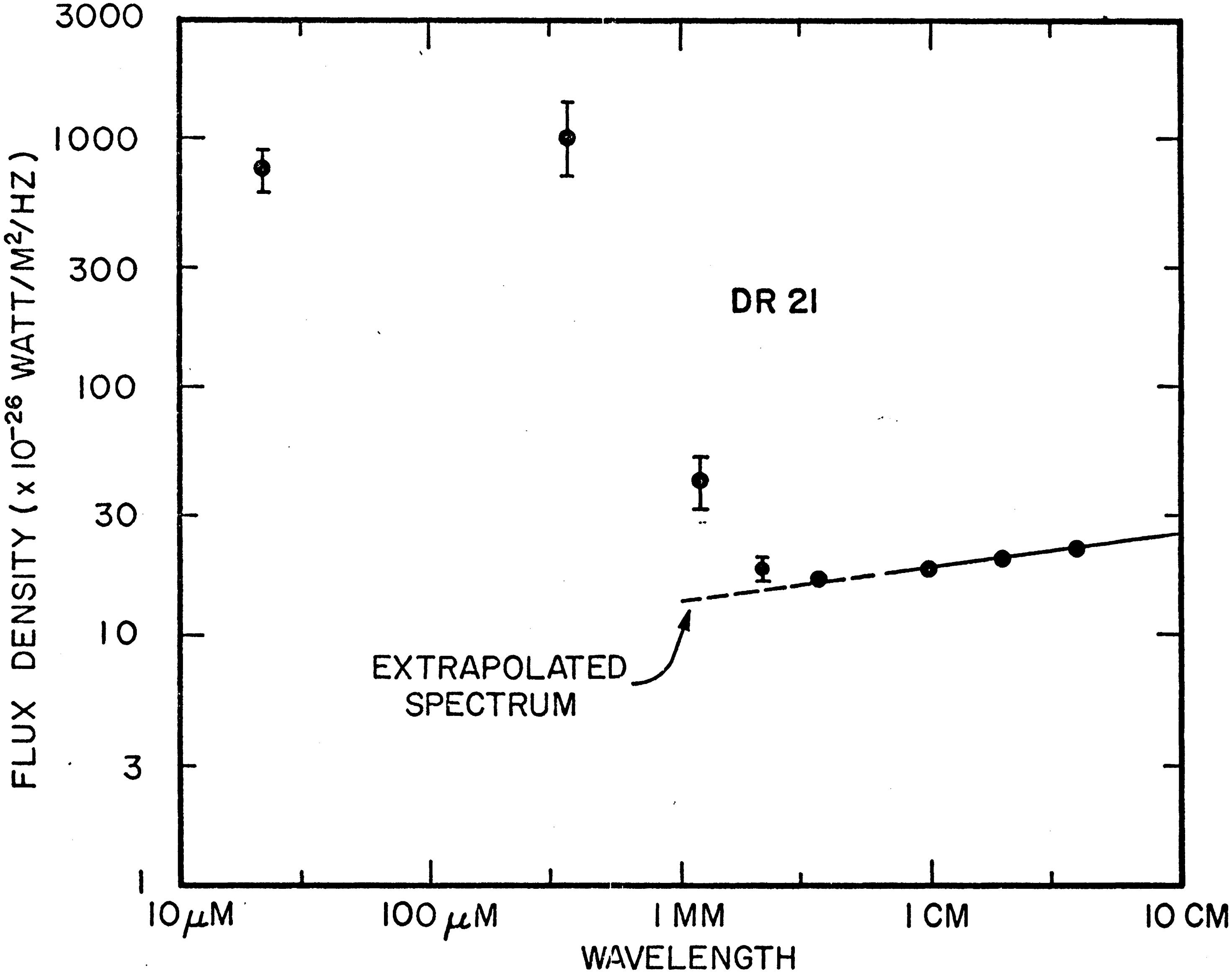
3000
1000
300
100
30
10
3
1
FLUX DENSITY (x 10^{-26} WATT/M^2/HZ)
DR 21
EXTRAPOLATED SPECTRUM
10 μM
100 μM
1 MM
1 CM
10 CM
WAVELENGTH

