25-m Telescope
Memo No. 87
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Regarding Specifications for Surface Plates

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1. Numbers (of plates and firms)

If we can afford it, I would suggest ordering two identical plates. First, on general grounds. Second, for our thermal tests to be done later: using different white paints, spraying foam on backside, or others. Simultaneous measurements are needed for proper comparison.

If possible, we should ask two or three firms, for comparing their cost estimates, before actually ordering any plates.

2. Dimensions

In his Note of March 25, 1977, Buck Peery suggests ordering a plate of ring No. 4; this seems a good choice, since this plate is just a little wider than average. Subtracting the width of the gap between panels (1.5 mm), the dimensions then are, see Fig. 1,

$$\begin{aligned} L &= 151.30 \text{ cm, length of plate} \\ A &= 91.69 \text{ cm, upper width,} \\ B &= 72.70 \text{ cm, lower width.} \end{aligned} \tag{1}$$

For some of the following estimates, we need the dimensions in between the four support points. Length and average width are

$$\begin{aligned} \ell_o &= 140.30 \text{ cm,} \\ b_o &= 70.80 \text{ cm.} \end{aligned} \tag{2}$$

What matters is the "effective length" of the diagonal, ℓ. Since gravitational deformations go with ℓ^4, thermal ones with ℓ^2, we calculate

$$\ell_{gr} = (\ell_o^{\,4} + b_o^{\,4})^{1/4} = 142.52 \text{ cm}, \tag{3}$$

$$\ell_{th} = (\ell_o^{\,2} + b_o^{\,2})^{1/2} = 157.15 \text{ cm}. \tag{4}$$

3. Surface Specs

a. Multiply by cos θ?

If specs are written for 1/2 the pathlength error, the surface deviations get multiplied by the cosine of the tilt angle θ, where $\theta = 0$ at the dish center, and $\theta = 30°$ ($\cos\theta = .866$) at the dish rim, see Memo No. 81. The requirements on the manufacturing accuracy are then somewhat relaxed, for example, by 13.4% at the dish rim.

I have calculated the average of $\cos\theta$ over the whole dish surface, using a parabolic illumination taper. The result is

$$\text{av}(\cos\theta) = 0.949. \tag{5}$$

Since this does not make much difference, relaxing the requirements by only 5.1%, we should disregard this factor and not multiply by $\cos\theta$.

b. Various degrees of best-fitting

Should we specify $\text{rms}(\Delta z) \leq 25\ \mu\text{m}$, or allow $\text{rms}(\Delta z - \overline{\Delta z}) \leq 25\ \mu\text{m}$? The latter could be called a one-parameter fit. Regarding the future adjustment of this plate on the telescope, the first case simply means that all four corners are to be adjusted exactly on the design paraboloid. In the second case, all corners are to be lowered by the same amount, $\overline{\Delta z}$; but $\overline{\Delta z}$ is then different for each plate. This can be done, but it is an inconvenience and might give rise to mistakes.

In principle, one could even make better fits with up to four parameters (= four corners). Using previous plate measurements, I have made a rough estimate for finding out by which amount the manufacturing requirements would be relaxed:

parameters	means, for adjustment,	relaxation (%)
1	parallel shift, up/down	10 - 17
2	plus tilt, lengthwise	2 - 3
3	plus tilt, sideways	1 - 2
4	plus internal bending	1 - 2

(6)

Comparing relaxation versus inconvenience, I would suggest no fitting at all, specifying

$$\mathrm{rms}(\Delta z) = \left\{ \frac{1}{N} \sum_{i=1}^{N} (\Delta z_i)^2 \right\}^{1/2} \leq 25\ \mu\mathrm{m}, \tag{7}$$

for a number $N \geq 121$ of surface points as suggested by Buck Peery's Note, demanding that all four corner points are adjusted exactly on the design paraboloid, and where Δz = measured - design, perpendicular to the plane going through the four corner points.

c. Subtract measuring error?

Regarding our specifications of the manufacturing accuracy (as well as any measurements of gravitational or thermal deformations), I would suggest not to subtract the measuring error. Subtracting it does not make much difference for small errors, but it introduces some uncertainty of the results for large errors. Also, not subtracting the error is a safety device, enforcing accurate measuring techniques.

