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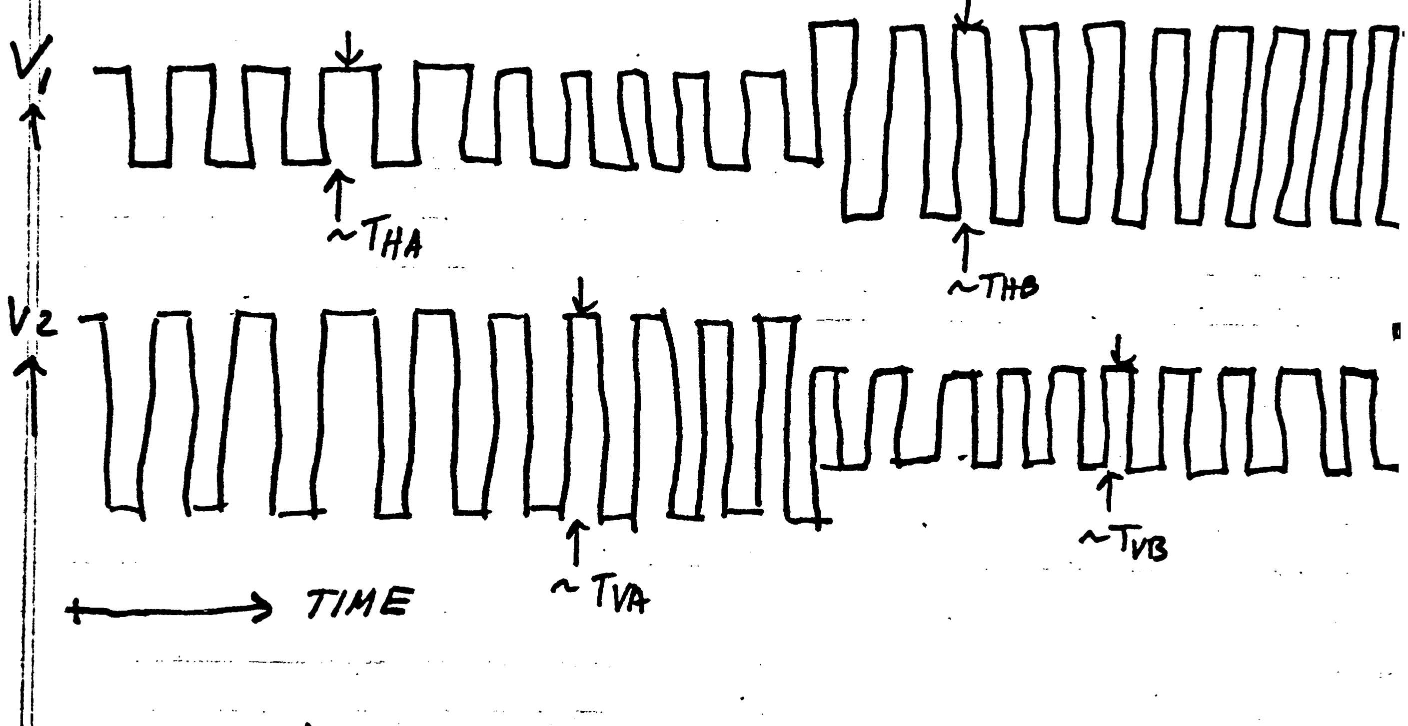
POLARIMETER NOISE CALCULATIONS

