25-m Telescope
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ACCURACY AND THERMAL DEFORMATION OF ESSCO PANEL

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1. History

In April 1976, Buck Peery got a letter from ESSCO, giving the equations for the theoretical paraboloidal surface coordinates. At ESSCO, the panel was measured at n = 86 points of a rectangular grid, and ESSCO quoted for the deviations

$$\text{ESSCO rms} = 1.58 \text{ mils}^{*} = 40 \ \mu\text{m}. \tag{1}$$

The panel was shipped to Green Bank, where Sidney Smith measured it five times (July and November 1976) with n = 72 points of a radial grid, obtaining a range of 3.3 ... 4.2 mils, with an average of

$$\text{GB rms} = 3.78 \text{ mils} = 96 \ \mu\text{m}. \tag{2}$$

This discrepancy between (1) and (2), a factor of 2.39, caused a great deal of concern on both sides. The panel was shipped back to ESSCO and was measured there four times, results ranging between 2.13 ... 2.40 mils, and averaging

$$\text{ESSCO rms} = 2.23 \text{ mils} = 57 \ \mu\text{m}. \tag{3}$$

Comparing (1) and (3), it could be that the panel had been damaged during the first shipping. But the discrepancy between (2) and (3), a factor of 1.70, was still unexplained and too large to be tolerated.

The solution finally was given in February 1977 by a report from ESSCO, giving for the first time the detailed equation of their data evaluation. If Δ are the measured deviations, Δ = measured - paraboloid, ESSCO had subtracted

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* 1 mil = 10^{-3} inch.

the average (which may be called a one-apremeter best fit) and had multiplied by cos θ (where $\theta = 27.68^{\circ}$ is the tilt angle of this panel on the telescope, between the panel and a plane normal to the telescope axis):

$$\text{ESSCO rms} = \cos\theta * \text{rms}(\Delta - \overline{\Delta}), \tag{4}$$

whereas our values (2) mean simply

$$\text{GB rms} = \text{rms}(\Delta). \tag{5}$$

The panel was again shipped to GB, and a new set of four measurements was taken by Sidney Smith in February and March 1977. When evaluated the ESSCO way of equation (4), the results ranged between 2.3 ... 2.7 mils, with an average of

$$\text{GB measurements:} \quad \cos\theta * \text{rms}(\Delta - \overline{\Delta}) = 2.58 \text{ mils} = 65\ \mu\text{m}, \tag{6}$$

which now comes close enough, with overlapping ranges, to the ESSCO value of (3). And when evaluated the GB way of equation (5), the range is 2.6 ... 3.9 mils, with an average of

$$\text{GB measurements:} \quad \text{rms}(\Delta) = 3.32 \text{ mils} = 84\ \mu\text{m}, \tag{7}$$

which agrees with our older data of (2), meaning that no damage has occurred during the last two shippings.

Since two different grids had been used by ESSCO and by us, ESSCO measured in their Report (Feb. 8, 1977) both grids. The difference, 2.40 versus 2.19 mils, is negligible as it should be. Our new GB measurements, from February 1977 on, were all done using the rectangular ESSCO grid.

2. The Meaning of RMS

In total, it took ten months to remove the discrepancy, which then turned out to be a question of semantics (science of meanings). The abbreviation "rms" means "root mean square." Mathematically speaking, this is a function, and each function needs an argument and is incomplete without it. For example, "sin = ..." is incomplete, one must write "sin ϕ = ...". Thus, one should never write "rms = ...", but always "rms(x) = ...", or "rms(x − $\bar{x}$) = ... ", or whatever the argument may be. Then, by definition

$$\mathrm{rms}(x) = \left\{\overline{x^2}\right\}^{1/2} \tag{8}$$

and

$$\mathrm{rms}(x - \bar{x}) = \left\{\overline{(x - \bar{x})^2}\right\}^{1/2} = \left\{\overline{x^2} - (\bar{x})^2\right\}^{1/2}. \tag{9}$$

Another technical term frequently used is the standard deviation σ, where subtracting the mean is already included in its definition:

$$\text{standard deviation of } x = \sigma(x) = \mathrm{rms}(x - \bar{x}). \tag{10}$$

The square of the standard deviation is called the variance, $\mu_2(x) = \sigma^2(x)$.

