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14 April 1976

25 Meter - Millimeter Wave Telescope
Memo # 41

Dr J Findlay
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Charlottesville
Virginia 22901

Dear John

Thanks for your letter and the picture. It is a shame the Griffolyn didn't last because its RF properties are really rather good. My impression from the photograph is that it's only the white plastic that has broken down, which is not surprising since it will obviously be much less protected from UV than the black. The trouble is that one would prefer to have the white plastic on the outside to reflect as much of the sun's heat as possible.

I enclose the results of tests we have had done so far by NPL. These have concentrated on a very light thin inner membrane which is not supposed to stand up to the weather and which could be replaced fairly often. However, because of the costs associated with the rotating astrodome, we have to continue to consider conventional radomes, so we are very interested in the Essco materials. It is my understanding that Essco are having some tests of their materials done at NPL, and I assume they would pass the results on to you. If, however, you have any doubts about them I think we could get some more measurements made fairly easily. NPL only need a few inches square as far as I know.

On the measurement problem we are still pursuing the rather heavy handed "engineering" solutions, as you will see from the paper on "UV method". There are some practical snags with this of course - such as how do you swing it round past the feed legs? - and I don't think such a machine would be practical for dishes much larger than ours, but I am fairly hopeful that it can do the job for us.

One other thing I have done recently is to calculate the effect of deformations of surface panels caused by the weight of a curvature measuring trolley. (See second half of "Note on Deformation...") I don't know how serious these effects are likely to be in practice, but I thought the sum was probably worth doing. Please don't take the first part of that note too seriously; it is pretty much an academic exercise. The point is, I think, that one of the main reasons for always supporting panels at their corners is to stop them blowing around in the wind. If you put the telescope in a radome this may no longer be the right thing to do.

Another example of the same argument, and probably a much more important one, is the question of the weight of the telescope. Obviously in a radome it can

be much lighter than if it has to stand up to the weather. The contrast between the new Bell Telephone antenna by Philco and the Essco one being made for U. Mass is very marked in this respect. However, it is not possible to cut the weight much unless one can find a way of making the panels light and this seems likely to be a problem with castings. Certainly the Bell Telephone ones were very heavy. We are trying to decide which lines to follow in doing some experiments. Did you go ahead with getting some more trial panels made? If you did, would it be possible to compare notes on the results? It would be good to avoid too much duplication of effort if possible.

You will have gathered that our project is pushing on, at least on paper, although of course it is going too slowly, and that we are trying to get some practical work started. The latter is made difficult by our lack of a proper in-house engineering base. The prospects for getting the full project approved are still none too good, with competition hotting up as the funds get scarce. I was impressed by the increase in astronomy funding in next year's US budget. What is the earliest you can now hope to build your 25 m telescope?

With best wishes,
Yours sincerely

Richard Hills

Enclosures: NPL membrane interim report
UV measurement
MMN/REH/4

An Investigation of Possible Millimetre Wavelength Radome Materials

Interim Report

This report presents a discussion of the preliminary results of an investigation into the suitability of various materials as radomes for a proposed millimetre wavelength telescope. The general requirements for the radome material can be summarised as follows:

- (i) A power transmission in excess of 0.90 from 100 to 300 GHz with a lower figure acceptable at frequencies up to the 900 GHz atmospheric window.
- (ii) A real relative permittivity less than 2.
- (iii) An optical thickness less than $\lambda/4$ at 375 GHz ie 0.2 mm .
- (iv) A rejection of near infrared and visible solar radiation of about 0.90 .
- (v) Mechanical strength and weathering resistance.
- (vi) Reasonable cost and availability.
- (vii) Ease of handling and mounting.

Polymers are the class of materials most likely to satisfy these requirements. Low-loss polymers are commercially available, have low real refractive indices ~ 1.4 to 1.5 ($\epsilon' \sim 1.96$ to 2.25), are easily woven to increase tensile strength and resistance to tearing and can be filled to improve their near infrared and visible rejection characteristics.

The part of the investigation reported here is the initial measurement of the millimetre wavelength power transmission of five commonly available low-loss polymers with various fillers and coatings. These results will then be used to eliminate obviously unsuitable materials according to the first requirement. The measurements were made using the techniques of Fourier transform spectrometry with a two-beam Michelson interferometer, phase modulation and a liquid helium cooled indium antimonide photoconductor used in the Rollin mode. The power transmission of each specimen was measured from 210 to 900 GHz (7 to 30 cm^{-1}) at ambient temperature ($\sim 290\text{K}$) in the collimated beam leaving the exit aperture of the interferometer. Table 1

lists the specimens studied.

MATERIAL	FILLER	COATING	THICKNESS (μm)
Polypropylene	-	-	20
Polytetrafluoroethylene	-	-	90
Polyethylene	TiO ₂	-	47.5
	Carbon black	-	72.5
Polyurethane	-	-	117 and 302
	Carbon black	-	62.5 and 612
Polyethylene terephthalate	-	Nickel-cadmium	25
	-	Aluminium	47.5
	TiO ₂	-	37.5
	-	Graphite	25
	-	Carbon black	32.5
Sailcloth			82.5 and 140
Parachute cloth			300
Plasticised PVC			762
Griffolyn			100

RESULTS AND DISCUSSIONS

(i) Polypropylene and polytetrafluoroethylene (PTFE)

These were included in the survey to show the extremely low level of transmission loss that can be achieved in commercially available polymers. In these thicknesses the losses in the clear polypropylene film are mainly reflective and the power transmission is greater than 0.95 between 10 and 30 cm⁻¹ (Fig 1). The PTFE specimen was considerably thicker and its transmission lies between 0.90 and 0.92 over this range (Fig 2). PTFE has a naturally white colour but would not reject sufficient visible and near infrared solar radiation in thin film form.

(ii) Polyethylene

Unfilled or uncoated polyethylene based radomes will be largely unsuitable due to the rapid degradation of polyethylene in ultraviolet radiation. The addition of a filler or coating to absorb the ultraviolet radiation will go some way in overcoming this, and in general loading with carbon black considerably improves the weathering properties of most polymers. The measurements on the TiO_2 and carbon black filled polyethylene specimens are of interest in showing that the addition of fillers to low loss polymers does not introduce substantial transmission losses (Figs 3 and 4).

(iii) Polyurethane

This is a more weather resistant material than polyethylene and the measurements were made on two thicknesses of each of clear and black F100 polyurethane film supplied by ICI and known as Daltoflex. The transmission measurements indicate that it is a fairly lossy material with the thinnest (117 μm) clear specimen having a transmission below 0.8 at 300 GHz. In addition all the specimens exhibited well-defined multiple-beam interference fringes (Figs 5 and 6).

(iv) Polyethylene terephthalate (PET)

This material, which is also known as Melinex, Mylar and Terylene, is the principal material used for the beam dividers of far infrared interferometers and its optical properties are fairly well-known. It is intensely absorbing above 1.5 THz (50 cm^{-1}) but transparent in thin film form. Figures 7 and 8 show the transmissions of two films that had been lightly metalised. One had been lightly aluminised and was fairly transparent (≥ 0.5) in the visible while the other was heavily coated with nickel-cadmium and was visibly opaque as judged by the eye. These had very low transmissions < 0.05 and were clearly unsuitable. Two coated PET specimens were measured, one with graphite and one with carbon black and their transmissions are shown in Figures 9 and 10. These were fairly high but not

to the required level. The final PET film used was filled with TiO_2 and had a transmission in excess of 0.90 below 300 GHz (Fig 11).

(v) Sailcloth and parachute cloth

These were supplied by Carrington Performance Fabrics, a subsidiary of ICI. Sailcloth is a woven terylene dipped in clear polyurethane for wind-and water-proofing and measurements were made on orange and white coloured 82.5 μm thicknesses and on a blue 140 μm specimen. The thinner specimens showed identical millimetre wavelength transmissions in excess of 0.95 at 300 GHz falling to about 0.80 at 900 GHz (Fig 12). The parachute cloth was a fairly open nylon weave and not wind-proofed. Its transmission, shown in Fig 13 is fairly high but only rises to about 0.65 by 300 GHz .

(vi) Plasticised PVC

This was a white composite supplied by Mr Shenton of the Appleton Laboratory from an American source. Although fairly transparent at millimetre wavelengths it has closely spaced interference fringes that render it unsuitable for radome use (Fig 14).

(vii) Griffolyn

This was a composite material again supplied by Mr Shenton. It comprises a black and a white polyethylene sheet glued together with PIB^{*} glue over a widely spaced ($\sim 8-10$ mm) grid of nylon threads. Its transmission was fairly constant at 0.80 from 210 to 900 GHz (Fig 15).

Of the specimens studied five can be recommended for further consideration as possible radome materials in the following descending order of preference.

(i) 82.5 μm sailcloths

These have extremely high transmission characteristics and their woven nature implies high strength and resistance to tearing. It is unlikely that, in their present form they would reject much visible and near infrared

* Polyisobutadiene

radiation but the measurements on filled polyethylene and polyurethane indicate that it should be possible to use a black polyurethane dip on the terylene instead of the present clear one. The major reservation about this material comes from its periodic structure and possible diffraction effects. The individual fibres used have a very small diameter $<10^{-2}$ mm but they are collected together in flat strips $\sim 10^{-1}$ mm width (Fig 16) which could give rise to considerable diffraction at 900 GHz . This point should be considered further.

(ii) Griffolyn

This has a rather low, but constant, transmission of about 0.80 from 210 to 900 GHz although from thin film theory one would expect this to increase to ~ 1.0 as the frequency decreases further. It is predominantly polyethylene although the fillers used should inhibit ultraviolet degradation, and the nylon reinforcing fibres ensure that it has considerable strength. If the terylene sailcloths prove to be unsuitable then this material should be considered in spite of its low transmission. The glue used for holding the two sheets together is known to be a lossy material and it might be possible to replace it with a lower loss one.

(iii) Carbon black and TiO_2 filled polyethylene, TiO_2 filled polyethylene terephthalate

These remaining possible materials all have excellent transmission characteristics and (especially the carbon black) should have good short wavelength blocking characteristics. They would, however, be much more difficult to mount and more liable to tear than the woven materials but should be considered in the absence of suitable woven alternatives.

In the final report on the complete investigation the power transmission measurements will be extended down to ~ 100 GHz and the total (specular and diffuse) transmission and reflectivity of the preferred materials measured from 0.35 to 2.1 μm .

JRS021 21 MAY 75
POLYPROPYLENE FILM

POWER TRANSMISSION

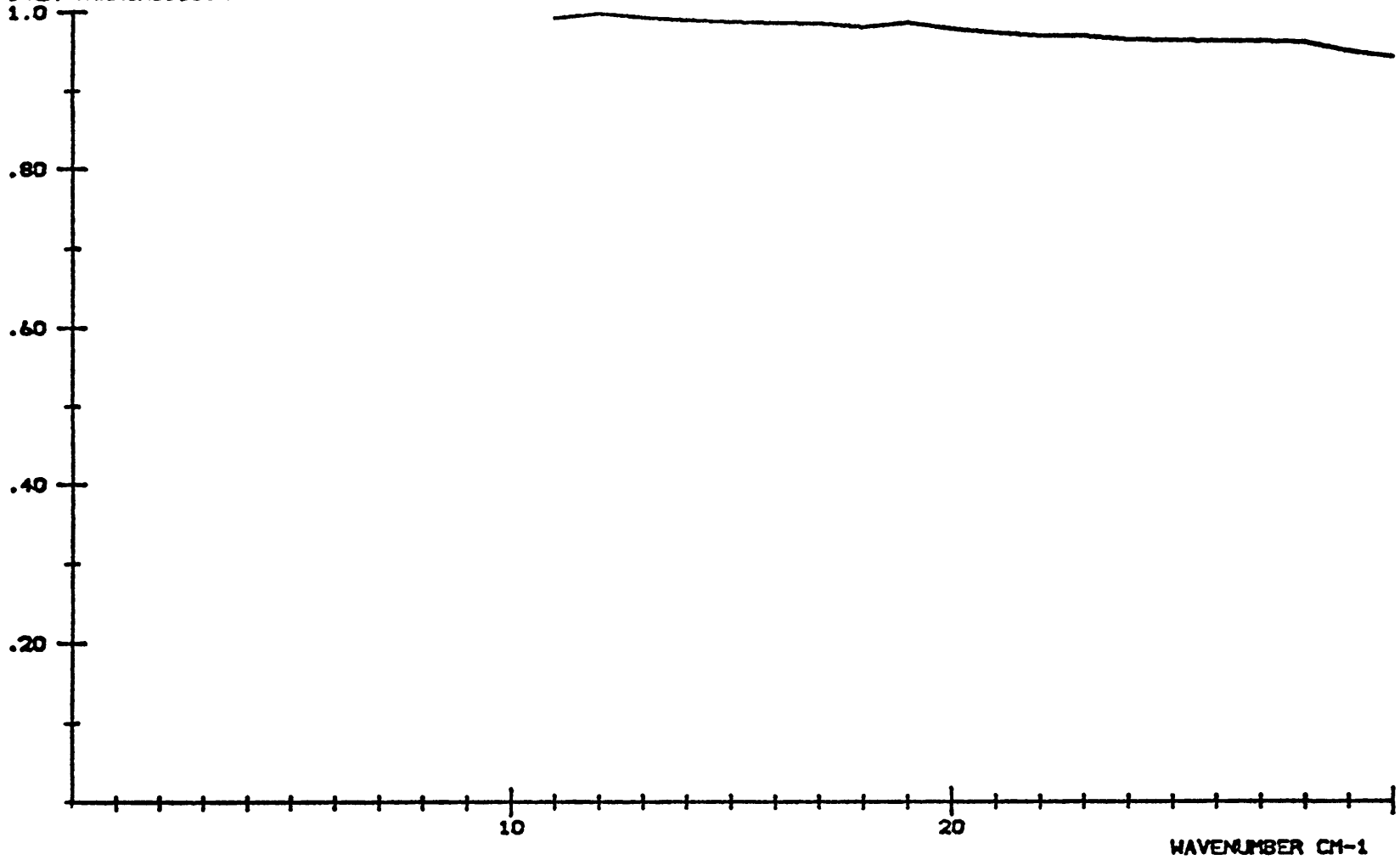


Fig 1.

