25-m Telescope
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Some Explanations Regarding Homology

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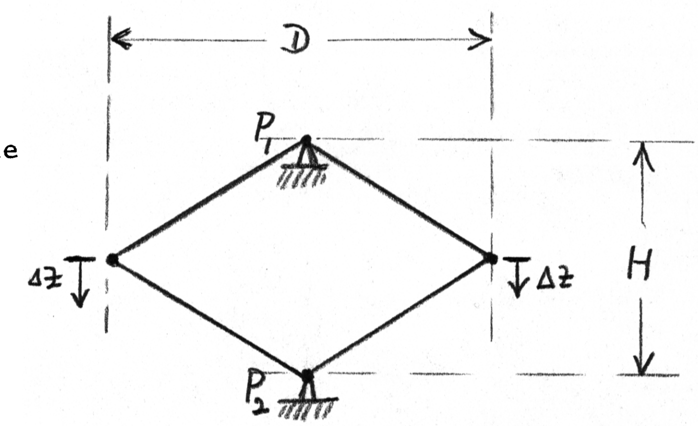
Unfortunately, it had taken almost a year to solve the discrepancy about the panel measurements, where ESSCO and NRAO had just meant different things by the incomplete use of the abbreviation "rms" (Memo 81). I would hate to see a new misunderstanding come up by the comparison of "stiff versus homologous," and I would like to try to clarify these concepts in the following. In summary: if "stiffness" is defined by minimizing the gravitational deformations, then there is a natural limit and nobody can pass it. But the deformations can be made harmless, or well-behaved (or "homologous", as I call it); and exactly that is what now everybody tries to approach, no matter which means are applied or which words are used.

1. Gravitational Stiffness



Let me explain the natural limit for gravitational stiffness with a crude example in two dimensions. Suppose we have a symmetric cantilever of four members (of bar area A, density ρ, and modulus of elasticity E). It is supported at points P_1 and P_2, and the task is to minimize the gravitational deformation Δz, by variation of height H and bar area A, for a given diameter D. After some very simple arithmetic, we find for the gravitational deformation the general formula

$$\Delta z \;=\; \frac{1}{8}\,\frac{\rho}{E}\,D^2 \left\{\frac{D}{H} + \frac{H}{D}\right\}^2 . \tag{1}$$

First, we see that the gravitational deformation is independent of A. The bar area, which is proportional to the stiffness for external loads, has no influence on the gravitational stiffness (because the weight or dead load goes also with A). Second, for given D and variable H, equation (1) has a minimum which cannot be surpassed; it is

$$\Delta z_{min} \;=\; \frac{1}{2}\,\frac{\rho}{E}\,D^2 \qquad \text{for} \qquad H = D. \tag{2}$$

Applied to tiltable telescope, I called this the "gravitational limit" in two papers in 1967. Third, the material chosen enters as ρ/E, which is amazingly similar for various cases, for example

$$\frac{\rho}{E} \;=\; \begin{cases} 3.7 \times 10^{-9}\ \text{cm}^{-1}, & \text{for steel,} \\ 3.8 & \text{aluminum,} \\ 5.8 & \text{wood.} \end{cases} \tag{3}$$

For aluminum or steel we then have

$$\Delta z_{min} \;=\; 1.9\ \text{mm} \left\{\frac{D}{100\ \text{m}}\right\}^2 \tag{4}$$

If we do it in 3 dimensions, with a somewhat more realistic geometry, adding some extra weight for secondary members and surface, we lose about a factor of two and obtain, roughly,

$$\Delta z \;=\; 4.0\ \text{mm} \left\{\frac{D}{100\ \text{m}}\right\}^2 . \tag{5}$$

This, give or take a few percent, is the natural limit for gravitational stiffness. It does not depend on the bar area, and very little on the material. A bad geometry can make it a lot worse, but a good one cannot make it any better.

2. Approach to Homologous Deformations

It is not the amount of the deformation which matters for a telescope, but only the deviation of the deformed surface from its best-fit paraboloid, and this can be minimized far beyond limit (5). Work done at NRAO since 1964 resulted in the definition of the goal, and in a method to achieve it.

The goal is to design a structure whose gravitational surface deformations transform one paraboloid of revolution into another one, permitting changes of focal length, vertex location, and axial direction. Mathematically, this is called a "homologous" transformation, from one member of a given family to another member of the same family, and this is why I have used that word.

One cannot say that one telescope deforms homologous while another one does not. This is a matter of gradual approach, and one telescope may approach the goal closer than another one. Even for a conventionally designed old telescope, the receiver looks automatically for the best-fit paraboloid, and the deviations from this one are smaller than the actual structural deformations are (by a factor 2 - 5). And even a telescope designed for perfect homology will show some small deviations from it, caused by manufacturing and erection tolerances. The goal, then, is to approach homology, as far as needed for a wanted application (shortest wavelength of observation), and in proper comparison with all other items of the error budget (thermal, wind, surface panels).

What matters is the deviation from homology, $H_{\phi\theta}$ = rms deviation from best-fit paraboloid when observing at zenith distance ϕ, if the telescope is perfectly adjusted at zenith distance θ. Equations for H and for the best adjustment angle θ, and a comparison of some telescopes, are given by von Hoerner and Wong (IEEE-AP 23, 689, 1975). The words "homologous deformations" were used to define the general goal, not our specific method for achieving it.

This goal of minimizing the deviations from homology can be approached in various ways. Even the old conventional telescopes approach homology to some extent; and more recent but still conventional designs, done with experience and intuition, will do it somewhat better. A fairly good approach to homology can be achieved by trial-and-error runs on a computer, using nothing but standard structural analysis, gradually changing from one run to the next a few joint locations and bar areas. This was done with good success for the 100-m Bonn telescope.

Finally, if a very close approach is wanted, there is a mathematical iteration procedure with extremely good convergence, developed in 1965 at NRAO and published in 1967. Our 65-m design was done this way, and so is our 25-m design. The convergence is so good that the practical limitation of the method is given by manufacturing and erection tolerances. We investigate these, after convergence has been achieved, in a separate program, using randomized structural changes within the tolerances specified for manufacture and erection. These gravitational residuals are then only a small fraction of the total error budget, whereas they were the most crucial contribution for older designs.

3. Maximum Stiffness Still Desirable

In addition to a good approach to homology, we still need maximum stiffness for other reasons. First, it is needed against wind deformations and survival loads for exposed telescopes (the only case where bar areas matter). Second, a high dynamical stiffness is needed for all tiltable telescopes regarding the pointing accuracy. Third, a telescope geometry designed for maximum gravitational stiffness will also minimize the thermal deformations for given temperature differences in the structure. Fourth, the residual deviations from homology, caused by the tolerances, are about proportional to the deformations Δz (not to the deviations H).