<u>Finite sample</u>. These equations refer to samples of infinite (or very large) size. If a finite (small) number n of measurements are used for obtaining both $\bar{x} = (1/n) \sum x_i$ and $\overline{x^2} = (1/n) \sum x_i{}^2$, equation (8) still holds, but equation (9) needs a correction:

$$\mathrm{rms}(x - \bar{x}) = \sigma(x) = \left\{\frac{n}{n-1}\left[\overline{x^2} - (\bar{x})^2\right]\right\}^{1/2}. \tag{11}$$

<u>Measuring error</u>. Equations (8), (9) and (11) refer to the measured values or readings. If each reading x_i consists of a true value x_{oi} and an error ε_i, with $x_i = x_{oi} + \varepsilon_i$, and if we want to know the true values (of our plate, for example), we need an independent determination of our measuring error, calling $\varepsilon_o = \mathrm{rms}(\varepsilon)$. Since $\bar{x} = \overline{x_o}$ whereas $\overline{x^2} = \overline{x_o{}^2} + \varepsilon_o{}^2$, we finally have, referring to the true values,

$$\sigma(x_o) = \text{rms}(x_o - \overline{x_o}) = \left\{ \frac{n}{n-1} \left[\overline{x^2} - (\bar{x})^2 \right] - \varepsilon_o^2 \right\}^{\frac{1}{2}} , \tag{12}$$

and

$$\text{rms}(x_o) = \left\{ \overline{x^2} - \varepsilon_o^2 \right\}^{\frac{1}{2}} . \tag{13}$$

If measurements with different methods are to be compared, and if the measurement errors are not negligible, the corrections given above should be applied.

3. Systematic Change

The last set of five GB measurements is shown in Table 1. The errors given are the (statistical) mean errors of each quantity, as obtained from the readings. Table 1 shows that the averages $\bar{\Delta}$ are significantly different from one day to the other; for example $\overline{\Delta_d} - \overline{\Delta_c} = 1.34 \pm 0.46$, with $1.34/0.46 = 2.91$. The values $\sigma(\Delta)$, however, stay much more constant, which means we did not just have "good and bad days."

Table 1. Last set of GB measurements (with mean errors).
Δ = measured height - paraboloid, in mils.

Date	Fig. 1	$\bar{\Delta}$	$\overline{\Delta^2}$	rms(Δ)	$\sigma(\Delta)$
Feb. 25	a	−1.57 ± .33	11.71 ± 2.39	3.42 ± .35	3.06 ± .43
" 28	b	−1.84 .34	13.12 2.91	3.62 .40	3.14 .51
Mar. 1	c	−2.49 .33	15.63 2.58	3.95 .33	3.09 .50
" 21	d	−1.15 .32	10.24 1.77	3.20 .28	3.00 .32
" 22	e	−1.64 .31	11.08 2.04	3.33 .31	2.91 .40

Fig. 1 shows the second moment $\overline{\Delta^2}$ and the standard deviation $\sigma(\Delta)$, as functions of the average $\bar{\Delta}$. We get the impression that some systematic deformation takes place between different days, but not monotonic with time (as a creeping of the epoxy would do).

The best check for a systematic change is to ask for the correlation between the differences of any two days, and those of any two other days; for example

$$M_a = \frac{1}{86} \sum_{i=1}^{86} (\Delta_{di} - \Delta_{bi})(\Delta_{ai} - \Delta_{ci}). \tag{14}$$

If M is significantly larger than zero, then the changes are systematic and not random. Since we have one more day of readings, we may also replace Δ_{ai} in (14) by Δ_{ei}, calling the result M_e. From the measurements mentioned in Table 1 we obtain

$$\begin{aligned} M_a &= 0.80 \pm 0.28, \quad \text{with } 0.80/0.28 = 2.9, \\ M_e &= 1.04 \pm 0.25, \quad \text{with } 1.04/0.25 = 4.2. \end{aligned} \tag{15}$$

This is significant enough to conclude that the changes indeed are systematic.