JRS021 21 MAY 75
PTFE SHEET 0.003 INCH

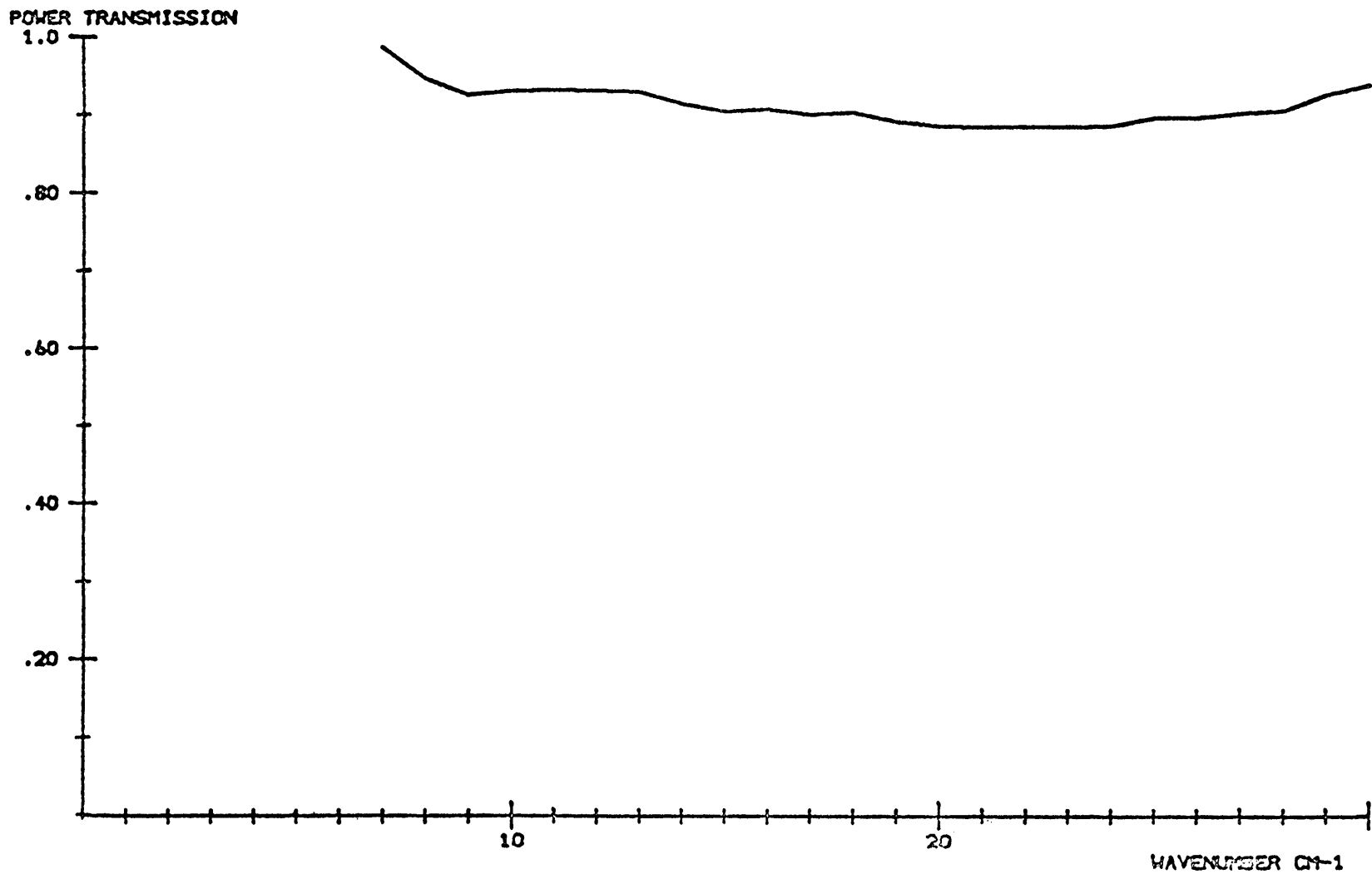


Fig 2

JRS021 21 MAY 75
TiO2 FILLED POLYETHYLENE

POWER TRANSMISSION

1.0

.80

.60

.40

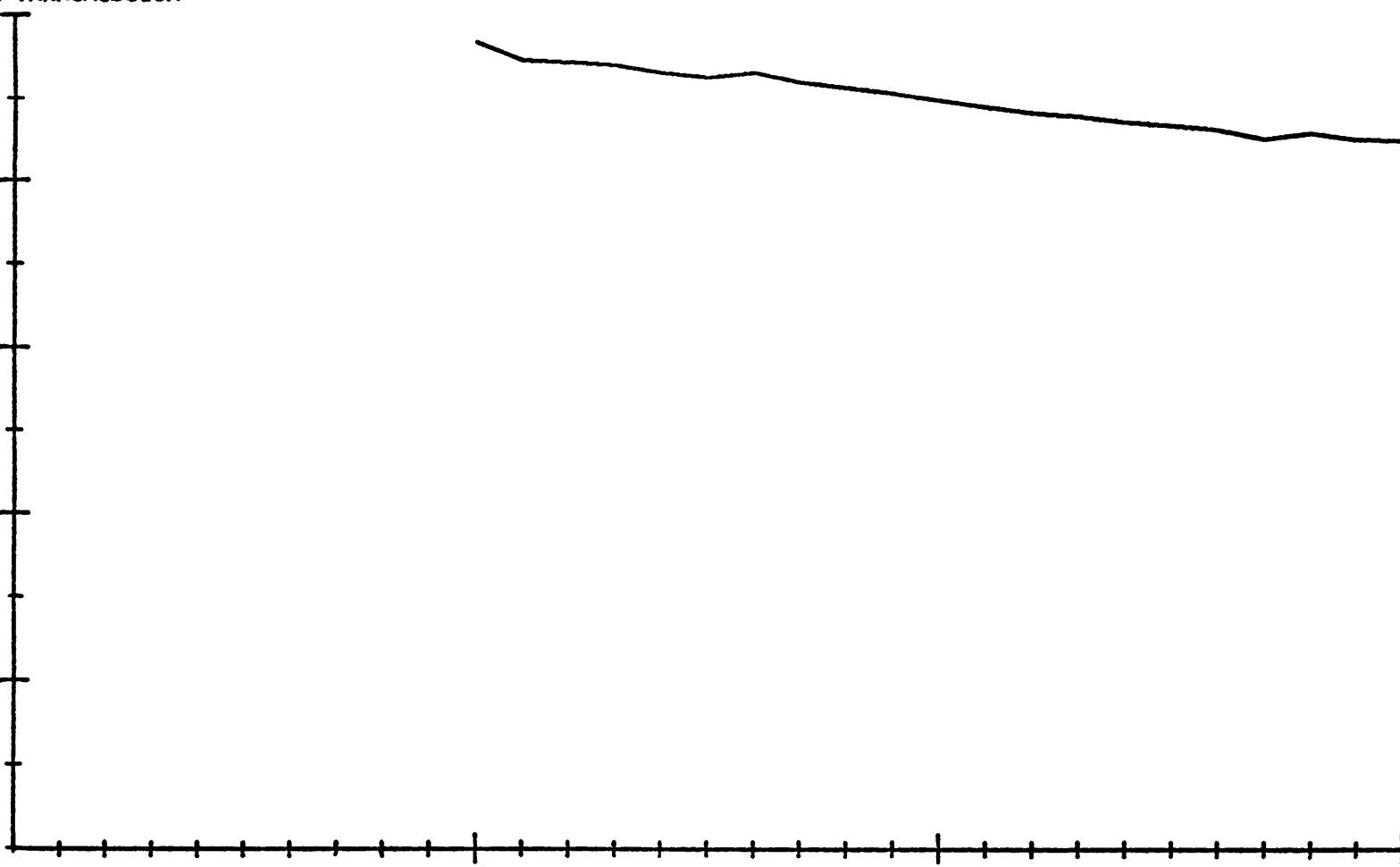
.20

10

20

WAVENUMBER CM-1

Fig 3



JRS021 21 MAY 75
CARBON BLACK FILLED POLYETHYLENE

POWER TRANSMISSION

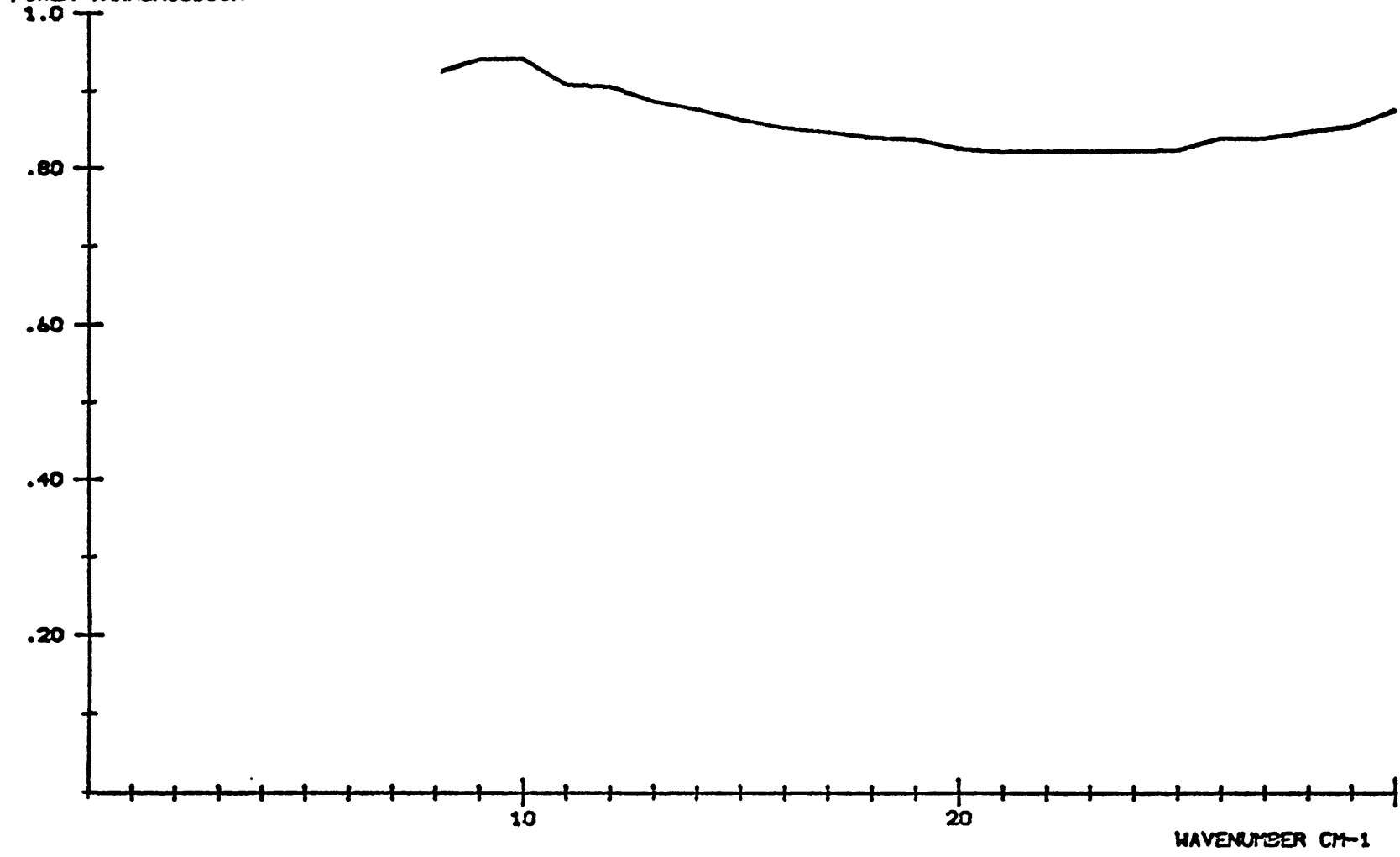


Fig 4

JRS021 21 MAY 75
F100 POLYURETHANE

POWER TRANSMISSION

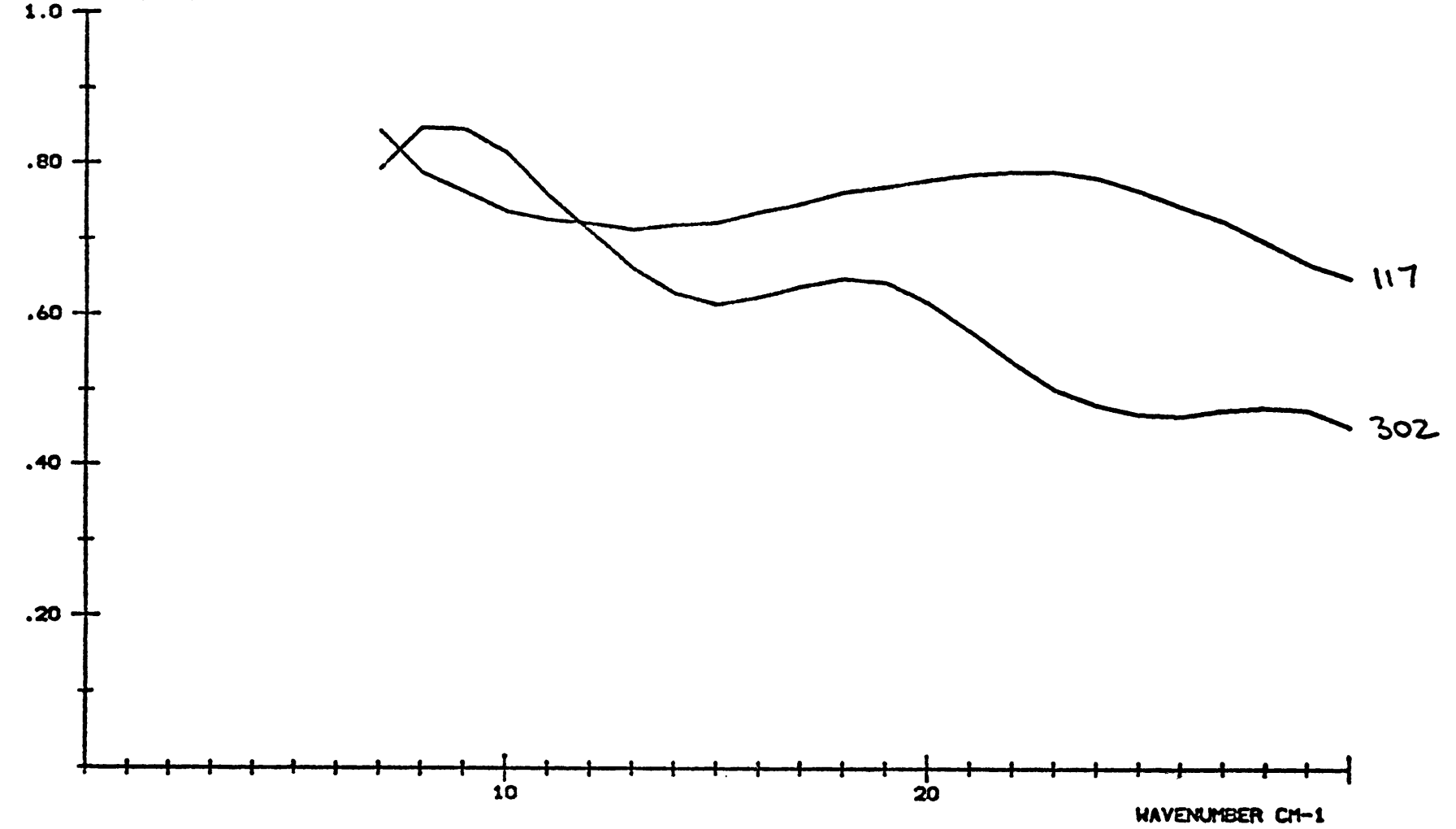


Fig 5

JRS021 21 MAY 75
F100 BLACK POLYURETHANE

POWER TRANSMISSION

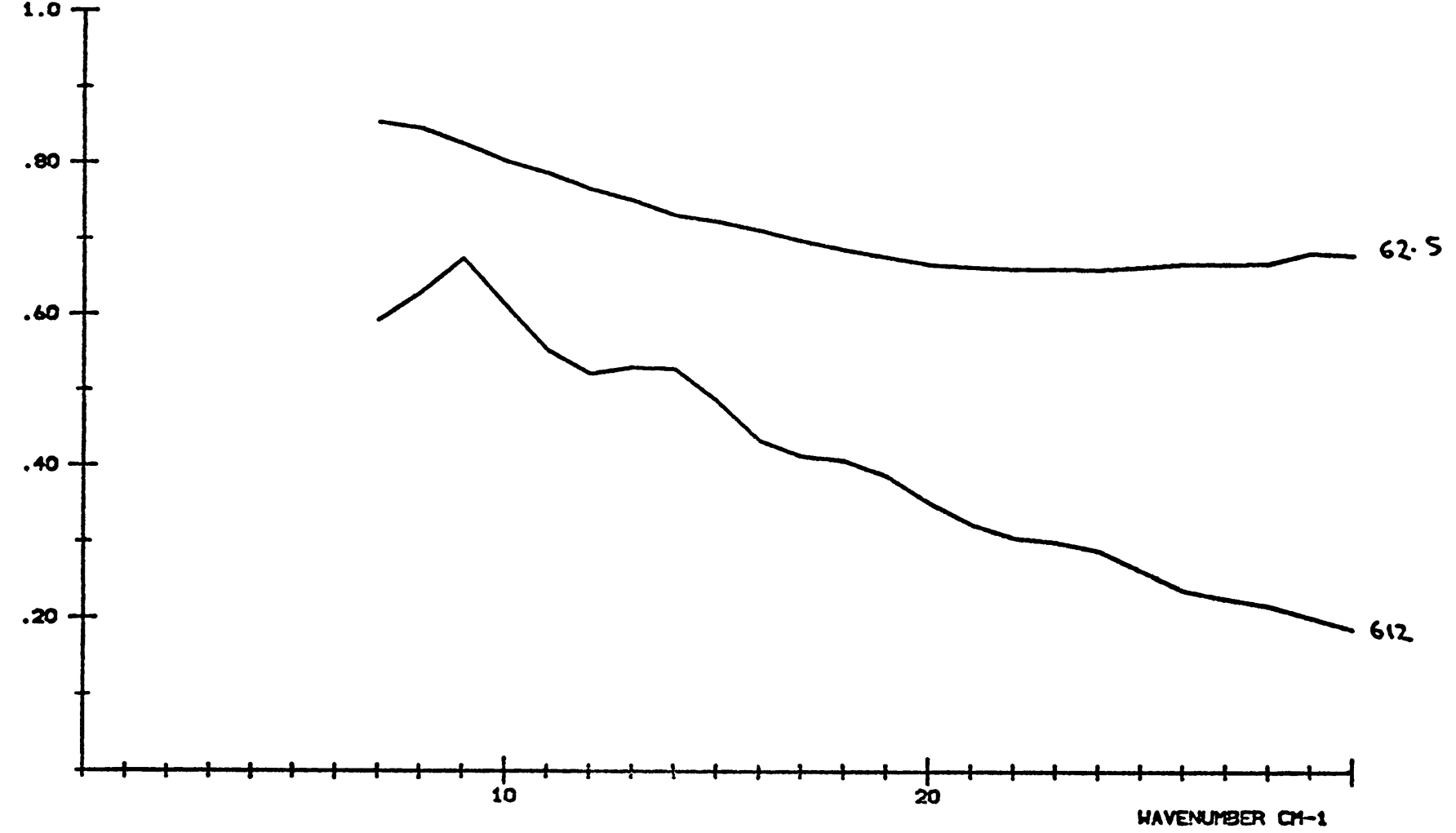


Fig 6

JRS021 21 MAY 75
NI-CD METALISED PET

POWER TRANSMISSION

.10

.08

.06

.04

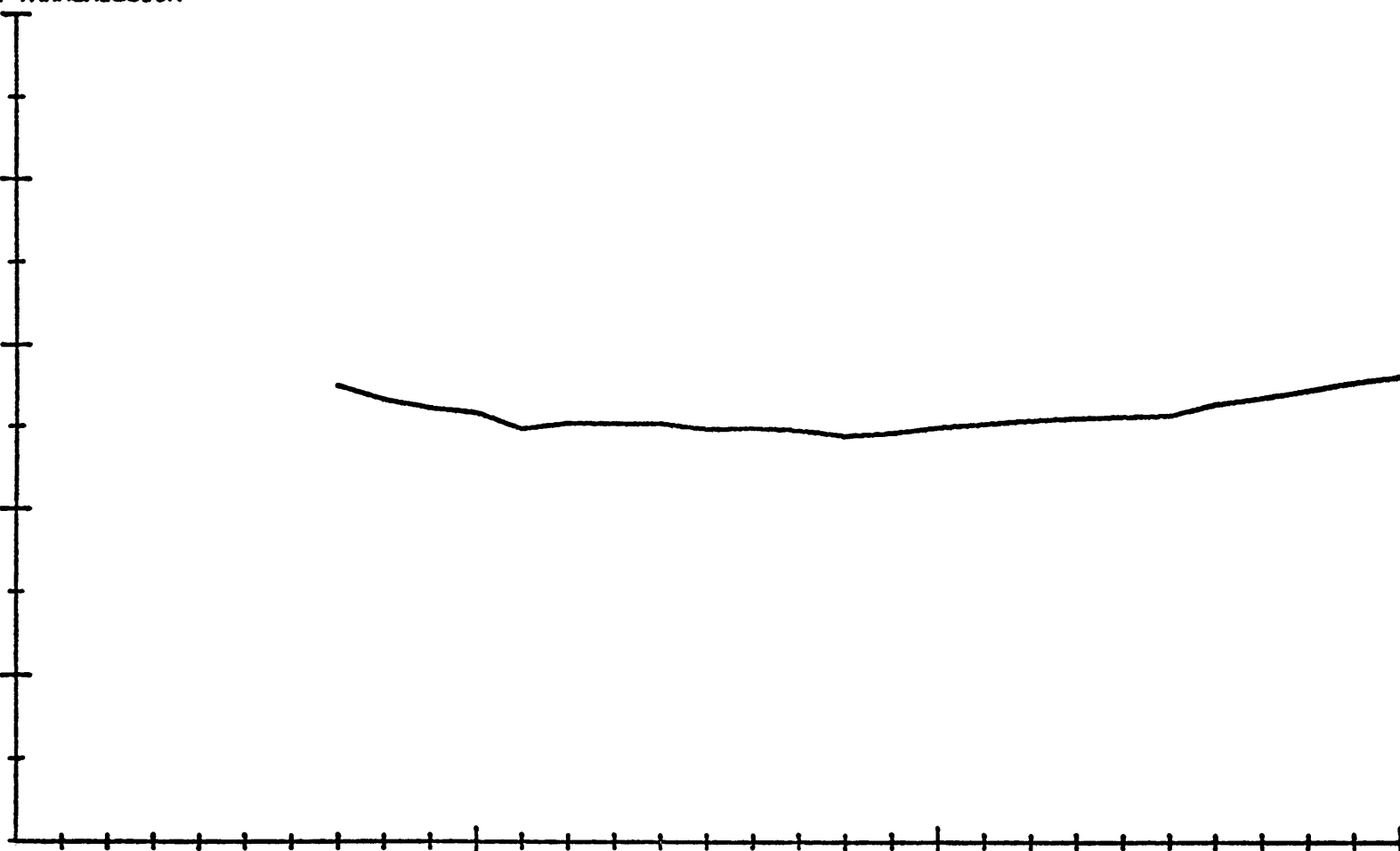
.02

10

20

WAVENUMBER CM-1

Fig 7



JRS021 21 MAY 75
FLASH ALUMINISED PET

POWER TRANSMISSION

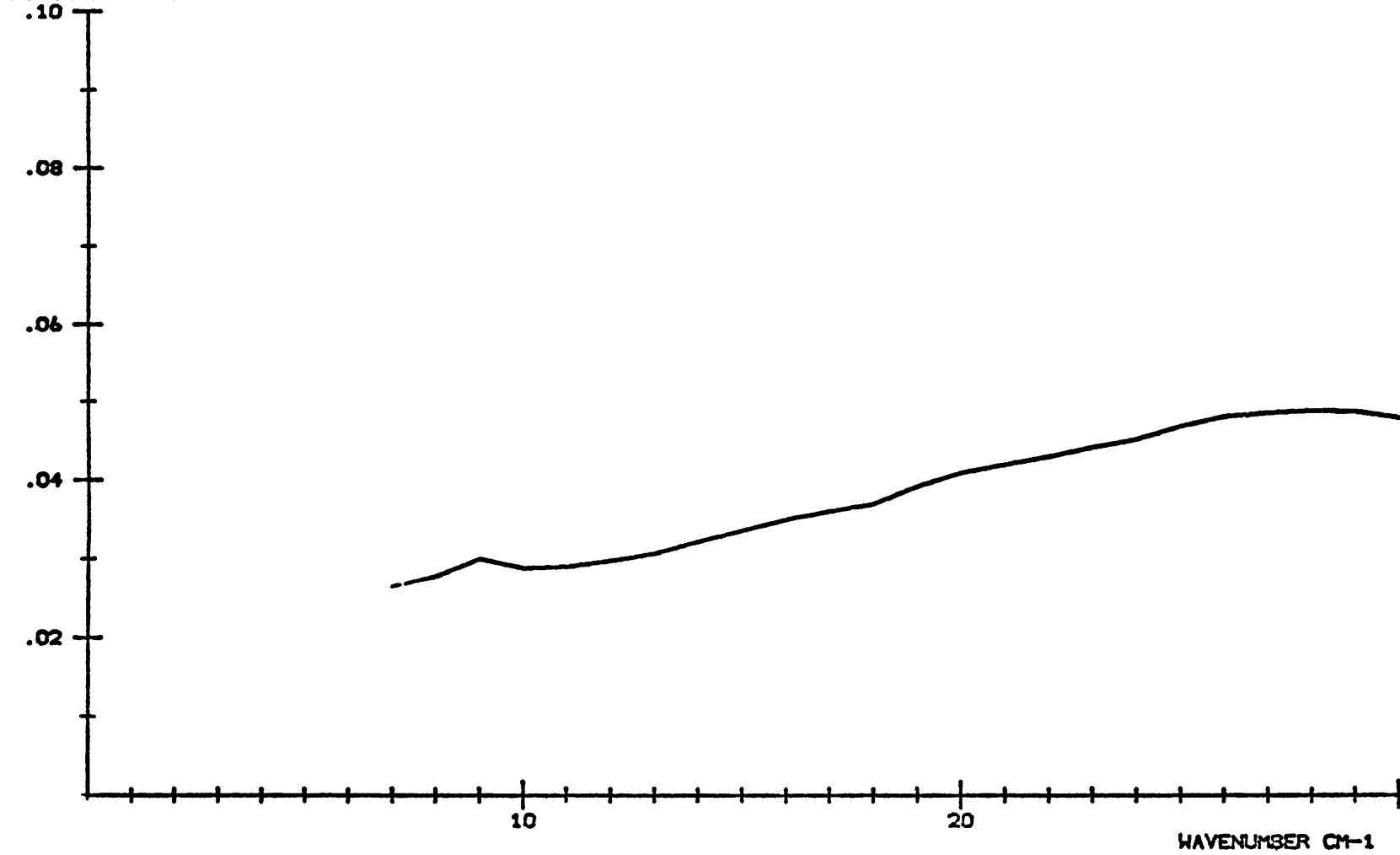


Fig 8

JRS021 21 MAY 75
GRAPHITE COATED PET

POWER TRANSMISSION
1.0

1.0

.80