FIGURE 1

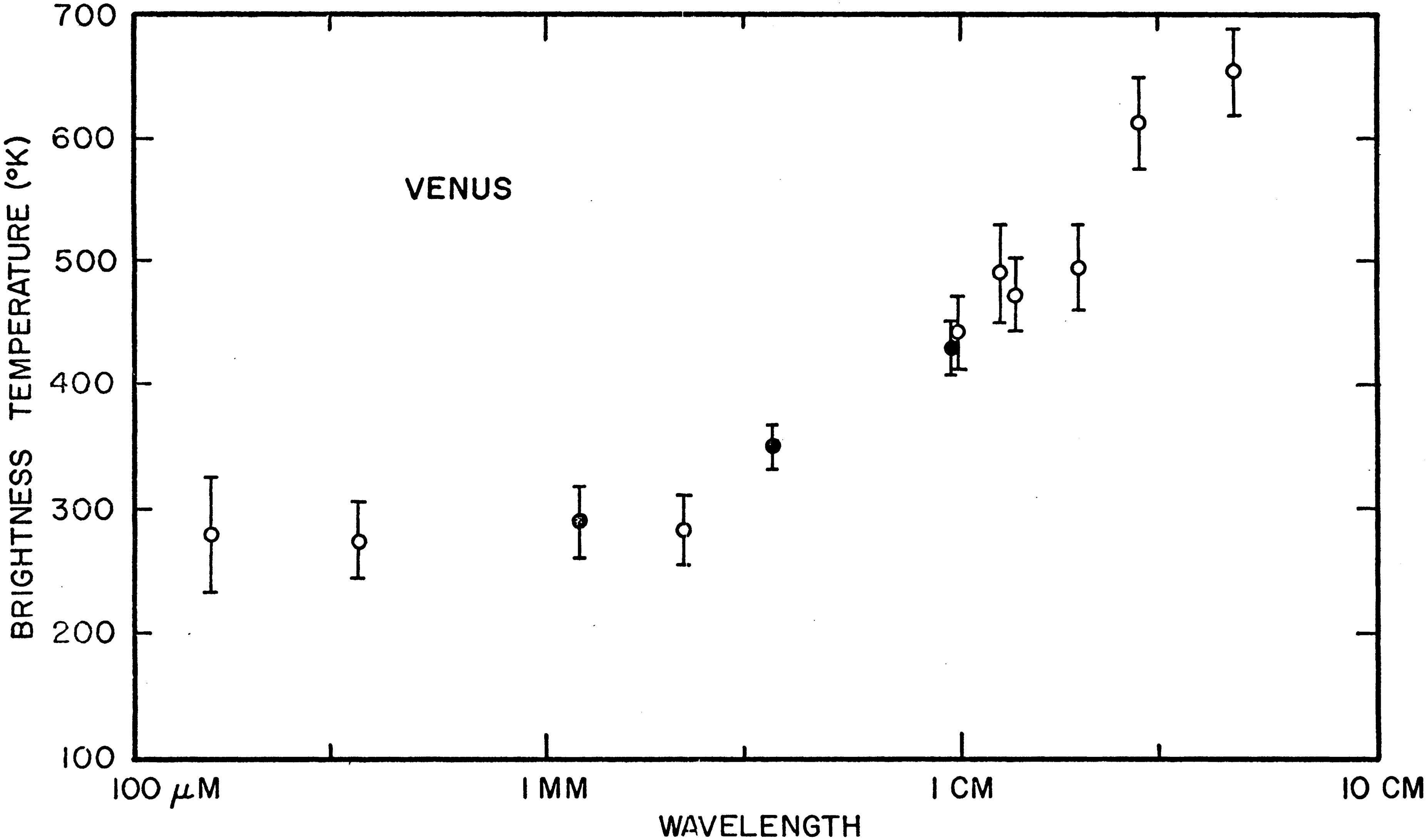
700
600
500
400
300
200
100
BRIGHTNESS TEMPERATURE (°K)
VENUS
100 μM
1 MM
1 CM
10 CM
WAVELENGTH