4. Our Measuring Error

From Fig. 1 we further conclude that cases e and a have about the same systematic deformation. Their difference then is a good measure for our intrinsic measuring error:

$$\frac{1}{86} \sum_{i=1}^{86} (\Delta_{ei} - \Delta_{ai})^2 = 2\, \varepsilon_o^{\,2}, \tag{16}$$

and from the measurements we obtain

$$\varepsilon_o = (0.81 \pm 0.08) \text{ mils} = (21 \pm 2)\ \mu\text{m}. \tag{17}$$

This error includes: the reading accuracy of each point as well as setting the scale exactly on the bench mark of this point, and adjusting the six adjustment points to $\Delta = 0$. For an optical method this is quite good, amounting to 1.7 arcsec total.

5. Thermal Deformations

The most probable explanation for the systematic changes seems to be thermal deformations. For this case we expect the deformations to be smallest close to the six adjustment points, to be largest in between and always of the same sign, but to be of opposite sign in the cantelevering piece. Fig. 2 gives the differences ($\Delta_{di} - \Delta_{ci}$) for all 86 points. Keeping in mind that the mean error of these differences is $\varepsilon_o\sqrt{2} = 1.15$ mils, we may conclude that the thermal expectations are fairly well fulfilled.

Thermal deformations occur when the skin has a temperature different from that of the lower part of the ribs underneath. The simple model of Fig. 3 gives a central maximum deformation of

$$z_{max} = \frac{1}{4} c_{th} \Delta T \frac{\ell^2}{h} , \tag{18}$$

and for the average $\bar{z}$, for aluminum and the dimensions of this panel,

$$\bar{z} / \Delta T = 2.69 \text{ mils}/^{o}\text{F} = 123 \ \mu\text{m}/^{o}\text{C}. \tag{19}$$

Between the extremes of Fig. 1, the observed difference is

$$\bar{z} = \overline{\Delta_d} - \overline{\Delta_c} = 1.34 \text{ mils} = 34 \ \mu\text{m}, \tag{20}$$

and with (19) we obtain for the temperature difference ΔT between skin and lower part of ribs:

$$\Delta T(\text{March } 21) - \Delta T(\text{March } 1) = 0.50 \ ^{o}\text{F} = 0.28 \ ^{o}\text{C}. \tag{21}$$

This seems possible, regarding the small distance between panel and door, and between panel and the heating units at the wall, enforced by the small size of the room.

This result emphasizes the need for rather <u>thick panels</u> for our 25-m design, to keep the thermal deformations sufficiently small.