.60

.40

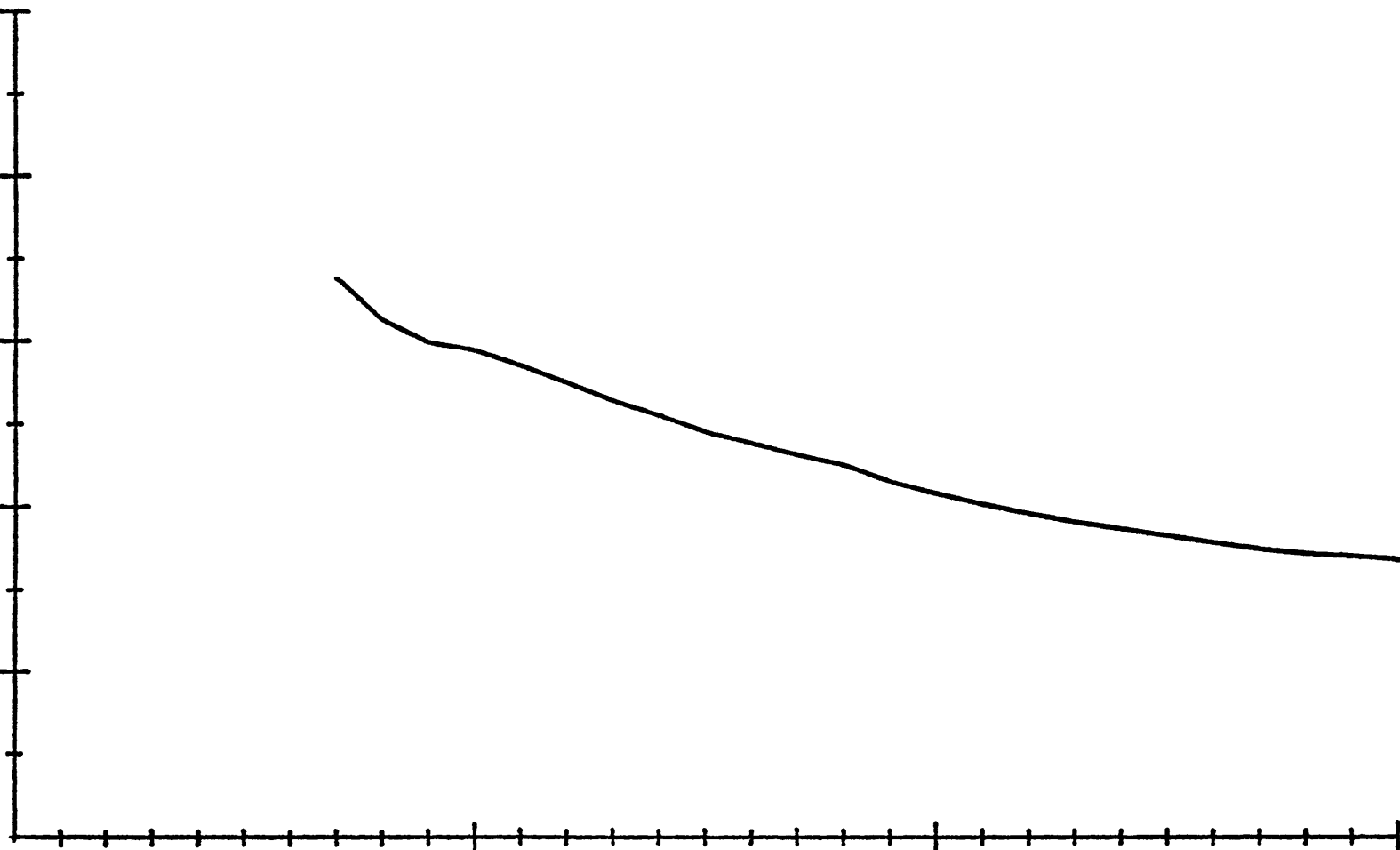
.20

10

20

WAVENUMBER CM-1

Fig 9



JRS021 21 MAY 75
PET CARBON COMPUTER PRINT OUT

POWER TRANSMISSION

1.0

.80

.60

.40

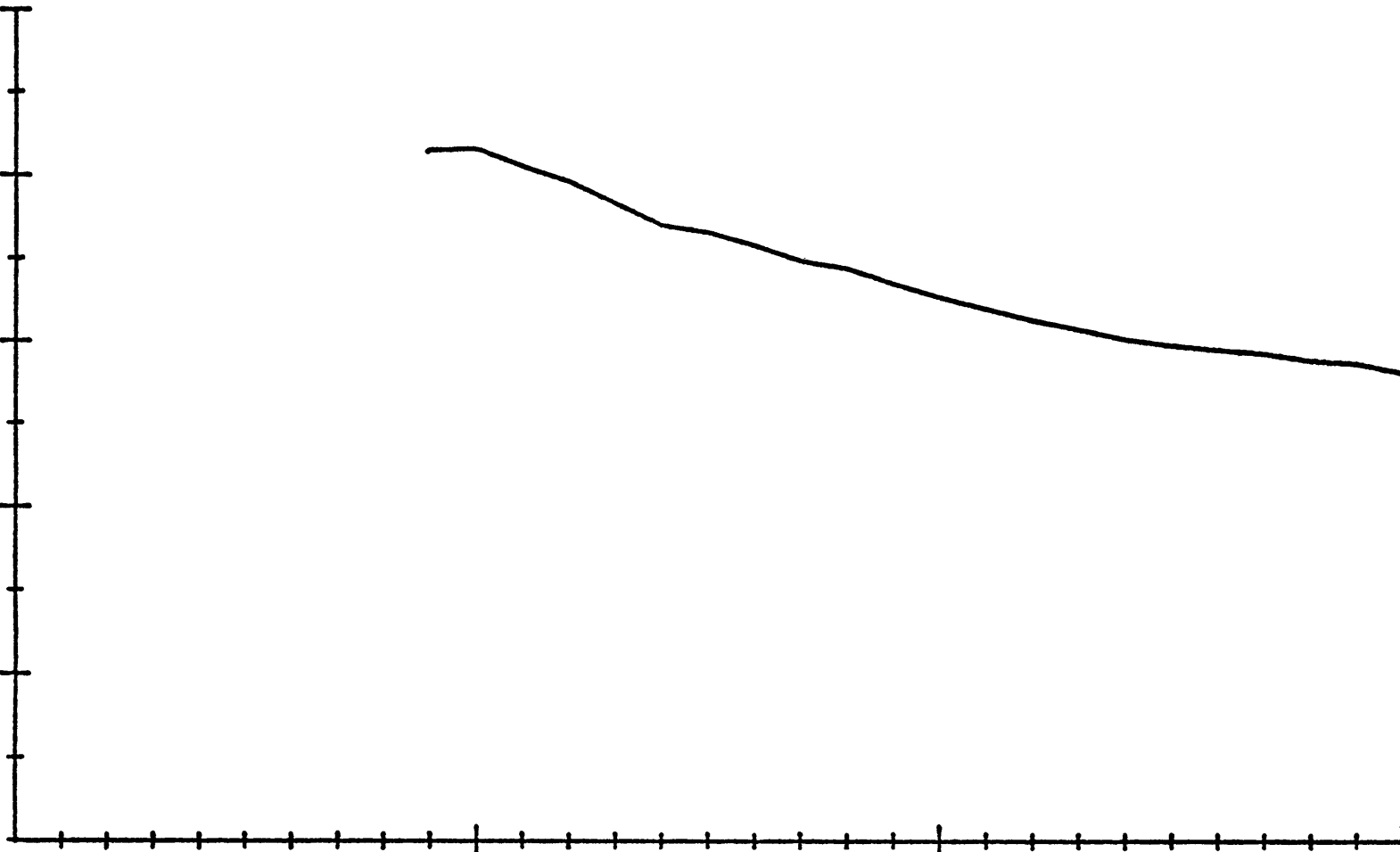
.20

10

20

WAVENUMBER CM-1

Fig 10



JRS021 21 MAY 75
TiO2 FILLED PET

POWER TRANSMISSION

1.0

.80

.60

.40

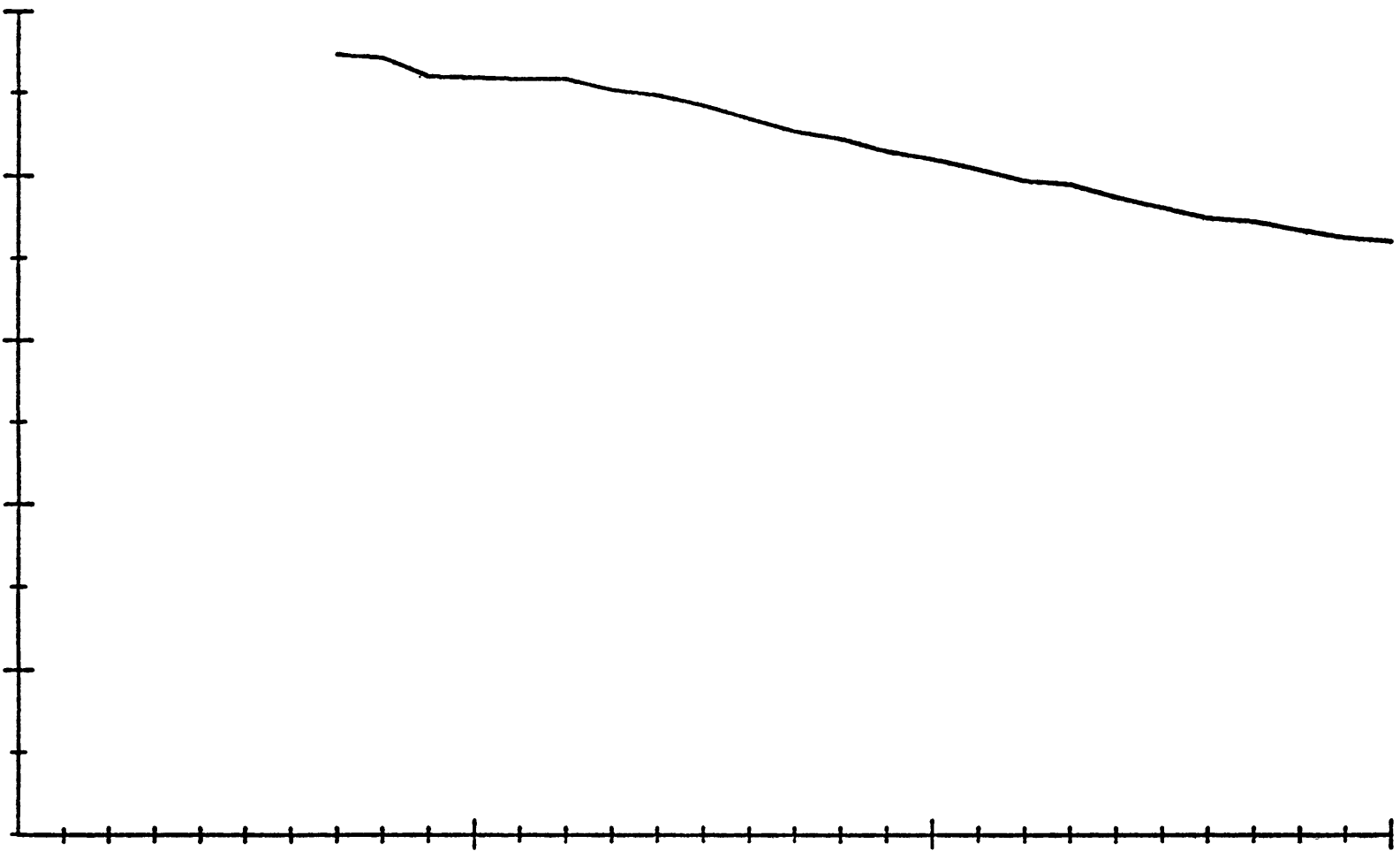
.20

10

20

WAVENUMBER CM-1

Fig 11



JRS021. 21 MAY 75
SAILCLOTHS

POWER TRANSMISSION

1.0

.80

.60

.40

.20

ORANGE
WHITE
BLUE

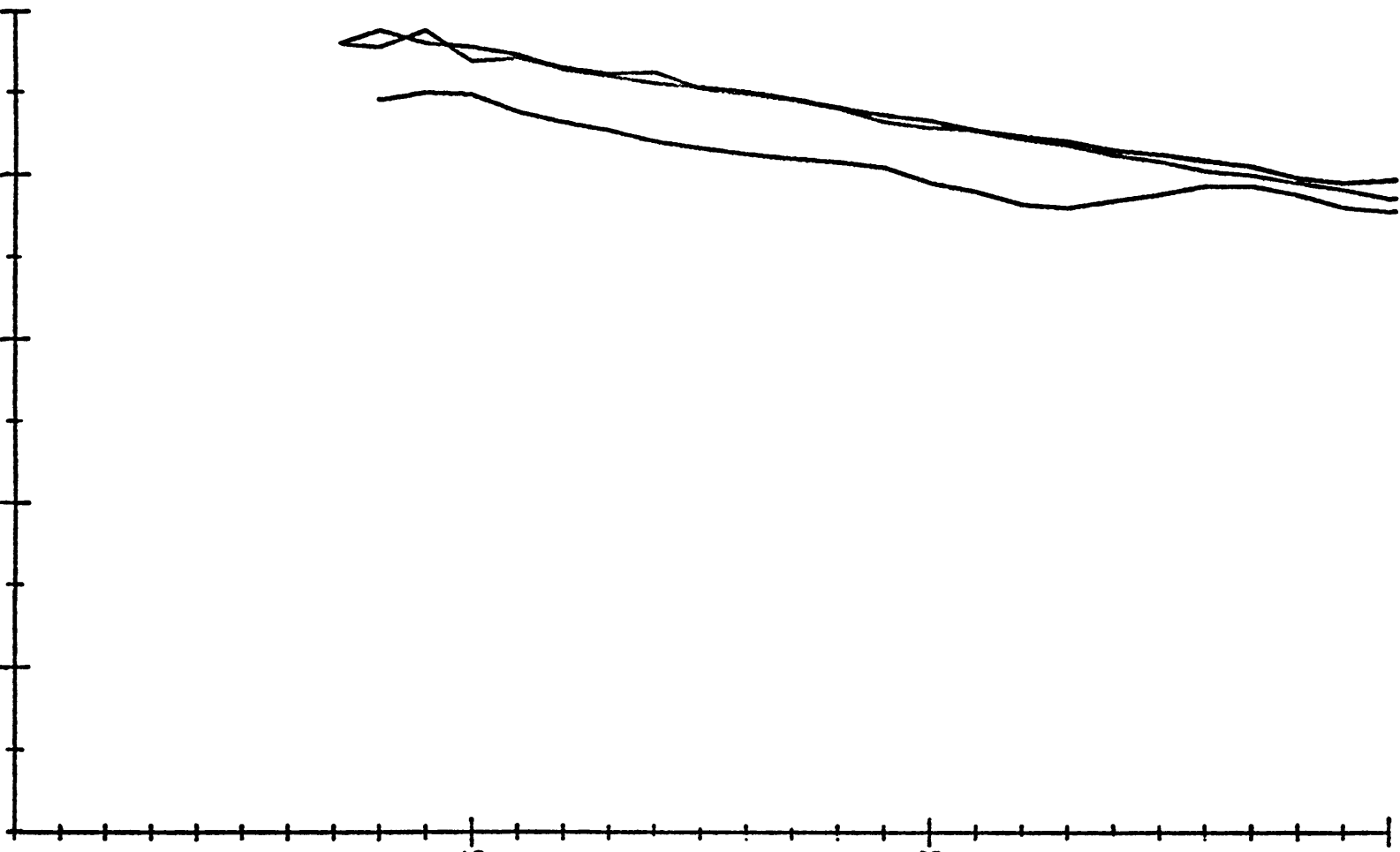


Fig 12

WAVENUMBER CM-1

JRS021 21 MAY 75
PARACHUTE MATERIAL

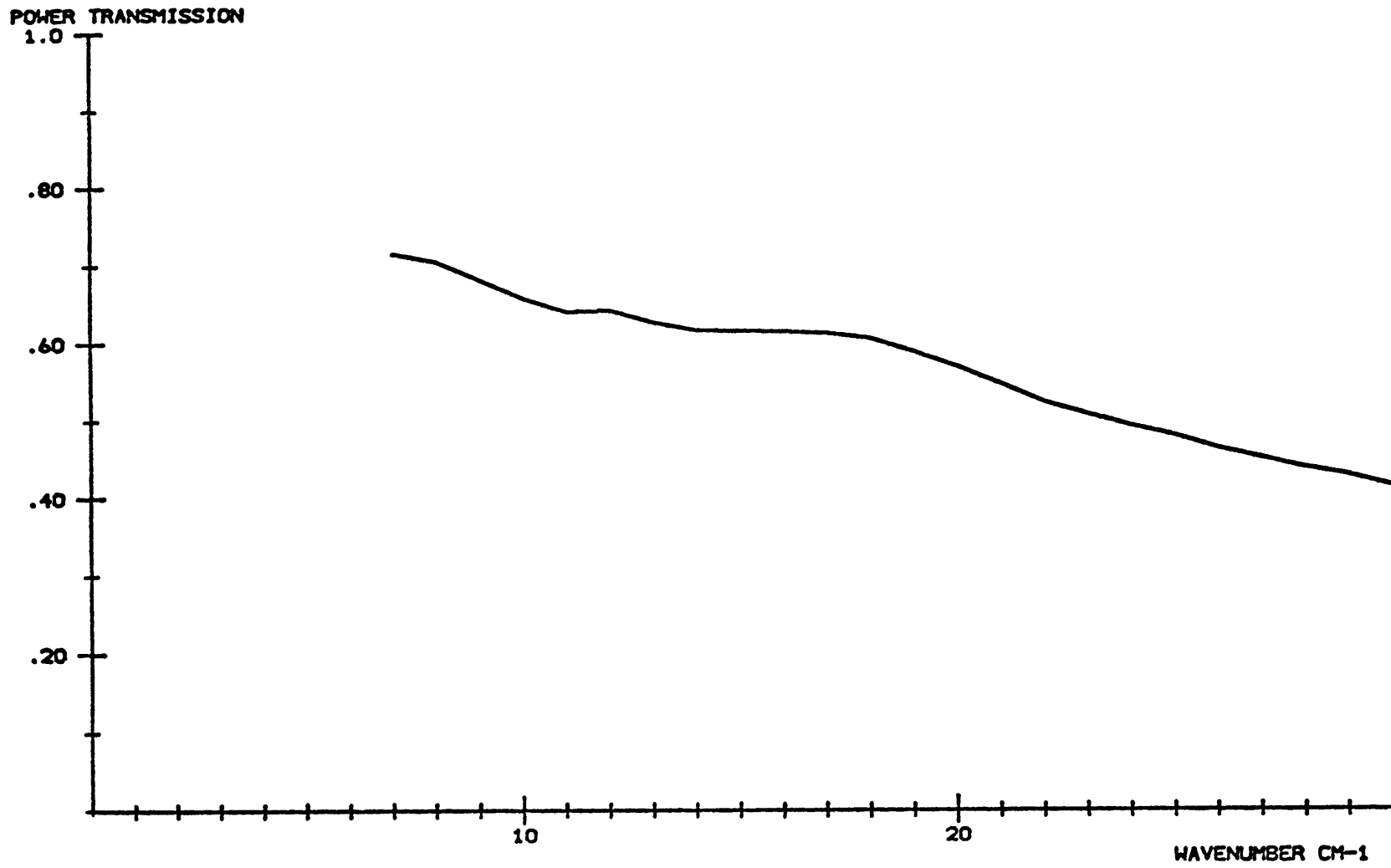


Fig 13

JRS021 21 MAY 75
WHITE COMPOSITE

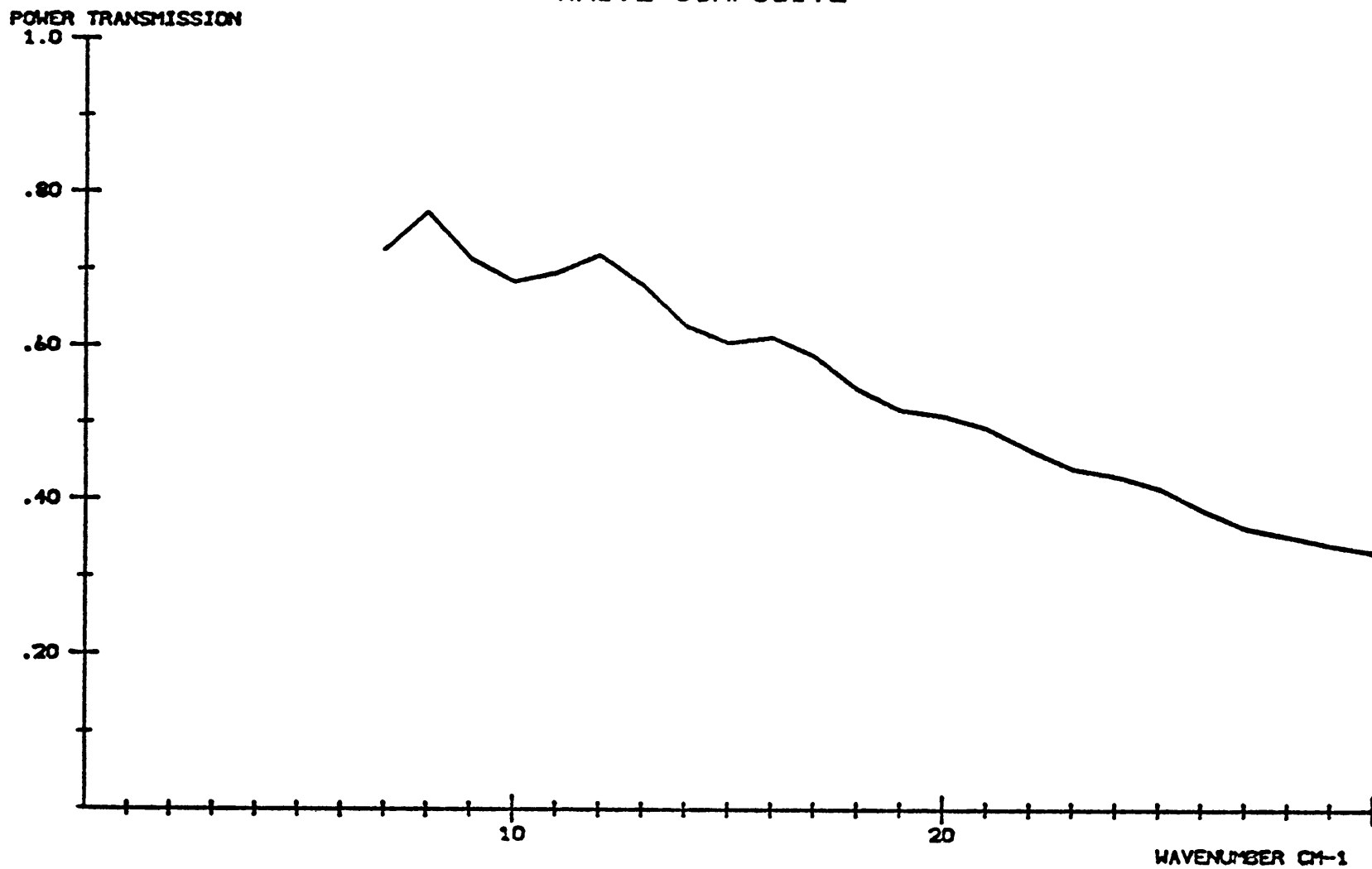


Fig 14.

JRS021 21 MAY 75
REINFORCED GLUED 2-LAYER SANDWICH

POWER TRANSMISSION

1.0

.80

.60

.40

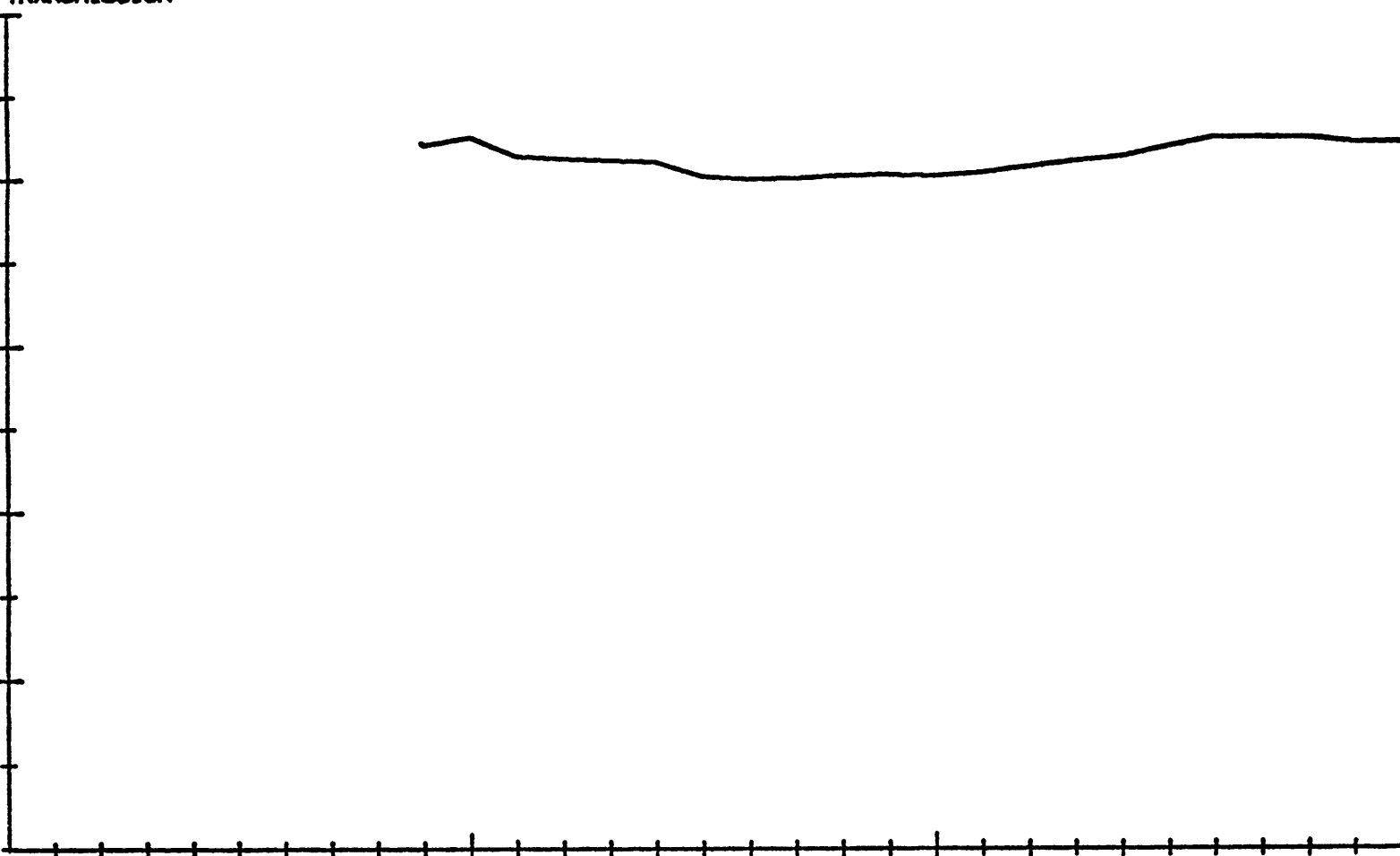
.20

10

20

WAVENUMBER CM-1

Fig 15