FIGURE 2

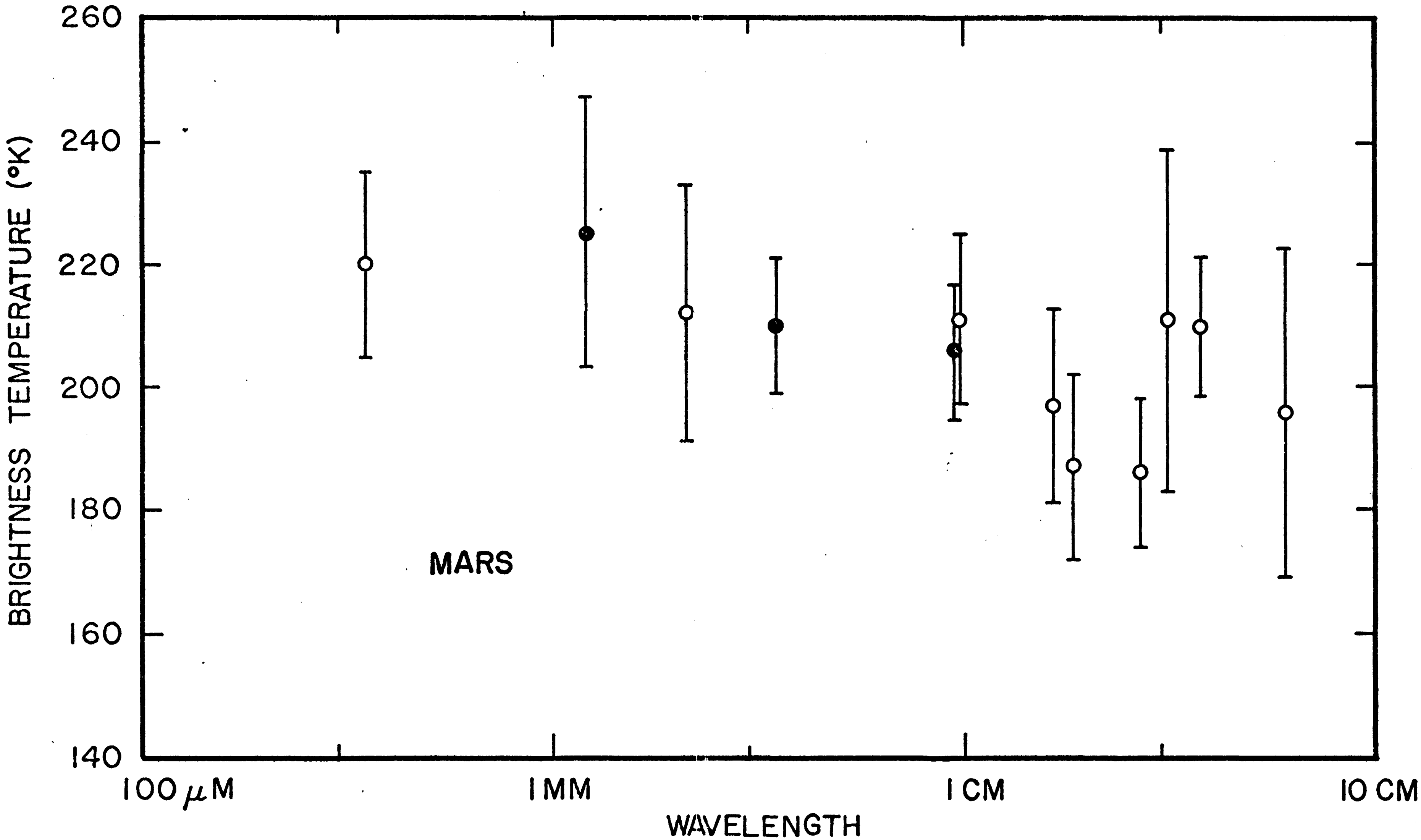
260
240
220
200
180
160
140
BRIGHTNESS TEMPERATURE (°K)
MARS
100 μM
1 MM
1 CM
10 CM
WAVELENGTH

