25-Meter Millimeter Wave Telescope
Memo #56

SURFACE SHAPE BY DISTANCE MEASURING

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1. Introduction

It may be that the measurement accuracy achievable by the trolley (Payne et al. Rev. Sci. Instr. 47, 50-55, 1976) will fall short of the accuracy needed to set the 25-m telescope. If this proves to be the case, we should be ready with either another system, or a support system for the task. In the present note we will consider the usefulness of the modulated laser distance measurer (LDM) (Payne, Rev. Sci. Instr. 44, 304-306, 1973*) in support of the trolley.

We do not reject the use of the LDM alone, but we do not, for the present, wish to forego the speed and practical simplicity of the trolley. In this present note we consider this accuracy likely to be achieved by using the LDM and its probable speed. We conclude with a note on what to do next.

2. The Accuracy Obtainable with the LDM

The LDM described and tested by Payne was designed for measuring a 65-m antenna. We will analyze the accuracy obtainable if the antenna size were 25 m. We suggest that the greatest range to be measured would be about 25 m. This is the range from the parabola vertex, via the focal point to a point on the edge of the dish (see Appendix 1).

(a) Atmospheric effects. Following Payne, we can say that to measure range over 25 m to $\pm$ 0.025 mm we need to know the air pressure to 2.4 mm of mercury and the air temperature to 0.9° C. The effects of relative humidity are unimportant.

Thus we conclude that we shall have to monitor air temperature and pressure, but that their effects can be allowed for.

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* Hereafter referred to as Payne.

(b) Phase measurement. In Payne, an accuracy of phase angle measurement of $\pm$ 0.10° at the final measurement frequency was achieved. In Payne's case this corresponded to a range accuracy of $\pm$ 0.08 mm.

We propose to increase the laser modulation frequency by a factor of 10. (We discuss this later in more detail.) There is no obvious reason why the phase accuracy should change, so that our range accuracy should be about $\pm$ 0.008 mm.

(c) System accuracy. It thus appears that the limits of range accuracy will be largely set by the geometry and method we adopt to measure the surface. Thus we turn next to this table.

3. A Typical Measurement System and Its Accuracy

Let us choose to measure distances VMP and VFP (Appendix 1) in order to measure the surface. We will not postulate at this stage that such a system is optimum, but it will serve as a vehicle to carry the discussion of accuracy. It will, however, be fairly near optimum since it has the desirable characteristics:

- The LDM can be mounted in a fixed position on the dish axis at or near its vertex.
- The two distances measured VMP and VFP are clearly closely related, each almost independently of the other to x and y.

We assume that P is defined by a carefully drilled hole in the reflector surface. This hole will carry one of two corner cube reflectors. Rotatable, tiltable mirror reflectors will be at M and F. F will be carried on the subreflector support legs, at a point near the actual focal point.

A list of questions which need answers show how the system accuracy can be assessed:

(a) How stable is the LDM zero point with time?

(b) How will path lengths change as a reflecting mirror is rotated and tilted?

(c) How stable will the position of F be?

(d) How well can we relate the point of reflexion in a corner cube to the reference point at P which we are measuring?

(e) Does the measurement of VMP and VFP lead satisfactorily to a measure of (x,y) of P?

We now consider these questions one by one.

(a) Zero point stability. Payne measured this (including atmospheric effects) over one hour and a 60 m range. He found a 1σ RMS of $\pm$ 0.043 mm. We thus suggest that a new instrument over a shorter range will be satisfactory. However, the point will be checked when a new LDM is built.

(b) Mirror tilt and rotation

(i) Consider a surface-silvered plane mirror rotated about Oy and tilted at an angle (ϕ) to Oy (Fig. 1). Let d be the distance from the surface to the point of rotation.

The change in path length of a ray reflected from the mirror as ϕ goes from 0° to ϕ is given by

$\Delta(\text{path}) = 2d\,(\sec\phi - 1)$.

To keep this small, for $\phi = 30.76^{\circ}$ which corresponds to the edge of the dish ray, set $\Delta(\text{path}) = 2\text{x}\Delta(\text{length}) = 0.02$ mm.
Hence $d \leq 0.06$ mm ($2.5\text{x}10^{-3}$ inch).

