Interoffice

National Radio Astronomy Observatory

Charlottesville, Virginia

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25 Meter Millimeter Wave Telescope Memo #76

To: D. S. Heeschen

From: Lee J Rickard

Subject: Relative observing efficiencies at Kitt Peak and Mauna Kea

In his memorandum on the relative observing efficiencies of the Kitt Peak and Mauna Kea sites for the 25-m telescope, Liszt considered only the atmospheric transmission and noise evaluated for sources at transit. In actual operation, though, the telescope user will observe sources at large hour angles from transit, often as far as the elevation limit of the telescope. The total signal-to-noise ratio (SNR) for such an observation depends strongly on the variation of atmospheric transmission and noise with hour angle. In order to see what effect this has on Liszt's analysis, I have constructed a computer model of a spectral line observer who attempts to reach some specified SNR for a set of test sources and lines. The times required to achieve the specified SNR (or, conversely, the sensitivities achieved in a fixed number of observing days) can then be compared for the two sites.

The details of the model used are outlined in the Appendix. In Table 1, I have given the results for the case of an observer trying to reach SNRs of 3, 4, and 5 for lines of HCN (90 GHz), CO (115, 230, and 345 GHz), and H_2CO (150 GHz) in the sources Orion A (δ=-5°24'), M17 (δ=-16°14'), Sgr B2 (δ=-28°22'), and NGC 6334N (δ=-35°45'). The values tabulated are the ratios of the observing times required at Kitt Peak to those required at Mauna Kea. For all cases, I have used T_{tel}=200 K, T_{atm}=273 K, zenith optical depths from Wade's memo, T_A^o=0.05 K, β=1 MHz, t_s=5 min, t_i=1 min, a_{min}=15°, and SNR_{min}=0.10. (These quantities are all defined in the Appendix.) Liszt's results are included for comparison.

Three comments: There is some noise in the values near 1 for ν=90 and 150 GHz, essentially the effect of quantizing the observing time. None of the Mauna Kea numbers quoted required more than one observing day; some of the Kitt Peak observations required two or more days. (Entries marked "∞" required seven or more days at Kitt Peak; several of these required more than one day at Mauna Kea.) The agreement between Liszt's values and those for SNR=3 is rather good; most of these observations do not require the observer to follow the source far from transit.

In summary, this more detailed treatment supports Liszt's analysis; if anything, his results tend to underestimate the time required for an observer at Kitt Peak to equal the sensitivity of an observer at Mauna Kea.

TABLE 1

	Frequency (GHz)	Orion A	M17	Sgr B2	NGC 6334N
SNR=3	90	1.0	1.0	1.0	1.5
	115	1.7	2.0	4.0	5.7
	150	1.5	1.5	2.0	3.0
	230	2.5	3.4	4.3	8.2
	345	28.	∞	∞	∞
SNR=4	90	1.4	1.0	1.3	1.7
	115	1.8	2.4	4.2	8.7
	150	1.7	1.7	2.3	3.6
	230	3.3	3.2	6.7	13.1
	345	∞	∞	∞	∞
SNR=5	90	1.2	1.2	1.5	2.0
	115	2.1	3.2	4.5	12.9
	150	1.7	2.0	2.4	4.4
	230	3.4	4.5	9.4	37.3
	345	∞	∞	∞	∞
LISZT	90	1.4		1.6	
	115	1.6		2.7	
	150	1.7		2.5	
	230	2.8		4.7	
	345	10.		37.	

APPENDIX

Suppose that the source is observed in a sequence of scans of length $2t_s$ (so that the on-source time per scan is t_s), spaced by times t_i. The observer presumably selects the hour angle at which he begins observing in order to minimize the amount of time spent at large zenith angles. In order to simplify the analysis, I assume that he does so by pairing scans around the transit time of the source. Thus, if the total observing time needed to reach the desired SNR is T_{tot}, the scans will begin at a time $T_{tot}/2$ before transit.

For N scans, the total SNR is $\langle T_A \rangle / \sigma_t$, where $N^2 \sigma_t^2 = \sum_{scans} \sigma_i^2$, σ_i=RMS noise for scan i, $\sigma_i = 2T_{sys_i}/\sqrt{\beta t_s}$. Here, β is the spectral resolution in Hz, T_{sys_i} is the system temperature during scan i, $T_{sys_i} = T_{tel} + T_{atm}(1-\exp[-\tau_i])$. T_{tel} includes all non-atmospheric noise, and T_{atm}=273 K.

The atmospheric optical depth during scan i is $\tau_i = \tau_o \sec z_i$, where z_i is the zenith distance during scan i, and is related to the site latitude, source declination, and hour angle of scan i by the usual formula.

The measured signal is actually the average of the signals measured in each scan, each weighted by the squared SNR of each scan:

$$\langle T_A \rangle = \frac{\sum_{scans} T_{A_i} (S/N)_i^2}{\sum_{scans} (S/N)_i^2} .$$

The $T_{A_i} = T_A^o \exp(-\tau_i)$, where T_A^o is the line antenna temperature outside the atmosphere. The SNR of each scan is

$$(S/N)_i = T_{A_i} / (2 T_{sys_i} / \sqrt{\beta t_s}) .$$

Thus,

$$\frac{\langle T_A \rangle}{\sigma} = \frac{N}{2} T_A^o \sqrt{\beta t_s} \frac{\sum_{scans} T_{sys_i}^{-2} e^{-3\tau_i}}{\sqrt{\sum_{scans} T_{sys_i}^2} \left(\sum_{scans} T_{sys_i}^{-2} e^{-2\tau_i} \right)}$$

Since the scans are placed symmetrically about the transit time, for an arbitrary quantity Q_i:

$$\sum_{scans} Q_i = 2 \sum_i Q(t_{(i)}) ,$$

where $t_{(i)}$ is the time away from transit. Clearly, the set of values $\{t_{(i)}\}$ is

$$\{t_s + \tfrac{1}{2}t_i ,\ 3t_s + \tfrac{3}{2}t_i ,\ 5t_s + \tfrac{5}{2}t_i , \ldots ,\ t_{max},\ t_s + \tfrac{1}{2}t_i ,\ 3t_s + \tfrac{3}{2}t_i , \ldots \} ,$$

where t_{max} is the time, measured from transit, at which the observer stops following the source. Usually, t_{max} is taken to be the time at which the source is found to have an elevation angle $\leq a_{min}$. In practice, the observer will often stop when the SNR for an individual scan is less than some minimum SNR_{min}.

In summary, the total SNR for the averaged scans is

$$\frac{\langle T_A \rangle}{\sigma} = \frac{1}{2\sqrt{2}} N T_A^{\circ} \sqrt{\beta t_s} \; \frac{\sum_i T_{sys}^{-2}(t_{(i)}) e^{-3\tau(t_{(i)})}}{\sqrt{\sum_i T_{sys}^{2}(t_{(i)})} \left(\sum_i T_{sys}^{-2}(t_{(i)}) e^{-2\tau(t_{(i)})} \right)}$$

This is evaluated for each element in the sequence $\{t_{(i)}\}$ until it exceeds the specified limit.