6. Best Possible Shape

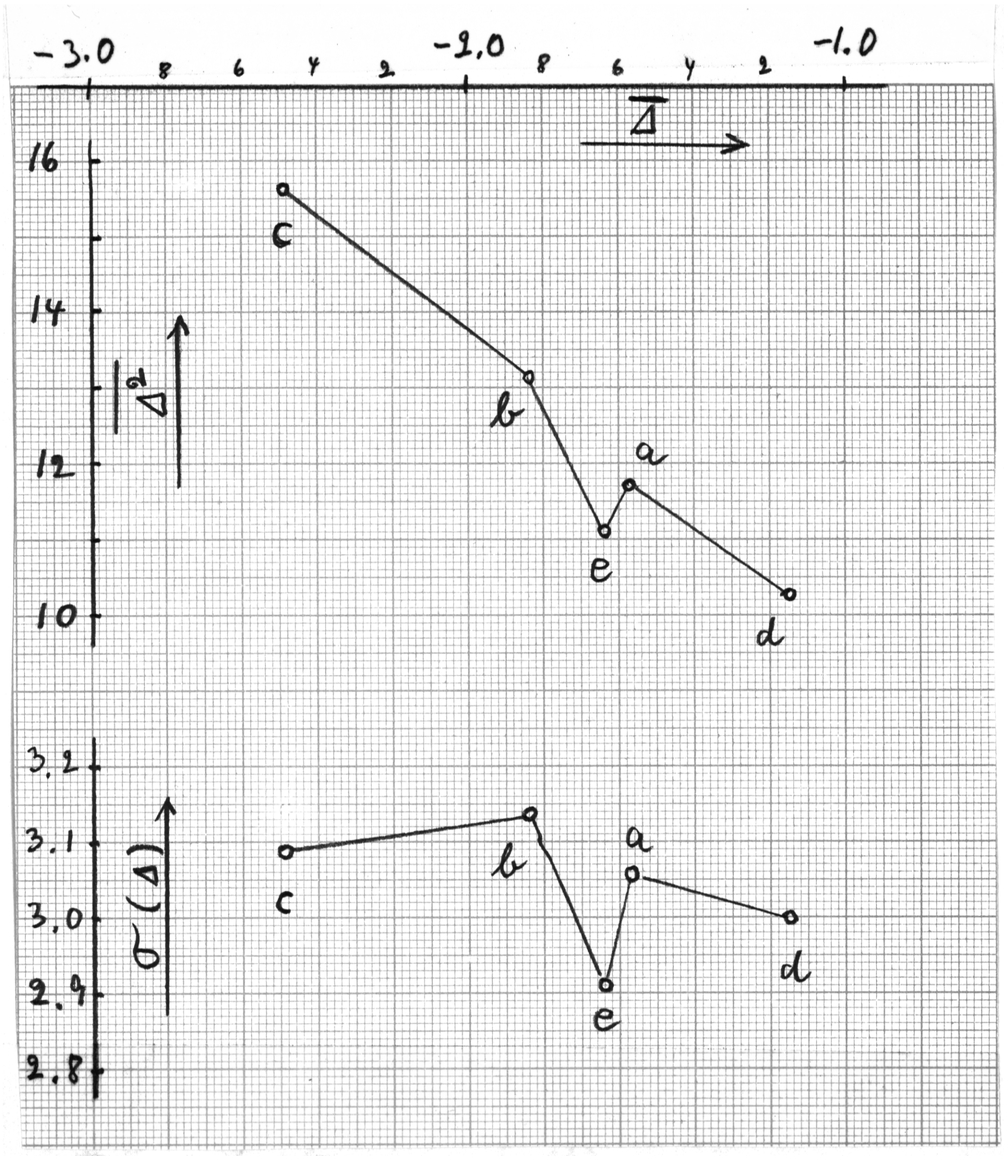
From Fig. 1 one may get the idea that there might be some thermal condition better than that of case d, extrapolated from c to d and beyond. Calling Δ_o the deviation of this best shape from the paraboloid, including correction (13) for our measuring error, it can be shown that

$$\overline{\Delta_o^{\ 2}} = \overline{\Delta_d^{\ 2}} - \left\{\overline{\Delta_d(\Delta_d - \Delta_c)}\right\}^2 \Big/ \overline{(\Delta_d - \Delta_c)^2} - \varepsilon_o^{\ 2} \tag{22}$$

and from the measurements we obtain

$$\mathrm{rms}(\Delta_o) = (3.03 \pm .31)\ \mathrm{mils} = (77 \pm 8)\ \mu\mathrm{m}. \tag{23}$$

Compared with $\mathrm{rms}(\Delta_d) = (3.20 \pm .28)$ mils from Table 1, we see no significant improvement. Which means that a change of thermal conditions can make it a lot worse but not much better.



<u>Fig. 1.</u> Last five GB measurments of ESSCO panel; second moment $\overline{\Delta^2}$, and standard deviation $\sigma(\Delta)$, as functions of average $\bar{\Delta}$. Δ = measured height - paraboloid, in 10^{-3} inch.

	-15	-12.5	-9	-4.5	0	4.5	9	12.5	15
4	0		-1	-1	0	0	-1		+1
14	+1		0	+1	0	+1	+1		+1
24	+4		0	+1	+3	0	0		0
34	+1		0	0	+1	+1	+2		+1
44		+2	0	+1	+1	+6	+2	+1	
54		+2	+2	+1	+1	0	+1	0	
64		+4	+4	+2	+1	+1	-1	0	
74		+1	+2	+2	+2	+1	+1	0	
84			+2	+2	+3	+2	+2		
94			+2	+2	+3	+2	+2		
104			+3	+1	+2	+1	+1		
114			+1	+2	+1	+3	+1		
124			+4	+1	+3	+1	+2		
131			+2	+3	+3	+4	+3		

<u>Fig. 2.</u> Rectangular measuring grid with n = 86 points, showing the difference $\Delta_{di} - \Delta_{ci}$ between March 21 and March 1.