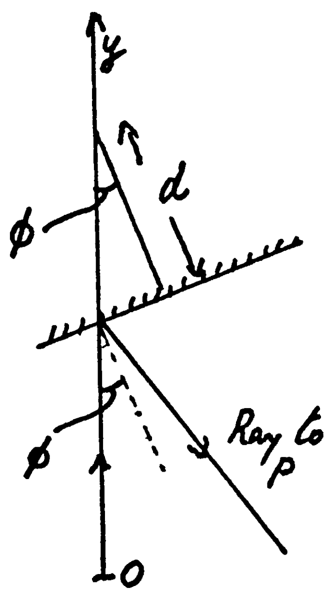

Fig. 1

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Thus the mirrors must tilt about an axis which lies within 0.06 mm of their reflector surface. This is quite easy to do.

(ii) Mirror rotation

To work a ring of target points P the mirror must rotate about Oy. However, in practice Oy will be the fixed axis of the LDM in its mounting. A mirror (specially the one near F) will rotate about an axis which will move if the subreflector support moves. Consider, therefore, the situation as shown in Fig. 2 will occur. The incident ray from the LDM follows Oy. It strikes the mirror M and is reflected to P. We must assume that the tilt of M has been set correctly to give the light to P. We show the rotation axis RR' of M as being moved away from Oy. Let the distance it has moved be d. Then the change in path is

$$\Delta(\text{path}) = d \tan\phi + d \sin\phi .$$

So $\Delta(\text{length}) \doteqdot d \tan \phi$.

Since $\phi \simeq 30^{\circ}$ this gives $\Delta(\text{length}) \sim d/2$.

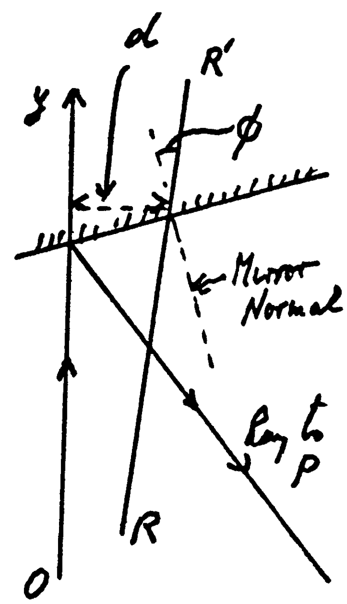

Fig. 2

This effect is by no means negligible. However, it can (and must) be taken care of as part of (c), the stability of the subreflector support.

<u>(c) Movements of F due to Subreflector Support Movements</u>. We will only consider movements of the point F which are slow (taking perhaps an hour to go one mm). Such movements can be separated into $(\delta F)_x$ and $(\delta F)_y$ motions.

We can measure both by the following observing technique.

Two rings of targets are used*. One (the inner) is a ring of 4 or 8 targets near V. The other is the "edge" ring--perhaps 24 in number corresponding to the 24 homologous edge points of the 25-m design.

The principle we shall propose is based on:

(i) Measurements of the inner ring show changes in the y position of F only.

(ii) Measurements of the edge ring show changes in the x position of F. (The y change in F is already known from (i).) The x position is found by requiring that the variations in range to the edge targets is the result of the x-motion of F.

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* Instead of the inner ring the LDM could be set to measure distances to the mirror F, set normal to Oy.

Use of this technique alone amounts to saying that the 24 edge targets remain in a plane normal to Oy (the LDM axis) and that changes in their ranges are due to movements of F.

It would, of course, be possible to measure or correct for the x-movements of F by having on the mirror at F a quadrant-detector. This needs further study; it might be valuable.

(d) Corner-cube design and errors. Our present choice would be to have two corner cubes for each target point P. Both would interchangably plug into the target hole. The first cube would be used in measuring the path VMP--let us first describe these measurements.

(i) Measuring VMP and thus x - First, we assume that all target point holes have been drilled and measured by a "tape and transit" method. Thus the surface distances and elevation angles are known. We will assume that these elevation angles are accurate and reproducible to 5 arc seconds. Then a measure of VMP can be converted to x by

$$x = (\mathrm{VMP}-\ell_M) \cos \alpha \ ,$$

where ℓ_M is the LDM zero point distance to M and α is the elevation angle of P from M. The error in x due to a 5 arc second uncertainty in α is approximately: $(\delta x)_\alpha \doteqdot x \sin \alpha \times \sin (5'')$. We choose M at a point about 2 m above the vertex V. Then the greatest value of α at the edge is 7.8°. The error in x at the edge is then 0.04 mm, which is acceptable. We therefore conclude that x can be derived from measures of VMP if ℓ_M is known.