FIGURE 3

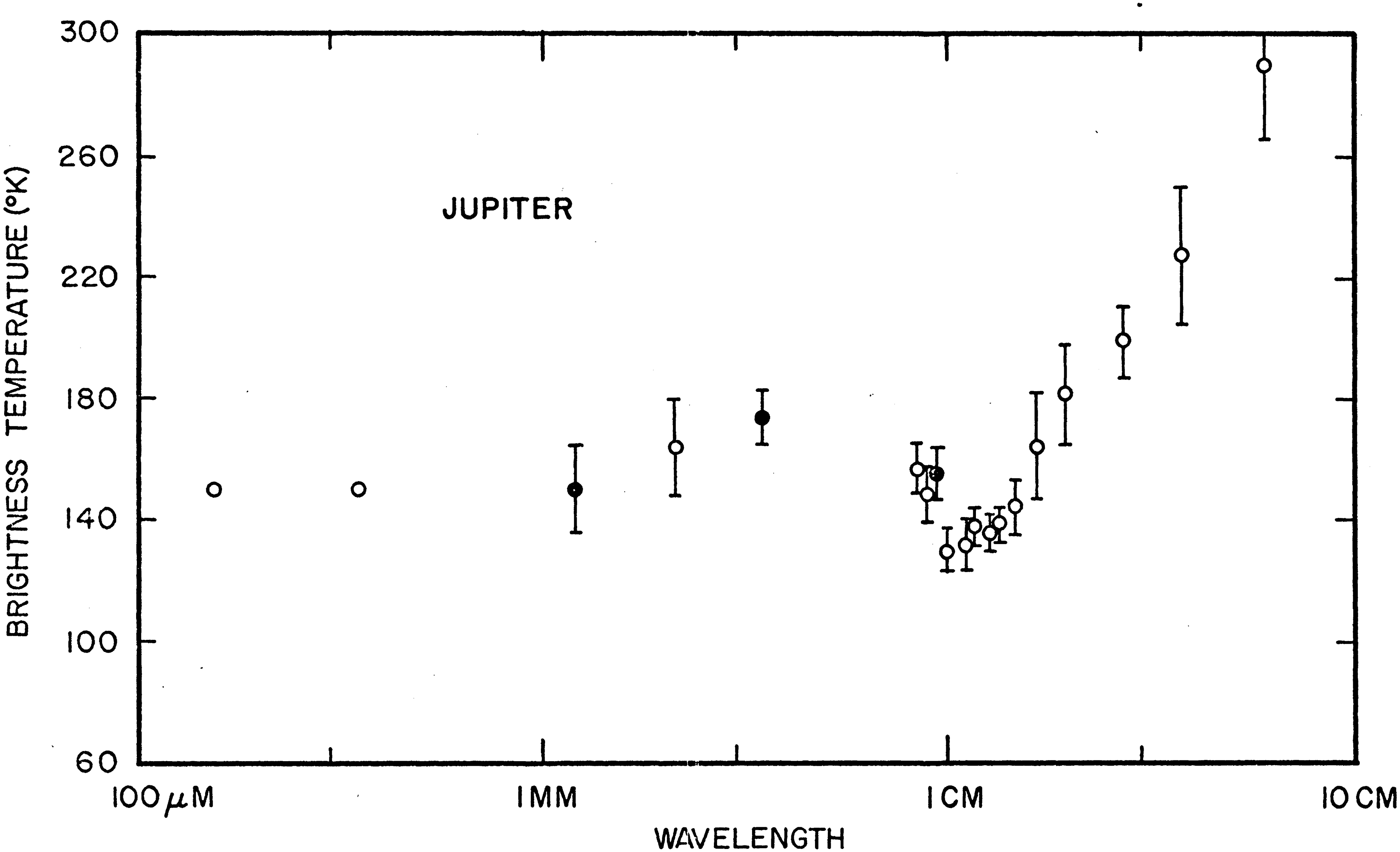
JUPITER
BRIGHTNESS TEMPERATURE (°K)
300
260
220
180
140
100
60
100 μM
1 MM
1 CM
10 CM
WAVELENGTH