4. Gravitational Deformations

a. Specification

For a theoretical evaluation, we call

$$\Delta z = z(\text{without gravity}) - z(\text{gravity, plate horizontal}). \tag{8}$$

If evaluated by measurements, the plate could be measured in horizontal position, first as usual, and second with a uniformly distributed additional load equal to the plate weight:

$$\Delta z \;=\; z(\text{without load}) - z(\text{with load}). \tag{9}$$

Again: should we specify rms(Δz) or rms($\Delta z - \overline{\Delta z}$)? All plate normals have about the same angle from the vertical direction of gravity, for any telescope pointing. If this were exactly the case, then rms($\Delta z - \overline{\Delta z}$) would be all that is needed, allowing a change of focal length $\Delta F = \overline{\Delta z}$. Actually, from dish center to rim, the angles of the normals vary by $\pm$ 30°, which would need a slightly more stringent requirement. On the other hand, equations (8) and (9) would mean that we observe right down to the horizon, while observations at shortest wavelengths actually are limited to about $\geq$ 20 above horizon. It is thus safe to demand only

$$\sigma(\Delta z) \;=\; \mathrm{rms}(\Delta z - \overline{\Delta z}) \;\leq\; 12\ \mu\mathrm{m}, \tag{10}$$

where

$$\sigma(\Delta z) \;=\; \left[\frac{N}{N-1}\left\{\overline{\Delta z^2} - \overline{\Delta z}^{\,2}\right\}\right]^{1/2}$$

$$= \left[\frac{1}{N-1}\sum_{i=1}^{N}\Delta z_i^{\,2} - \frac{1}{N(N-1)}\left\{\sum_{i=1}^{N}\Delta z_i\right\}^2\right]^{1/2}. \tag{11}$$

b. <u>Estimate of rib height</u>

Fig. 2 gives the deformation at plate center under dead loads (meaning neglecting the skin's weight and stiffness):

$$\Delta z_m \;=\; Q\,\frac{\rho}{E}\,\frac{\ell^4}{h^2} \tag{12}$$

where ρ = material density, E = modulus of elasticity, and $\rho/E = 3.8 \times 10^{-9}\ \mathrm{cm}^{-1}$ for aluminum. If the manufacturer calculates it theoretically, he should use the actual details of his plate design. For our present estimate, we replace the actual cross section of the rib by two extreme models: a rectangular beam and a symmetric truss (equal upper and lower bars). This yields

$$Q = \left\langle \begin{array}{l} 0.156, \text{ rectangular beam} \\ \\ 0.125, \text{ symmetric truss} \end{array} \right\} = 0.140 \text{ average.} \qquad (13)$$

A channel will be somewhere in between these models, probably closer to the truss.

The surface adds both weight and stiffness, but the asymmetry introduced by it must yield larger deformations. Some rough estimates showed that the result might be the average of (13), increased by about 30%, and we adopt Q = 0.180. Since

$$\mathrm{rms}(\Delta z - \overline{\Delta z}) = \frac{2}{3\sqrt{5}}\,\Delta z_m = 0.298\,\Delta z_m\,, \qquad (14)$$

our demand (10) and equation (12) yield for the rib height $h \geq 8.4$ cm = 3.3 inch, or roughly

$$h \geq 3.5 \text{ inch} = 8.9 \text{ cm.} \qquad (15)$$

5. Thermal Deformation

a. Problem

In Memo 86, we derived $h \geq 4$ inch for the rib height, using $\ell = 165.6$ cm for the diagonal, and $\Delta T_{sr} = 0.29$ °C for the temperature difference between skin and lowest part of rib. We used $\sigma = 0.35\,\Delta z_m$ assuming an average location on the telescope. We also mentioned the problem that ΔT_{sr} will actually depend on the height and thickness of the ribs, and on the method of their attachment to the skin.

This problem means that we really cannot specify the deformations Δz, because they are proportional to ΔT_{sr} which depends in a complicated way on the design. In the following we give a better derivation of h and a suggested procedure.

b. Estimate

Since during our measurements the plate in the tent was tilted south, towards the sun, the measured ΔT_{sr} corresponds already to a plate location at the center of the dish (worst case), where $\sigma = .298\ \Delta z_m$ may be used. A detailed treatment of the whole telescope was done, comparing Δz_m and $\overline{\Delta z}$ for various locations, and assuming different ways in which the radiation onto the plate could be a function of the location. Omitting the details, the result is that demanding

$$\mathrm{rms}(\Delta z - \overline{\Delta z}) \leq 16\ \mu\mathrm{m} \tag{16}$$

for the test plate is on the safe side (by 5 - 10%) regarding the deformation of the whole telescope and its average.