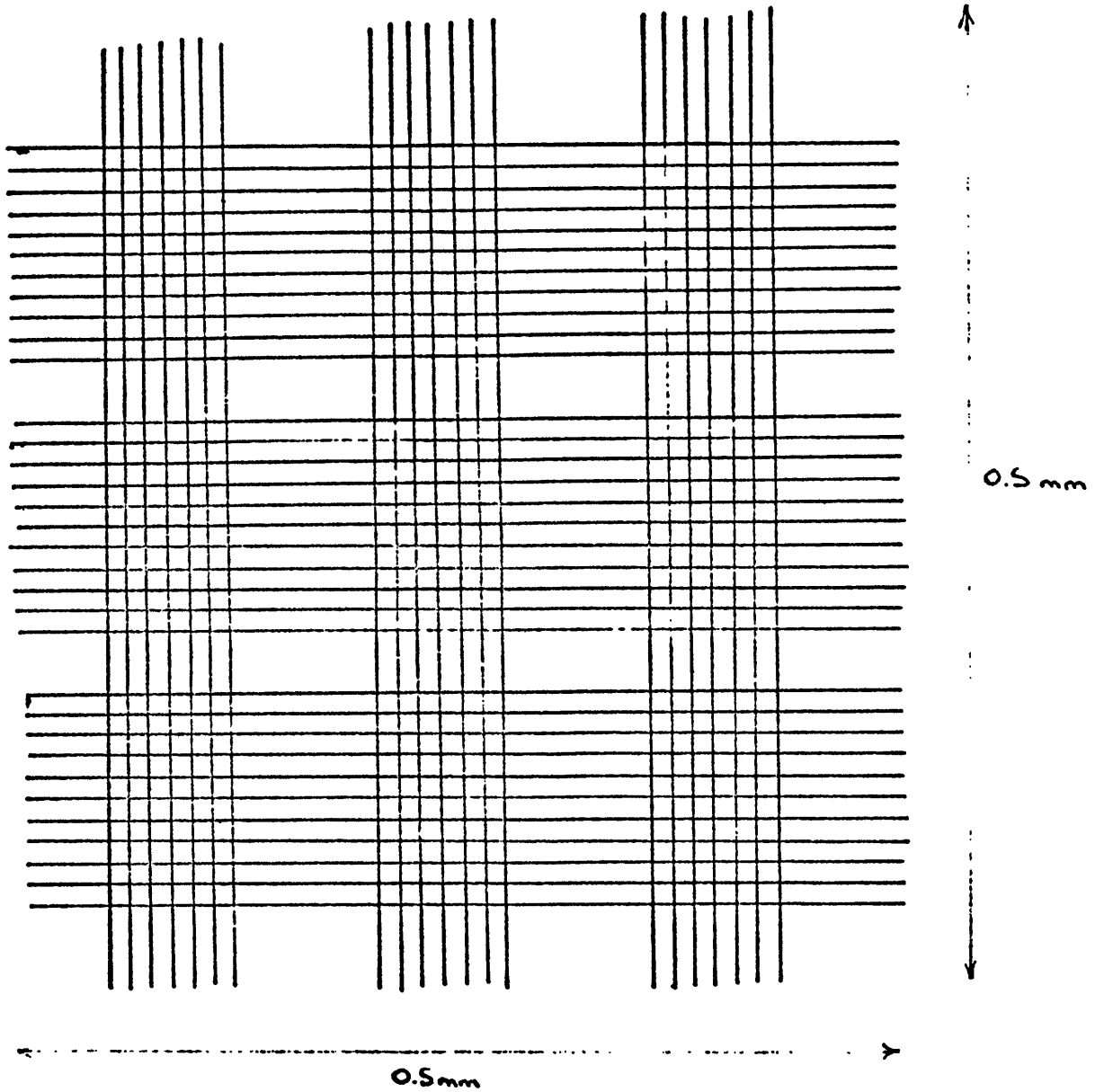


Fig 16. Sailcloth weave. Dimensions are approximate.

The UV Method of Dish Alignment

Description and Outline of Possible Implementation

In this scheme a trolley runs along a straight inclined track from the vertex of the dish to the edge. The trolley carries a probe which measures the distance from the straight line to the surface of the dish. We call this the UV method to distinguish it from other geometries such as X-Y, r- θ etc (see figure 1). It is intended that this method be used only to align and check the surface with the telescope in the zenith position. However, it should be possible to reinstall it and make further checks after the telescope has been in operation for a while. Measurements of an arbitrary set of points on the surface should be possible and a complete survey of the surface (\approx 5 points per panel) would be obtained in a few hours, under computer control. A direct read-out of a single point with the probe positioned on it would be used during adjustment of the surface.

The principal advantages of this method of measurement are:

1. The origin of the measuring system and the position and direction of the axis of symmetry are all defined by components located in the strong part of the telescope near the vertex.
2. The measuring probe is short (\sim 0.7 m) and nearly normal to the telescope surface. This simplifies construction and eases the tolerances on some of the lengths and angles to be measured.

The main disadvantage would appear to be that the track and the probe do not lie in the horizontal and vertical directions, so that slightly more elaborate arrangements of rollers might be needed to ensure smooth motion.

Geometry

We measure the co-ordinates of a point on the dish surface (u,v) in a frame rotated with respect to the axis of the telescope (z) by an angle θ .