We can measure ℓ_M at any time by setting the mirror M normal to the LDM beam.

We propose to measure x values to all target rings, and probably to do these measurements at a range of different ambient temperatures. We thus should have x values for all targets as a function of ambient temperature of the telescope.

We sketch the corner-cube design in Fig. 3 (Figure 3 is 2-dimensional only), which we would use for x-measures.

The vertex of the cube is exactly ($\pm$0.02 mm) above the center of the pin. The cube is made by injection-molding plastic--the surfaces are surface silvered by vacuum sputtering. (It will be remembered that the path length in and out of a cube is the same as the path to the vertex.) Thus, to get the geometry right, h_o must also be known to about $\pm$ 0.02 mm.

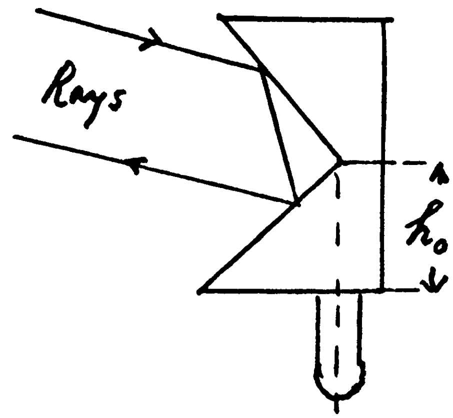


Fig. 3

(About 2x actual size)

(ii) Measuring FP and thus y. We measure FP as the difference

$$FP = (VFP - \ell_F)$$

where ℓ_F is the distance from the zero-point of the LDM to F. This is measured just as ℓ_M was measured, by direct reflexion from F set normal to the beam.

We now have x and FP for the points to be measured. The corner-cube needed to measure VFP is shown in Fig. 4 (2-dimensions only).

Again, the vertex should be closely above the pin center and h_o should be known to 0.02 mm.

It is an interesting (fairly simple) problem to devise a method by which corner-cubes could be measured to see that they are manufactured to the required accuracy.

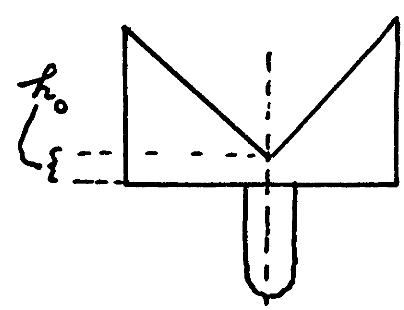


Fig. 4

(e) Deriving the surface errors from the measurements. To derive y from the measured FP values we suggest the following. First let us suppose the mirror F is at a distance f' from the vertex, where f' is not equal to the focal length f.

As has been described, the x value (x_P) is already measured for all points P. Then any measure of FP, called $(FP)_M$, satisfies:

$$(FP)_M^2 = x_P^2 + (f'-y_P)^2$$

where y_P is the actual y value of P.

If all points P lay on a perfect surface of focal length f the FP values would be $(FP)_D$ and would satisfy:

$$(FP)^2 = x_P^2 + (f-y_D)^2$$

where y_D is the value of y which satisfies

$$x^2 = 4fy.$$

We therefore compute $(FP)_D$ for every point and then best-fit all points by varying f' until $(FP)_M^2 - (FP)_D^2$ is a minimum. Having found the best fit values of f' we can then find the values of $\Delta y = y_P - y_D$ and proceed.

4. Some Practical Considerations

(a) Overall accuracy. The previous discussion suggests that of the several sources of error there is no single one where the range measure error should be greater than (say) $\pm$ 0.02 mm. A more careful error budget assessment is needed, but it does seem possible that the method could meet our needs for accuracy.

(b) Speed. Speed requires adequate light and quick control of the steering mirrors and quick data recording. In Payne, 2.7 seconds per point was the measurement cycle. A good small computer could read and store the data in milliseconds. The mirrors have to be positioned (in 2 angular coordinates) to about 1 arc minute (an error of 5 mm at 12.5 meters). These movements should also be computer controlled and should take about 1 second for each axis. Thus 1 target in 5 seconds (720 per hour) seems reasonable. Thus it is too early to decide whether the system should be an add-on to the trolley or should stand alone.