FIGURE 4

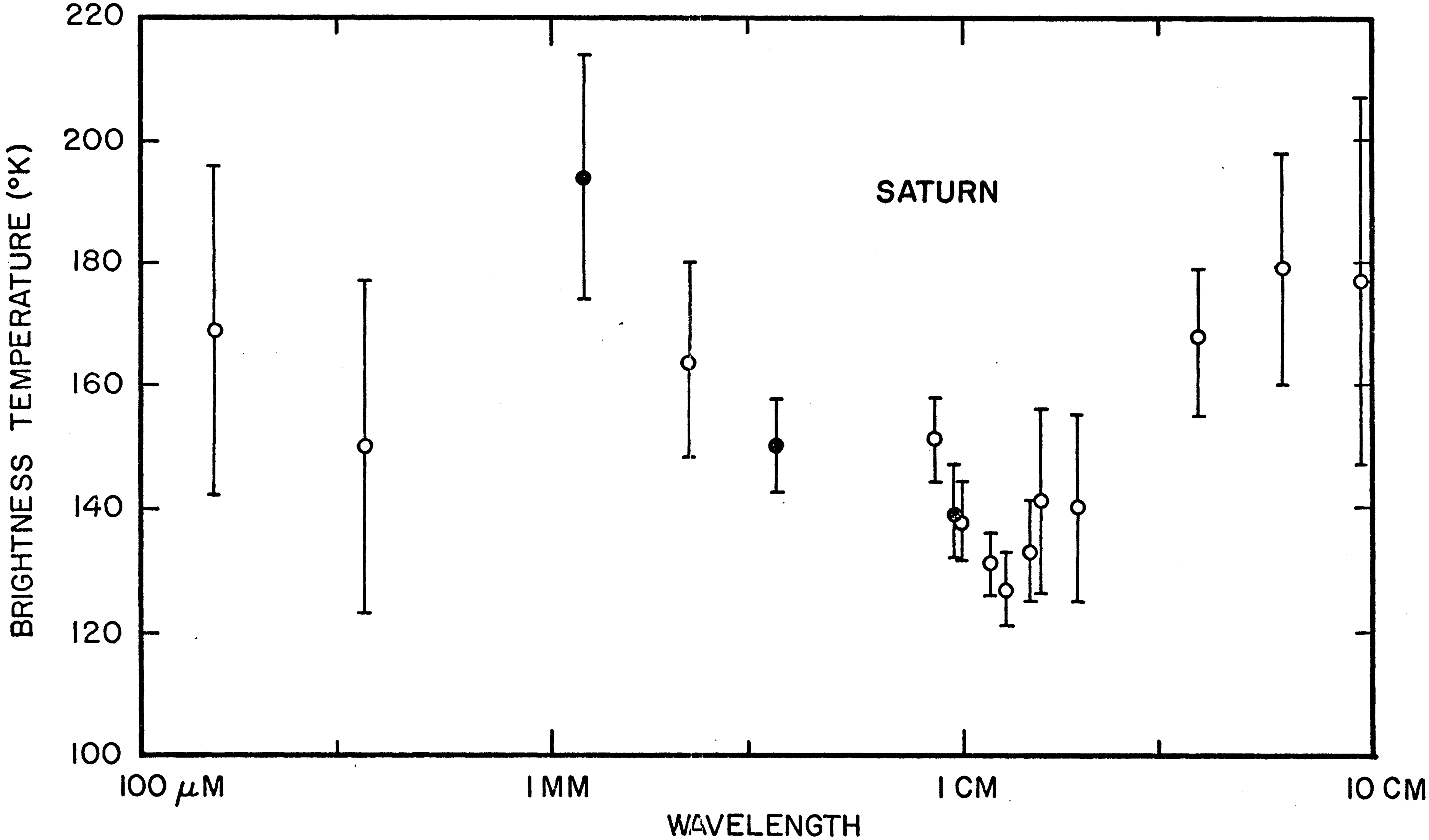
SATURN
BRIGHTNESS TEMPERATURE (°K)
220
200
180
160
140
120
100
100 μM
1 MM
1 CM
10 CM
WAVELENGTH