The frames are connected by

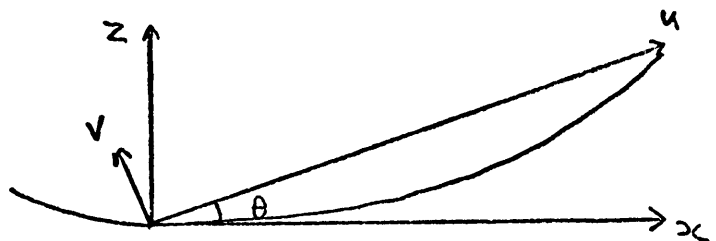
$$u = xc + zs \quad s = \sin\theta$$

$$v = -xs + zc, c = \cos\theta$$

and conversely

$$x = uc - vs$$

$$z = us + vc$$



In general, if we know the desired shape of the surface in the form $z_s = z_s(x)$, we can measure u and v, convert these to x and z and calculate the normal distance

from the required surface

$$\Delta n = \frac{z - z_s}{\left[1 + \left(\frac{x}{2f}\right)^2\right]^{\frac{1}{2}}}$$

In the particular case of a paraboloid, $z = x^2/4f$, one can, after some manipulation, find v as a function of u :

$$v = \{u + 2 \operatorname{cosec}\theta [f - (f + u \tan\theta \sec\theta)^{\frac{1}{2}}]\} \cot\theta$$

Generally the origin of the u, v frame could be offset from the dish vertex but for the present we neglect this and we also assume that the u axis touches the telescope at its edge. We then have $\theta = \tan^{-1} \frac{D}{8f}$ where D is the dish diameter

$$= 19^{\circ}.1 \text{ for } \frac{f}{D} = 0.36$$

Error Analysis

Errors in the measurements will occur due to errors in the lengths u and v , in setting the angle θ and in maintaining the probe which measures v perpendicular to the u -axis. The tolerances allowed are given in Table 1. The values of " Δn allowed" are such that random errors of magnitude Δv , Δu , etc, would each produce a weighted rms measuring error of $14 \mu\text{m}$. If all four had random errors of this magnitude the resultant error would be $28 \mu\text{m}$ (which is unacceptable). It can be seen that, because the u axis runs roughly parallel to the surface of the dish, the accuracy requirement is reduced so it is never worse than $\Delta u/u < 10^{-5}$. Again the short length of the probe measuring v means that $\Delta v/v < 1.6 \times 10^{-5}$. These two effects combine to ease the tolerance on Ψ , the angle of the probe; when v is large the probe is most nearly normal to the surface. The most difficult requirement to meet would appear to be $\Delta\theta < 0.5$ arc sec. It therefore makes sense to ensure that all the other errors are held well below their respective limits to allow most of the error budget for $\Delta\theta$.

The above errors will occur during measurements along a single radius. When a new radial direction is to be measured random shifts in the origin of the $u-v$ measuring frame relative to the dish must be limited to about $30 \mu\text{m}$ in the radial direction and $10 \mu\text{m}$ in the vertical co-ordinate. Again these tolerances seem relatively easy to meet, with precision bearings for example, but the requirement on $\Delta\theta$, which is still ~ 0.5 arc sec, will need special attention.

Finally there are systematic errors which arise if there is uncertainty in the actual value of θ and in the location of the origin of the $u-v$ frame with respect to the axis of rotation. These errors result in a telescope with the wrong shape,

but much of the error can be removed by adjusting the focal length. A fitting program will have to be run to check the errors allowed here but they are probably about 2 arc sec for θ and 50 μm for u .

Implementation

The general scheme is shown in figure 2. A rotating sweep arm has replaced the overhead gantry used in earlier proposals. This substitution is based on the need to keep the measuring machine accurately located relative to the hub of the telescope, rather than to any external datum point. (The use of an independently supported gantry would almost certainly make it necessary to provide servo mechanisms to compensate for movements between it and the telescope.) It might perhaps be necessary to provide some support to the outside of the sweep arm when, for example, one wanted to work on the trolley.

It would obviously be desirable to have an instrument which, once calibrated, gave an absolute read-out of position. That is to say, one could turn on the measuring system with the probe in any position and read its co-ordinates. The machine illustrated in figure 2 attempts to meet this aim by employing mechanical means to measure the lengths u and v . It is believed that the accuracies indicated in the previous section (~ 1 in 10^5) could be achieved. One possibility that can be imagined is the linear equivalent of an optical shaft encoder, with a coded set of lines etched onto the side of a solid bar. These possibilities have not been investigated in detail since alternative solutions described below seem more attractive at present.

The straightness and the direction of the u -axis are defined by a laser beam. According to the manufacturer's claims it should be good enough to use an alignment laser and pentaprism as illustrated in figure 2. However, in view of the doubts about the directional stability of lasers expressed by NPL, a slightly more complex scheme, illustrated in figure 3, would probably be necessary. In this an optical flat replaces the laser as the primary reference for the direction of the telescope axis. By using detectors on the trolley which measure the positions of both the incident and the reflected beams and averaging their outputs, one obtains a reading which is independent of the alignment of the laser. The difference in the readings of these two detectors gives a measure of the laser mis-alignment and the output of a third detector on the pentaprism reads the error in the height of the light beam. A second pair of detectors on the trolley could be used to check tilting of the trolley.

The pentaprism is mounted on a precision bearing and it is the axis and plane of this bearing which define the position of the axis of the telescope and the origin of the z -co-ordinate; only the direction of the axis is referred to the optical flat.

There are two problems with the use of mechanical devices for measuring the lengths u and v ; they are likely to be expensive, and their accuracy is likely to be only just adequate. In particular they will suffer from thermal deformations and will require very careful mounting and handling. We therefore also consider incremental methods of measurement, in particular the HP laser interferometer. This has an intrinsic accuracy of better than 1 part in 10^6 and so can achieve our goal of reducing the errors in u and v to essentially negligible levels. The range of distances to be measured, up to about 30 feet, falls comfortably within the specifications of the instrument (max range 200 ft). However, the laser interferometer is a fringe counting device and can only measure changes in the distances between the optical components (see figure 4). It is therefore essential to provide a means of setting the zero points of the axes and to ensure that the beam is not broken by lateral motion of the trolley or tilting of the laser. The allowable tolerances on misalignment are ± 2.5 mm, which corresponds to about 1 arc minute at the end of the track, so the requirements for engineering the measuring machine are not too severe. It might be necessary to provide a separately supported walkway so that adjustments could be made to the machine without deflecting the track too far. With luck this would only be needed during the setting up of the instrument.

To provide an accurate zero point it would be best to drive the probe back onto the optical flat which defines the dish axis. If this is difficult to arrange, a machined ring, rigidly attached to the hub of the telescope, could be used. An initial set of measurements would be needed to find the orientation of this ring with respect to the optical flat, but the zero point would then be available at any radial position of the arm.

If the laser interferometer is used to make the length measurements, then it seems logical to try to use it for defining the u -axis as well. HP does make a straightness measuring device, which works by measuring the differences in two slant paths to a pair of mirrors (see figure 5). The straight line is defined by the bisector of the angle between the two mirrors. The claimed accuracy is very good - below 0.1 arc sec. There are two problems with using the device as it stands. First, HP makes the straightness interferometer in two versions, short range (0.1 to 3 m) and long range (1 m to 30 m). We need an intermediate range device, say 0.3 to 10 m. Secondly, and more fundamentally, there is no obvious way of keeping the pair of mirrors at a fixed angle to the optical flat with an accuracy of better than 0.5 arc sec. Figure 6 illustrates a possible way round this. We replace the pair of mirrors by viewing the optical flat through a pair of prisms whose deviation angles must be accurately matched to those of the Wollaston prism. This combination is in turn seen through the pentaprism, which defines the angle θ . This

pentaprism would need to be specially made since it would need an entrance aperture about 4 inches by $\frac{1}{2}$ inch. It would probably be best made from two mirrors rather than out of solid glass. In all these optical components it will be necessary to pay attention to their polarising properties since the separation of the two orthogonal polarisations is vital to the performance of the HP instrument.

Finally we must consider how the probe is to contact the surface of the dish. Ideally one would reflect the light directly off the surface panels, but this is ruled out by surface roughness and the gaps between the panels. Experiments with a strip of aluminised mylar, which can be held down to the surface by electro-static attraction, showed that this did provide a sufficiently smooth surface to give a usable fringe in the interferometer, but the laying down and taking up of these strips would be rather awkward. An attractive possibility is to roll a steel sphere over the surface and to focus the laser beam on the back of the sphere (figure 7a). Slightly less attractive is a probe with a small retro-reflector attached (figure 7b). The probe tip would have to slide over the surface in this case. If neither of these works then the use of an electronic probe and normal corner reflector (figure 7c) seems practical, although slightly more complex.

R E Hills

9 March 1976

Addendum

I am indebted to Dr D M A Wilson for pointing out that, although one cannot expect a single bearing to keep the rotating and fixed parts parallel to better than a second of arc, one might be able to achieve this with a rigid shaft and two well separated bearings (figure 8). The line through the centres of the bearings then defines both the position and direction of the telescope axis. It should then be possible to mount the HP "straight-edge" on this shaft so that the pentaprism, prisms and optical flat of figure 6 are no longer needed. One is left with the problem of not having a version of the interferometer device to cover the range of distances needed.

In order to achieve an accuracy of 0.3 arc seconds one would need bearings accurate to 1 micro-metre on a shaft 1 metre long. This is a stiff requirement and experiments will be needed to see if it can be achieved.

Table 1

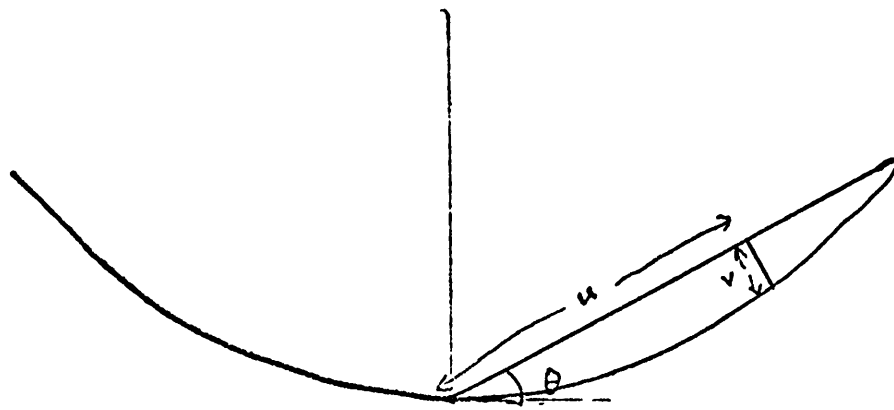
x(m)	v(m)	$\theta - \theta_n^\circ$	Δn allowed (μm)	$\Delta v(\mu\text{m})$	$\Delta u(\mu\text{m})$	$\Delta\theta''$	$\Delta\Psi''$
0.5	.153	16.4	10	10	35	4.1	47
1	.284	13.8	10	10	42	2.1	31
1.5	.393	11.2	10	10	51	1.3	27
2	.481	8.6	10	10	67	1.0	29
2.5	.546	6.1	11	11	104	0.9	39
3	.590	3.6	11	11	175	0.75	61
4	.612	1.2	11	11	525	0.57	177
5	.546	5.7	13	13	130	0.53	49
6	.393	10.0	15	15	81	0.51	43
7	.153	13.8	18	19	75	0.52	101
7.5	0	15.6	22	23	81	0.59	-

Notes

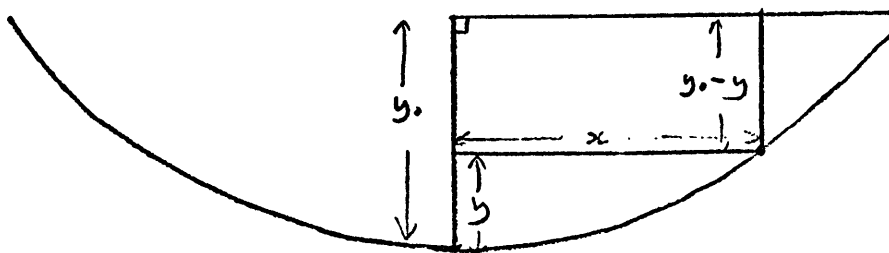
The quantities Δv , Δu , $\Delta\theta$ and $\Delta\Psi$ are the errors in v , n , θ and Ψ which would each produce the measuring error " Δn allowed" normal to the surface. Ψ is the angle between the probe which measures v and the u -axis; it should be a right-angle. Expressions are

$$v = -xs + x^2c/4f \quad \theta = \tan^{-1} \frac{\Delta}{8f} \quad \theta_n = \tan^{-1} \frac{x}{2f}$$

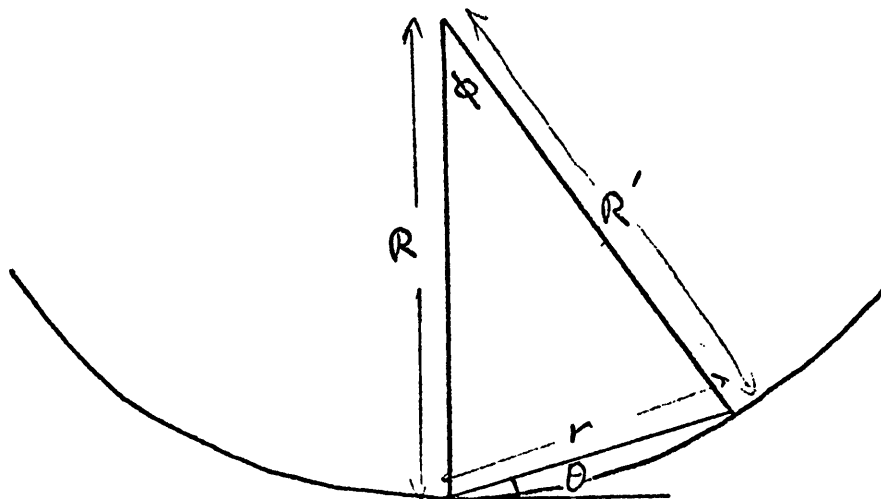
$$\begin{aligned} \Delta v &= \Delta n \sec(\theta - \theta_n) & \Delta u &= \Delta n \operatorname{cosec}(\theta - \theta_n) \\ \Delta\theta &= \Delta n / (2f + z) \sin\theta_n & \Delta\Psi &= \Delta n \operatorname{cosec}(\theta - \theta_n) / v \end{aligned}$$