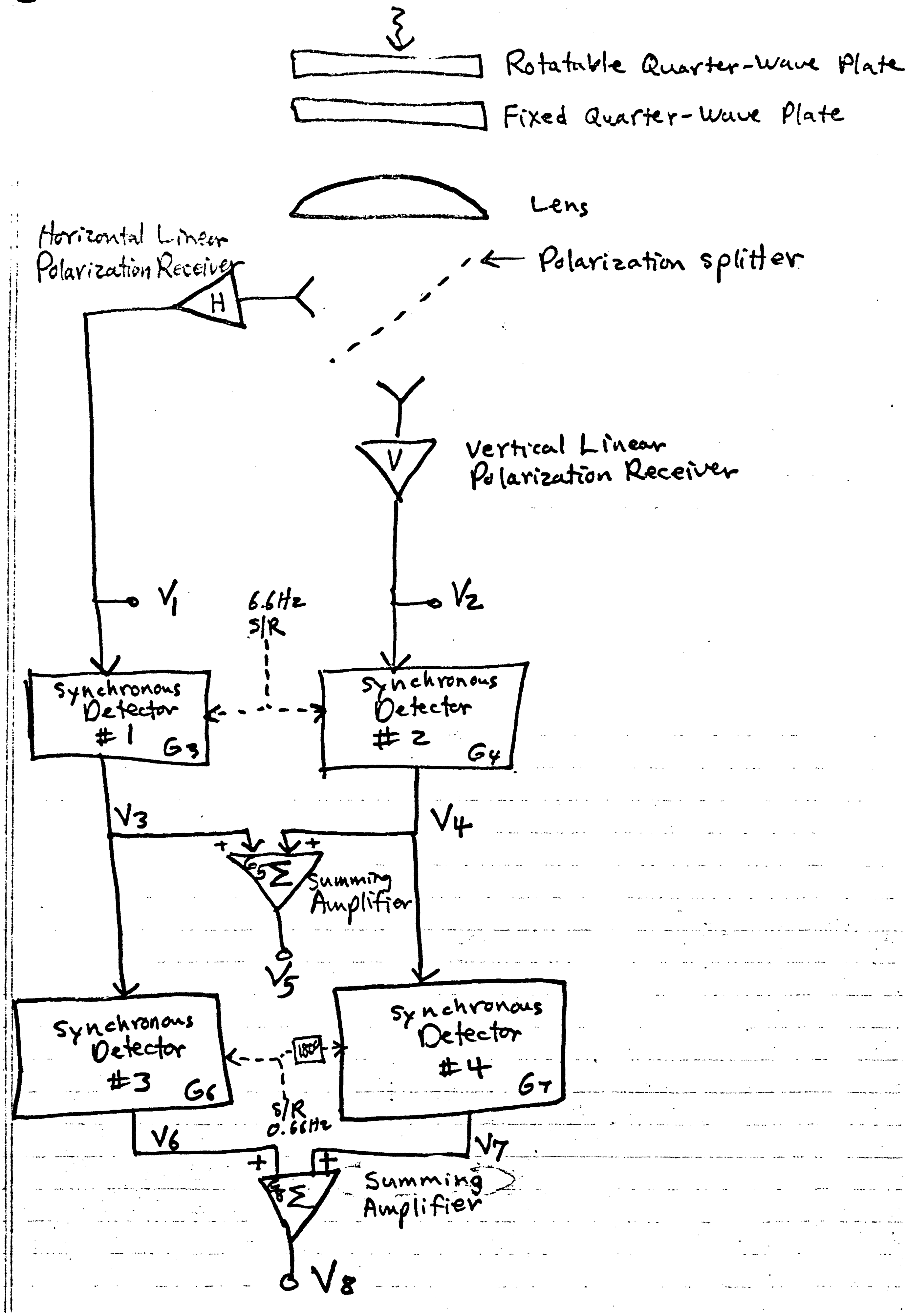
Consider the system of Fig. 1 used to measure the intensity and linear polarization of a radio signal at the focus of a reflector antenna. Synchronous detectors #1 and #2 are operated at a high S/R frequency (6.6 Hz) corresponding to the nutation rate of the beamswitching subreflector. Synchronous detectors #3 and #4 are operated at a slower rate (0.66 Hz) corresponding to the frequency of the rotating quarter-wave plate. The rotation angle is chosen to produce a 90° rotation in the output polarization of a linear input signal.

For a linearly polarized source, V_1 and V_2 might look like this:



Consider first a completely linearly polarized source.

FIGURE 1: SYSTEM BLOCK DIAGRAM



Let T_{HA} ≡ source signal received by horizontally polarized receiver for quarter-wave plate position 'A'

T_{HB} ≡ source signal (antenna temperature) received by horizontally polarized receiver for quarter-wave plate position 'B' (which differs from position 'A' by 90°)

T_{VA} and T_{VB} are similarly defined for the vertically polarized receiver.

Since the H and V receivers are orthogonal, and positions A and B are orthogonal, we know that

$$T_{HA} = T_{VB} \quad \text{and} \quad T_{HB} = T_{VA}.$$

In the 'A' position:

$$V_3 \equiv G_3 T_{HA}$$

and

$$V_4 \equiv G_4 T_{VA}.$$

In the 'B' position:

$$V_3 \equiv G_3 T_{HB}$$

and

$$V_4 \equiv G_4 T_{VB}.$$

The sum output V_5 is given by:

$$V_5 \equiv G_5(V_3 + V_4)$$

$V_5 = G_5(G_3 T_{HA} + G_4 T_{VA})$ in the 'A' position, and

$V_5 = G_5(G_3 T_{HB} + G_4 T_{VB})$ in the 'B' position.

Now, if (and only if) $G_3 = G_4$:

$V_5 = G_5 G_3 (T_{HA} + T_{VA})$ in the 'A' position, and

$V_5 = G_5 G_3 (T_{HB} + T_{VB})$ in the 'B' position.

We know that $T_{HA} + T_{VA} = T_{HB} + T_{VB}$, so
$V_5 = G_5 G_3 (T_{HA} + T_{VA}) = G_5 G_3 (T_{HB} + T_{VB})$, or

$$V_5 = G_5 G_3 (T_I) = G_5 G_3 T_{H+V} = G_5 G_3 T_{A+B}$$

where T_I is the antenna temperature representing the total intensity of the source (assumed to be linearly polarized).
The outputs of the second set of synchronous detectors are given by:

$V_6 \equiv G_6 G_3 (T_{HA} - T_{HB})$ and

$V_7 \equiv G_7 G_4 (T_{VB} - T_{VA})$.