FIGURE 5

APPENDIX

This Appendix describes the calculation of the flux densities observed from the calibration sources in Table I. The Planck radiation law, relating the flux density to the brightness temperature of a uniform disk, is

$$S = \frac{2h\nu^3\Omega F_D}{c^2} \frac{1}{e^{\frac{h\nu}{kT_B}} - 1} \quad (1)$$

where S = Observed flux density (watt/m^2/Hz)

h = Planck's constant (6.6252×10^{-34} joule-s)

ν = Frequency (Hz)

c = Speed of light (2.9979×10^8 m/s)

Ω = Solid angle of planetary disk (steradian)

F_D = Disk resolution factor

k = Boltzmann's constant (1.3805×10^{-23} joule/K)

T_B = Disk brightness temperature (K)

The solid angle of the disk Ω is given by

$$\Omega = 7.3841 \times 10^{-11} SD_E SD_P \quad (2)$$

where SD_E = Equatorial semidiameter (")

SD_P = Polar Semidiameter (")

The mean semidiameter is

$$SD_M = \sqrt{SD_E SD_P} \quad (3)$$

where SD_M = Geometric mean of equatorial and polar semidiameters (")

Assuming a uniformly bright planetary disk, the observed flux density is the true flux density reduced by the disk resolution factor

$$F_D \approx 1.0 - 1.32\left(\frac{SD_M}{HPBW}\right)^2 \qquad (4)$$

where HPBW = Half-power beamwidth of antenna (")

At 250 GHz Equation 1 reduces to

$$S = \frac{1.701\ SD_M{}^2 F_D}{e^{12.00/T_B} - 1} \qquad (5)$$

and at 85 GHz the observed flux density is

$$S = \frac{0.06686 SD_M{}^2 F_D}{e^{4.078/T_B} - 1} \qquad (6)$$

At longer wavelengths the Rayleigh-Jeans approximation to Equation 1 may be used

$$S = \frac{2k\nu^2 T_B \Omega F_D}{c^2} \qquad (7)$$

The error in using Equation 7 at 85 GHz is as much as 2% even for temperatures greater than 100 K. The observed flux density at this frequency is

$$S = 0.01639 T_B SD_M{}^2 F_D \qquad (8)$$

At 31.4 GHz Equation 7 is

$$S = 0.002236 T_B SD_M{}^2 F_D \qquad (9)$$

The antenna temperature is related to the observed flux density by

$$T_A = \frac{\eta A_G}{2k} S \qquad (10)$$

Where T_A = Antenna temperature (K)

η = Antenna aperture efficiency

A_G = Antenna geometrical area (m^2)

For the NRAO 36 foot antenna Equation 10 reduces to

$$T_A = 0.03425\eta S \tag{11}$$

The observed flux densities of DR21 are given in the last line of Table I. The antenna temperatures of the Sun and Moon are simply the brightness temperatures times the beam efficiency of the antenna at the frequency of observation. This beam efficiency is essentially the integral of the antenna power pattern over a uniform disk of the same angular diameter as the Sun divided by the same integral over 4π steradians. At 85 GHz this beam coupling efficiency to the Sun or Moon is about 68%; values at other frequencies are uncertain at the present.