U-V method



X-Y method



$r-\theta$, RR' , r , RQR' etc methods

Figure 1 Geometry for some measuring methods

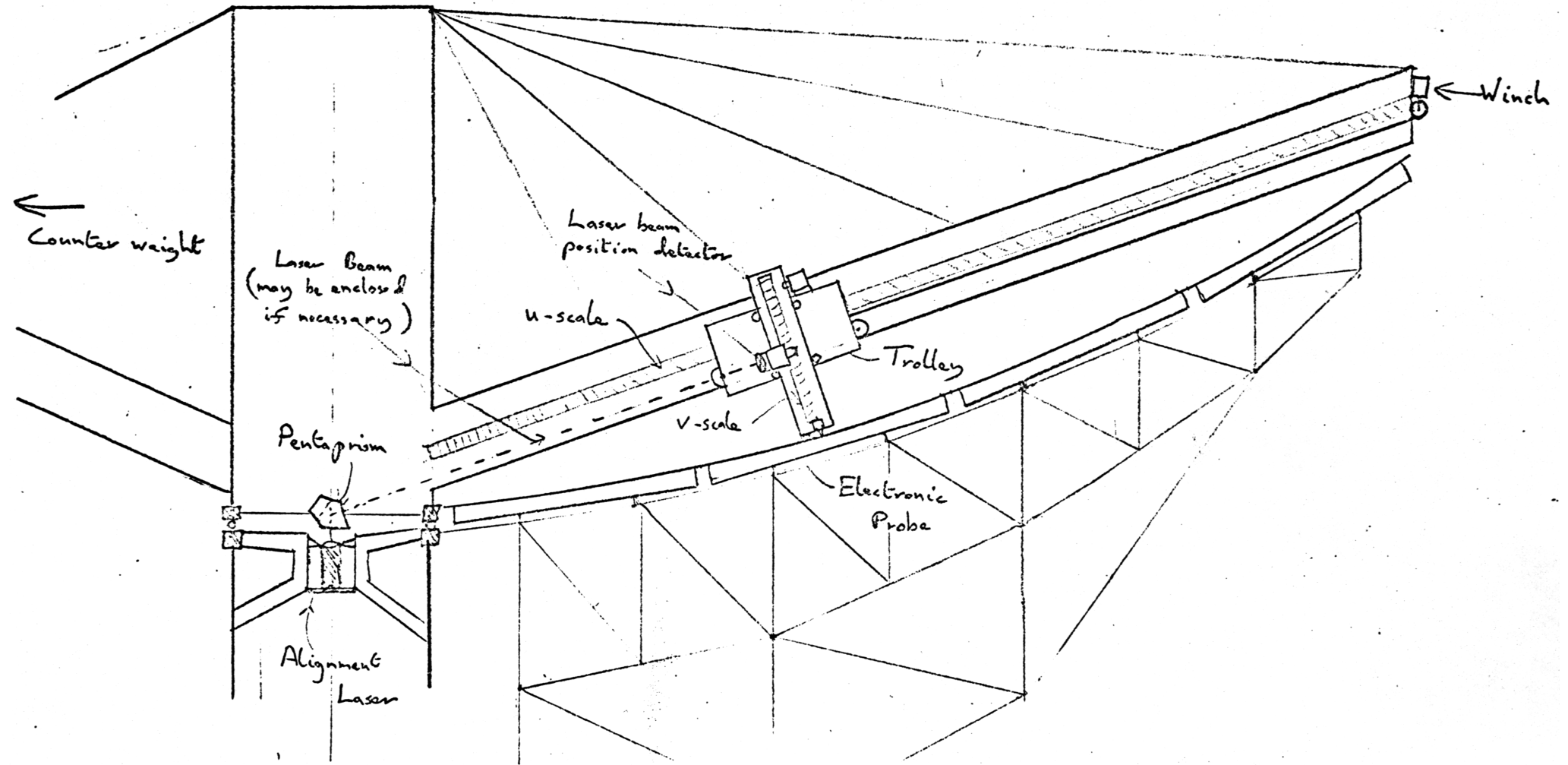
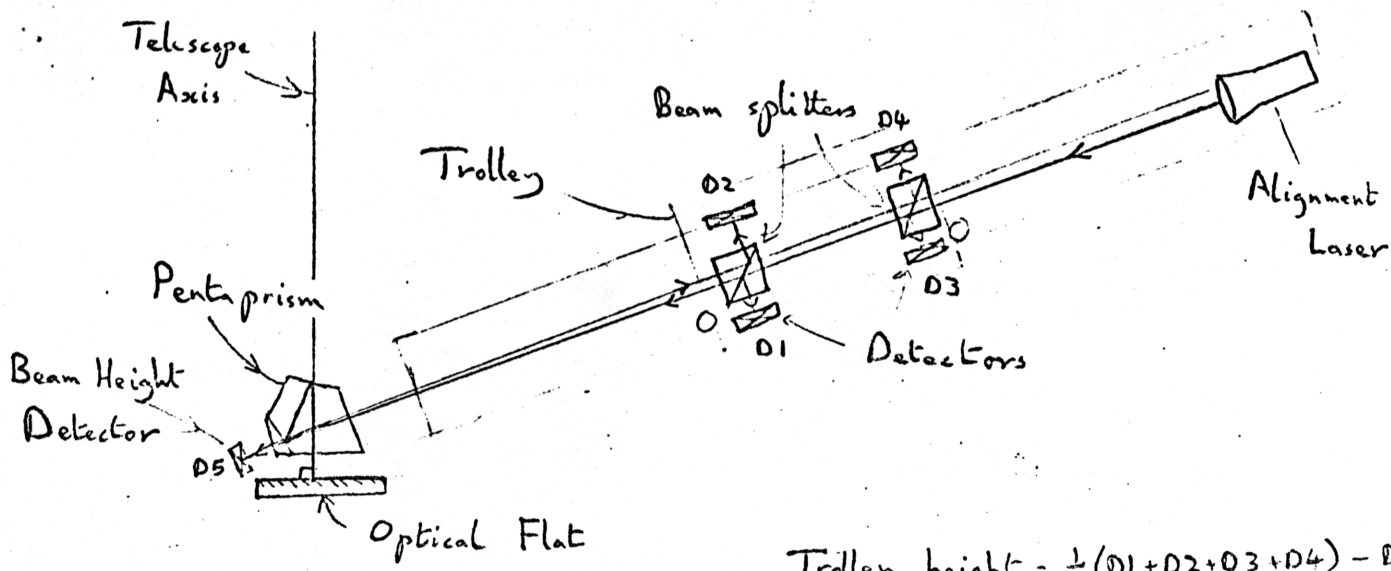


Figure 2 General Layout of U-V scheme.



$$\begin{aligned} \text{Trolley height} &= \frac{1}{4}(D1 + D2 + D3 + D4) - D5 \\ \text{Trolley slant} &= \frac{1}{2}(D1 + D2 - D3 - D4) \\ \text{Laser Misalignment} &= \frac{1}{2}(D1 - D2 + D3 - D4) \end{aligned}$$

Figure 3 Use of Optical Flat to Define Telescope Axis.

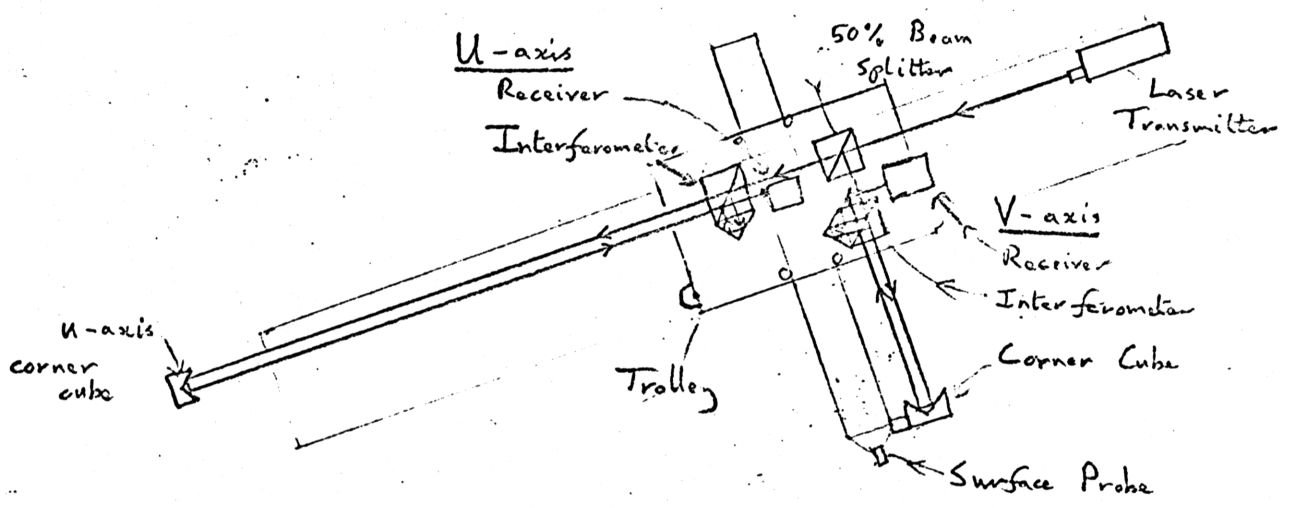


Figure 4 Use of Hewlett Packard Laser Interferometer to measure U and V.