Note that the phase of the reference signal to synchronous detector #4 is 180° different from that to synchronous detector #3.

The summing output V_8 is given by:

$$V_8 \equiv G_8 (V_6 + V_7)$$

$$= G_8 G_6 G_3 (T_{HA} - T_{HB}) + G_8 G_7 G_4 (T_{VB} - T_{VA})$$

Now, if (and only if) $G_6 G_3 = G_7 G_4$,

$V_8 = G_8 G_6 G_3 (T_{HA} - T_{VA} + \{T_{VB} - T_{HB}\})$.

Since $T_{HA} = T_{VB}$ and $T_{HB} = T_{VA}$;

$$V_8 = 2G_8 G_6 G_3 (T_A - T_B) \equiv 2G_8 G_6 G_3 T_{A-B}$$

where $T_A \equiv T_{HA} = T_{VB}$

and $T_B = T_{HB} = T_{VA}$.

Note the factor of 2. Thus V_8 is proportional to twice the difference in the received signals in positions A and B.

For accurate calibration, we must assure that $G_3 = G_4$ and that $G_6 = G_7$ by observing the linearly polarized noise tube and adjusting the gains so that V_5 is independent of the relative noise tube position angle, and that V_8 varies with position angle according to

$$V_8 = 2V_5 \cos 2\theta$$

where θ is the angle between the noise tube and position 'A'.

Now we consider the noise levels of the various outputs.

Let $\Delta T_H \equiv \dfrac{T_{SYSTEM(H)}}{\sqrt{B_{(H)} \cdot 1}}$ for the horizontal

receiver and $\Delta T_V \equiv \dfrac{T_{SYSTEM(V)}}{\sqrt{B_{(V)} \cdot 1}}$ for the vertical receiver.

The RMS noise (in terms of antenna temperature) on the outputs of the first synchronous detectors are:

$$\Delta T_3 = 2\Delta T_H \quad \text{and} \quad \Delta T_4 = 2\Delta T_V.$$

also $\Delta T_5^2 \equiv \Delta T_I^2 = \Delta T_3^2 + \Delta T_4^2 = 4(\Delta T_H^2 + \Delta T_V^2)$

$$\Delta T_I = 2\sqrt{\Delta T_H^2 + \Delta T_V^2}$$

If $\Delta T_H = \Delta T_V = \Delta T$, then

$$\boxed{\Delta T_I = 2\sqrt{2}\,\Delta T} = \Delta T_{A+B}.$$

Thus adding the two signals (V_3 and V_4) together increases the RMS noise by $\sqrt{2}$. For an unpolarized source, V_5 is twice as large as for a linearly polarized source (100%), and thus the S/N ratio for an unpolarized source is improved by $2/\sqrt{2} = \sqrt{2}$ by adding the two receiver outputs.

The RMS noise on the outputs of the second set of synchronous detectors is:

$$\Delta T_6 = 2\Delta T_3 \quad \text{and} \quad \Delta T_7 = 2\Delta T_4$$

$$= 4\Delta T_H \qquad\qquad = 4\Delta T_V.$$

The output of the second summing amplifier has an RMS noise given by:

$$\Delta T_8^2 = \Delta T_6^2 + \Delta T_7^2 = 16(\Delta T_H^2 + \Delta T_V^2)$$

$$\Delta T_8 = 4\sqrt{\Delta T_H^2 + \Delta T_V^2}$$

If $\Delta T_H = \Delta T_V = \Delta T$:

$$\Delta T_8 = 4\sqrt{2}\,\Delta T$$

Now we also know that $\Delta T_{A-B} = \frac{1}{2}\Delta T_8$, so

$$\boxed{\Delta T_{A-B} = 2\sqrt{2}\,\Delta T},$$

which is the same result we obtained previously for ΔT_{A+B}. Thus the RMS noise on the 'A+B' output is the same as on the 'A-B' output.

A partially polarized source can be modelled as the superposition of an unpolarized source and a completely polarized source.

For the unpolarized source:

$$T_{A+B} \equiv 2T_u$$

$$T_{A-B} \equiv 0 .$$

For the polarized source:

$$T_{A+B} \equiv T_p$$

$$T_{A-B} = T_p \cos 2\phi$$

where ϕ is the angle between the source position angle and position 'A'.

For the superposed case:

$$\boxed{\begin{aligned} T_{A+B} &= 2T_u + T_p \\ T_{A-B} &= T_p \cos 2\phi \end{aligned}} \quad [\phi \equiv \theta_{SOURCE} - \theta_A]$$

By varying the angle of position 'A', we can vary ϕ and thus determine T_p and the source position angle from the T_{A-B} data alone. We can next determine T_u, the % polarization, and the total intensity from the T_{A+B} data.

Maximum Intensity $\equiv T_u + T_p$

% Polarization $\equiv \frac{T_p}{T_u + T_p}$

Source Position Angle $\equiv \theta_{source}$.

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P.S. I did this very quickly & there may be mistakes. If you find any or have comments, please contact me.