At the six points marked ⊗, the panel is adjusted to the correct paraboloid.

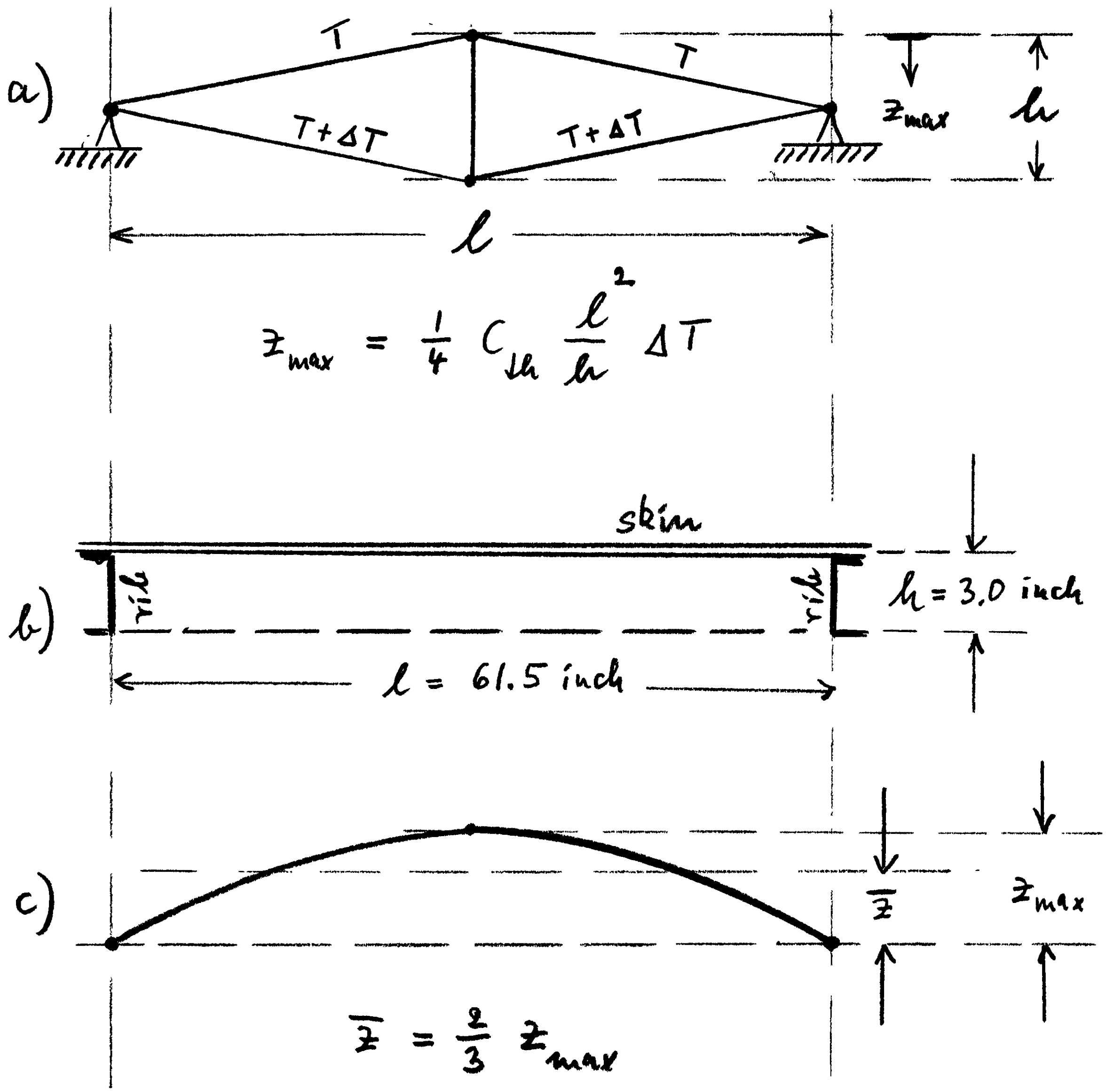


Fig. 3. Thermal deformation of narrow beam or plate.

a) Simplified model, and central deformation z_{max};

b) Skin and ribs of panel, length ℓ for diagonal;

c) Average deformation $\bar{z}$.