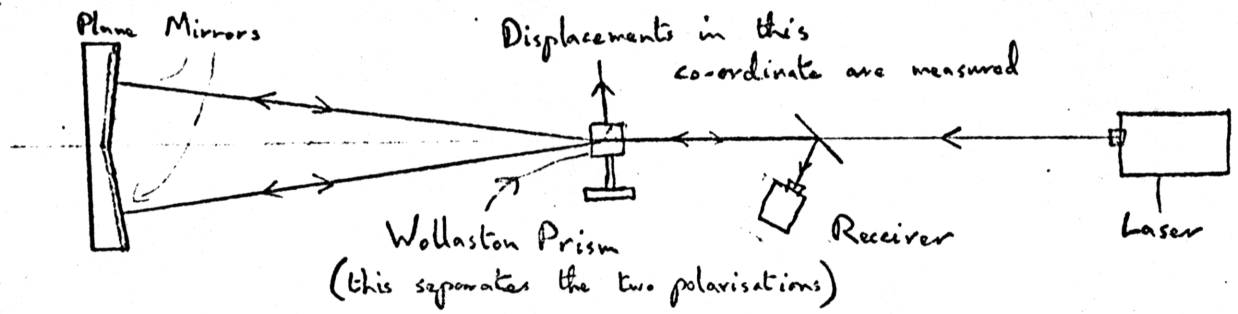


Figure 5 HP Straightness Measuring Device

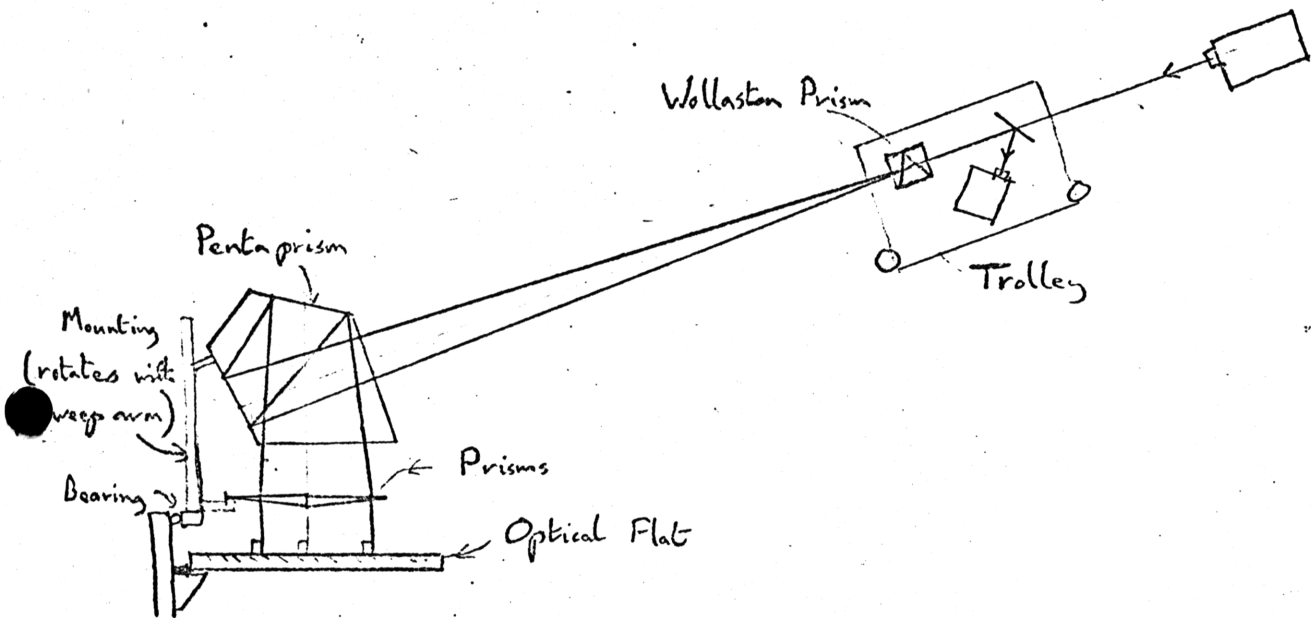


Figure 6 Modification to Relate V-axis to optical flat

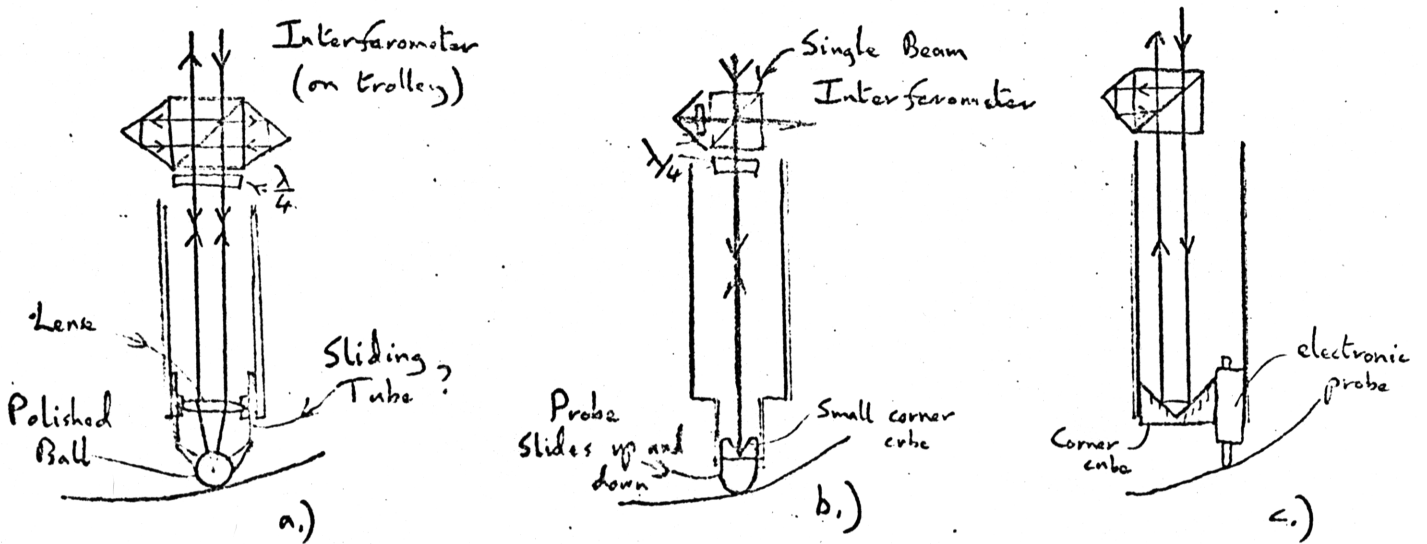


Figure 7 Details of V-axis measurement

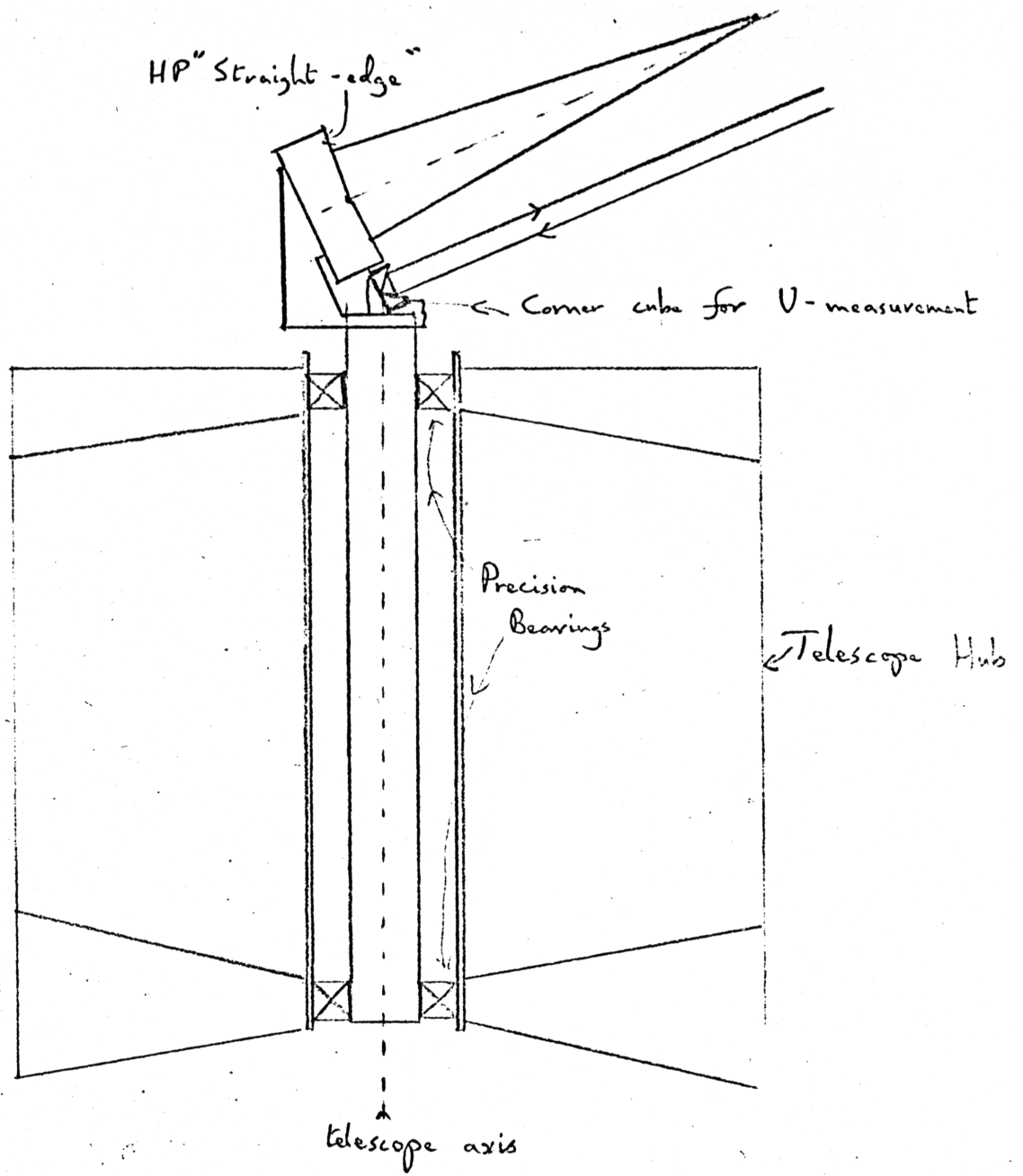


Figure 8 Use of two bearings to define axis

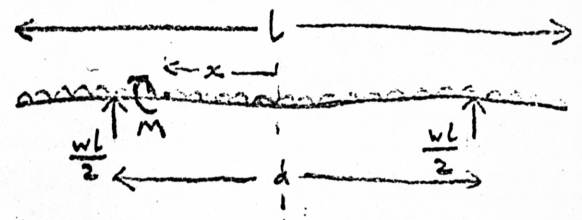
A note on selection of support points for surface panels
and on their deflection under the weight of a measuring trolley

On most telescopes the surface panels are approximately rectangular and are supported at their four corners. This has obvious advantages from the point of view of economy in the supporting structure and it also minimises any tendency for the panels to twist when buffeted by wind. For a protected high precision dish, however, the deformations of the panels under their own weight become more important and the question arises as to whether other points of support should be used. We here use a very simple approximation to estimate the advantage that might be gained. The same simple model is also used to calculate the effect of panel deformations on the method of measuring the shape of the surface described by Findlay and Payne (Rev.Sci.Instruments, 47, 50, 1976 January) which uses a trolley driven over the surface.

Unless the panels are almost square most of the sag will occur along their length rather than across their width, so that we can represent them as a simple beam supported at two points. We chose these to be equidistant from the centre. If the panel is fairly thin (say thickness $< 1/10$ length) we can concentrate on the effect of bending and neglect that of shear. One then proceeds in the usual way by taking moments about a point at a distance x from the centre of the beam.

$$M = -\frac{w}{2}\left(\frac{l}{2} - x\right)^2 \quad x > \frac{a}{2}$$

$$M = \frac{wl}{2}\left(\frac{a}{2} - x\right) - \frac{w}{2}\left(\frac{l}{2} - x\right)^2 \quad x \leq \frac{a}{2}$$



where l is the length of the beam, w the weight per unit length and a is the separation of the supports. By relating the moment to the local curvature:

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad , \text{ in which } E \text{ is Young's modulus and } I \text{ the second moment of}$$

area of the cross section, and the integrating twice one obtains the deflected shape

$$y = -\frac{wl^4}{384 EI} d(x, a)$$

where the dimensionless geometric factor d has the form

$$d = \left(\frac{2x}{l}\right)^4 - 4\left(\frac{2x-a}{l}\right)^3 + 6\left(1 - \frac{2a}{l}\right)\left(\frac{2x}{l}\right)^2 \quad x > \frac{a}{2}$$

$$= \left(\frac{2x}{l}\right)^4 + 6\left(1 - \frac{2a}{l}\right)\left(\frac{2x}{l}\right)^2 \quad x \leq \frac{a}{2}$$

Note that the deflection scales as the fourth power of the length, which is why

there is a fairly sharp upper limit to the possible size of panels. Generally it will also be inversely proportional to the square of the thickness, and with our assumptions independent of the width. The form of d is shown in the figure for various positions of the support.

At the end of the beam the value of the deflection relative to the centre is

$$d_e = d(x = \frac{l}{2}) = 3 - 12(\frac{a}{l})^2 + 4(\frac{a}{l})^3$$

for support at the end ($a = l$), $d_e = -5$, at the centre ($a = 0$), $d_e = 3$, and for support at the mid-points ($a = l/2$), $d_e = \frac{1}{2}$.

The minimum value of the downward deflection is obtained when the deflection at the end equals that in the centre $d_e = 0$. This is found by trial and error to occur when $a = 0.554l$. The highest point is then at $2x = 0.57l$ and has the value $d = -0.105$.

Thus one could in principle obtain a peak deflection which is nearly 50 times smaller than that of a panel of the same length supported at the ends. Alternatively one could make the panels twice as long and still have about 3 times less peak deflection. In practice of course the simplifying assumptions we have made do not justify these claims. However, it is probably reasonable to suppose that a factor 10 reduction in deflection could be obtained by supporting the panels at 2 points about midway from the centre to the edge. Thus we could probably go from 4 rings of panels 1.8 metres long, requiring 5 rings of support points, to 3 rings of panels 2.4 metres long requiring 6 supporting rings.

A numerical calculation of the rms deviations of the shapes has been made and this shows that a similar gain in the rms deviation is also found. Except for values of a/l close to 0.55 the rms deviation is about 0.3 times the peak.

A final point on this topic is that if the telescope is to behave homologously then ideally the surface panels should deflect in the same sense as the telescope. The telescope is supported in the centre, so that its focal length tends to increase when it points at the zenith. Panels supported at their ends are the worst choice in this respect, since they deflect in the sense which reduces their focal length. By fitting a parabola to the form of $d(x)$ it was found that in principle it should be possible to gain about an order of magnitude by choosing the deflections of the panel to match those of the dish. This analysis would suggest supporting the weight of the panel at a point as near as possible to its centre of gravity and having only fine adjusting screws at its edge.

Deflection due to a measuring trolley

For the rest of this note we assume that the above speculations come to nothing and consider only panels supported at their ends. In this case the largest deflection occurs when the trolley is in the middle of the panel and it amounts to

$$y_T = - \frac{W_T \ell^3}{48 EI} = \frac{8 W_T}{5 W_P} y_P$$

where W_T is the weight of the trolley, W_P that of the panel and y_P is the deflection of the panel under its own weight. By itself this would not be serious provided the weight of the trolley was small compared to that of the panel. However, with the Findlay-Payne type of trolley, which measures the local curvature of the surface and integrates twice to obtain the shape, these deflections produce a cumulative systematic error. This is a result of the additional curvature caused by the sag of the panels which is always in the same sense.

If the trolley is short compared to the panel the additional curvature is just

$$\frac{d^2 y}{dx^2} = \frac{W_T}{YI} \frac{x(\ell - x)}{\ell}$$

When the trolley moves from one end to the other and the shape is calculated the error will be

$$y|_{x=\ell} = \frac{W_T \ell^3}{12 EI} = 4 y_T$$

Furthermore the angle of slope is then in error by

$$\left. \frac{dy}{dx} \right|_{x=\ell} = \frac{W_T \ell^2}{6 EI}$$

This leads to a further error so that after the trolley has traversed a second panel the position error is

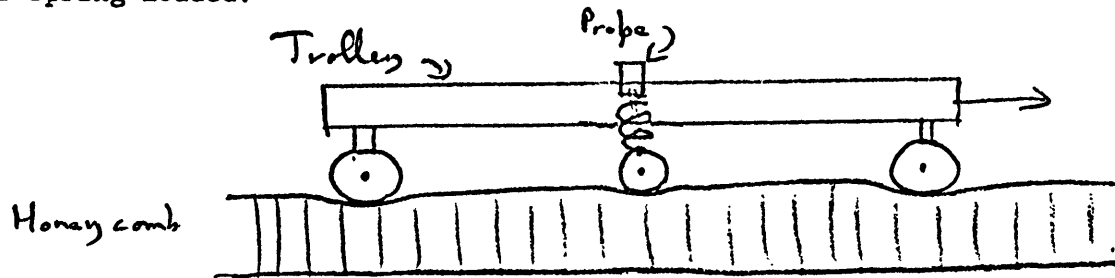
$$y|_{x=2\ell} = 16 y_T$$

These errors continue to build up as the square of the number of panels

$$y|_{x=n\ell} = 4n^2 y_T = \frac{32 n^2 W_T}{5 W_P} y_P$$

On a large dish it will therefore be necessary to keep the trolley very light. It should be noticed, however, that the error is a parabolic one so that, to the extent that all the panels are the same and the deformations are as calculated, the result is only a change in focal length. It would also be possible to account for these deformations by running the measurements twice with weights added to the trolley for the second run. The difference in the runs would provide the correction to

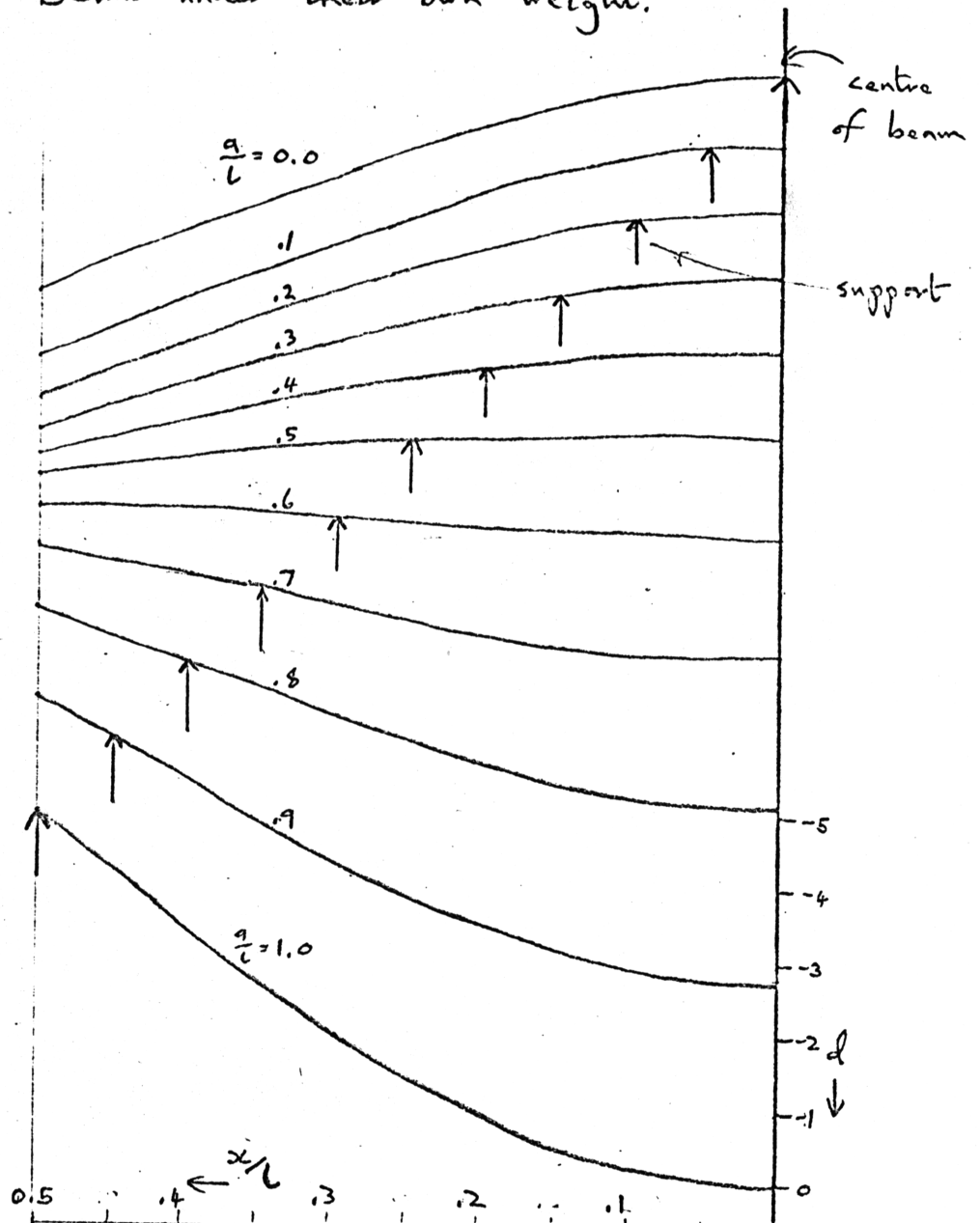
be made for deformations. One would still have to be careful that the load on each of the wheels was increased in the same proportion because there may be local deformations in the surface, particularly with honeycomb panels. It is hard to see how this controlled increase in loading could be easily achieved if the centre wheel is spring loaded.



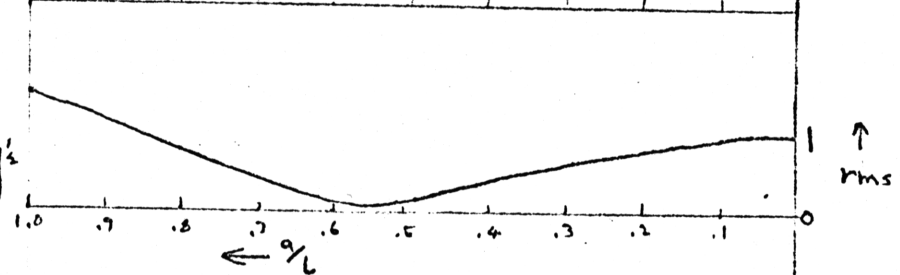
R E Hills
D B Shenton
7 April 1976

Bending of Beams under their own weight.

→
the function $d(x)$
for various values of
 a .



→
rms value of d
 $= [\langle d^2 \rangle - \langle d \rangle^2]^{1/2}$
as a function of a .



rms residual from
a parabolic fit
 $= [\langle (d - bx^2 + c)^2 \rangle]^{1/2}$
 b and c chosen
by least squares.