(c) Structure stability. We are assuming, in the measurement scenario described, that the x and y measures are made separately. Runs to measure x would be made under various stable temperature conditions (which would be measured to 0.1° C). An x value of 12.5 m on a steel structure will change by 0.13 mm/$^{\circ}$C so that it will be possible to measure the repeatability of the x-measures as a function of temperature.

A set of y-measures will then be associated with the correct (same temperature) set of x-measures.

Of course, if the structure does not behave well as temperature changes, nor will it behave well as a mm-wave telescope.

5. Things to Do

(a) Review the method. This is a first shot at defining the method of measurement. Comments (particularly on the ways of taking and handling the data, where the discussion is not good) would be welcome.

(b) Design a new modulated laser distance measurer. Much of Payne's computing and display circuitry is probably still usable. The move from 500 MHz to 5 MHz as a modulation frequency is well within the present-day practice. See for example the article on an 11 GHz Lithuim Tantalate modulator (Standley and Mandeville, App. Optics 10, 1022-23, 1971). However, how do we get a modulator?

The first steps of the mechanical design have already been taken. The main problem will be to ensure structural rigidity (to 0.01 mm) and to avoid the RF from the modulator (watts) leading into the photo diode receiver (where the level is nanowatts or less). Both Payne and JWF are needed here, plus some engineering.

(c) Design and build a mirror rotator and tilter. One will do to begin with. It can be used separately, in tests, at M and at F. This present note gives an adequate design specification. It must be computer controllable to $\pm$ 1 arc minute (16-bit encoders) and settle down in 1 second. Payne can do the servo; who can do the mechanical design?

(d) Find and test a source of corner-cubes. For tests we can use commercial glass cubes. However, we should investigate whether any system of injection molding plastic would work.

(e) Get some help and some money. Comments and offers would be welcome.

(f) Build and test a system. Test components indoors--Green Bank basement is OK. Finally test on the 140 foot? Or some other telescope.

Appendix 1

The Geometry of a Perfect 25-m Antenna

Choose D = 25.0 m

f = 10.5 m

so that f/D = 0.42.

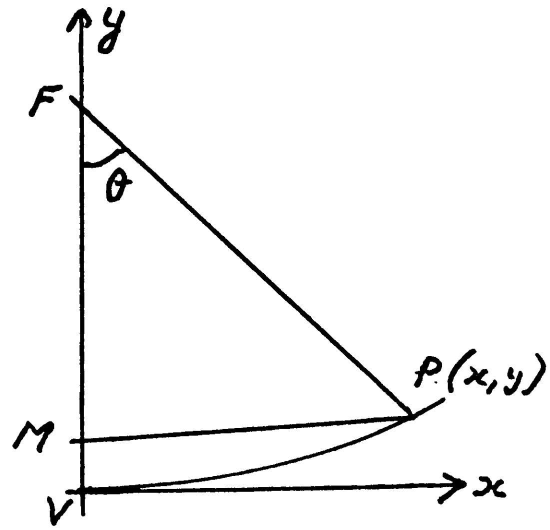
Then in the (x,y) system shown:

$x^2 = 42.0\,y$ ....(1)

At x = 12.5 m, y = 3.72 m.

At any point P on the surface:

$(FP)^2 = x^2 + (f-y)^2$....(2)



$$FP = \left\{\frac{x^4}{1764} + \frac{x^2}{2} + 110.25\right\}^{1/2} \qquad (3)$$

At P, $\tan\theta = x/f\text{-}y$ .... (4)

If M is 2.0 m above V, the elevation angle α of P seen from M is given by

$$\tan\alpha = \frac{y-2.00}{x} \qquad (5)$$

Table of Values

xm	ym	α^o if MV = 2.0 m	FP m	θ^o
1.0 m	0.02381	-63.1°	10.5238	5.45°
3.0	0.21429	-30.8°	10.7143	16.26°
5.0	0.59524	-15.7°	11.0952	26.79°
7.0	1.16667	- 6.8°	11.6667	36.87°
9.0	1.92857	- 0.45°	12.4286	46.40°
11.0	2.88095	+ 4.58°	13.3810	55.29°
12.0	3.42857	6.79°	13.9286	59.49°
12.5	3.72024	7.84°	14.2202	61.53°