But the problem is that we cannot just specify (16). In Memo 86 we found for the plate center

$$\Delta z_m = \frac{1}{4}\, C_{th}\, \Delta T_{sr}\, \ell^2/h. \tag{17}$$

With $C_{th} = 2.34 \times 10^{-5}/°C$ for aluminum, and $\ell = 157.15$ cm from (4), using $\sigma = .298\ \Delta z_m$, demand (16) yields

$$h \geq 26.9\ \mathrm{cm}\ \Delta T_{sr}/°C. \tag{18}$$

The Philco-Ford plate in the tent had ribs which were short (h = 2.75 inch) and thick (t = 1/4 inch), and which were cast together with the skin; with this plate we measured $\mathrm{rms}(\Delta T_{sr}) = 0.29$ °C during sunshine. For a future plate design, we may have h = 4 inch height and, say, t = 1/8 inch wall thickness, and the ribs will be attached to the skin by epoxy. All this means that ΔT_{sr} must be larger, but how much? If we guess an increase of, say, 30%, then ΔT_{sr} = 0.38 °C, and demand (16) means

$$h \geq 4 \text{ inch} = 10 \text{ cm}. \tag{19}$$

c. Suggested procedure

All this is not very satisfying. If nobody comes up with a better suggestion of how to treat the thermal deformations, I would suggest the following procedure.

1. We specify that the ribs should be channels (preferably of shape H, not U), with h = 4 inch and t = 1/8 inch. This satisfies also the gravitational demand of (15).

2. When delivered, we test the plate in a tent on Mauna Kea, measuring ΔT_{sr} (on clear sunny days) as a function of v, the speed of air circulation.

3. We then specify for the astrodome a circulation speed v such that $\Delta T_{sr} \leq 0.38$ °C.

4. We cross our fingers that $\Delta T_{sr} \leq 0.38$ °C also in the open dome during clear nights (measurements with exposed plate on Mauna Kea).

5. If that does not help, we spray the backside with foam and try again. Fulfilling the demand then is a question of the layer thickness of the foam.

Maybe it would help to repeat the thermal measurements in a tent, using our ESSCO plate, where the ribs are attached with epoxy, having h = 3 inch and only t = 0.040 inch.

6. Skin Thickness

a. Specification

It seems essential that we premit walking on the surface. This eases considerably the measuring, adjusting and painting of the surface. Most important:

we do not yet know the final method for accurate measurements, and maybe it requires walking on the surface.

This means the weight of one man, distributed over the area of one heel, say,

$$\begin{aligned} W &= 250 \text{ lb} = 115 \text{ kg, load,} \\ r &= 1 \text{ inch} = 2.5 \text{ cm, radius of area,} \end{aligned} \tag{20}$$

where this load should only give a negligible permanent deformation of, say,

$$\Delta z \leq 8 \ \mu\text{m}. \tag{21}$$

The manufacturer should make a proper theoretical calculation for his plate design, and an experimental measurement after its fabrication, to be repeated by NRAO.

b. <u>Estimate</u>

For the theoretical estimate we need information about the permanent deformation (set) of the aluminum which is used for the skin, as a function of the stress. If a rod of length ℓ obtains a permanent elongation $\Delta\ell$ after application of stress S, we call

$$p = \Delta\ell/\ell = p(S). \tag{22}$$

Unfortunately, we need this function p(S) for extremely small deformations ($p < 10^{-5}$) at low stress levels ($S < 20$ ksi = 20,000 lb/inch^2), and the available information does not reach so far down. The following estimate may serve as a suggestion for a <u>method</u> of solution, but its numerical result is very uncertain.

We used the Alcoa Structural Handbook and the Metals Handbook of ASM, and Buck Peery phoned the Research Center of Alcoa who then sent a booklet called "Technical Informations." We selected the alloy 6061-T6. Fig. 3 shows the results. The only available numerical table stops already at $p = 10^{-3}$. In addition, we found some graphs which may be read (with low and decreasing accuracy) to about $p = 10^{-4}$. Through the large scatter of these data, we draw a smooth average curve and extrapolate it down to $p = 10^{-5}$. This then is our adopted function p(S).

The stress in a thin plate of thickness t under load W is given in "Formulas for Stress and Strain" (p. 225, No. 37). For a rectangular plate of width b and length a = 2b, supported at all edges, with a uniform load W distributed over the area of a concentric circle of radius r, the stress S in the extreme fiber has its maximum at the plate center, of

$$S = \frac{3\ W}{2\ \pi\ m\ t^2} \left\{ 1 + (m+1)\ \ln \frac{2b}{\pi r} - \beta m \right\} \tag{23}$$

where $m = 1/\nu = 3$, and $\beta = 0.042$. With values (20) and b = 6 inch between ribs, we have

$$S = \frac{248}{t^2} . \qquad \left| \begin{array}{l} S \text{ in lb/inch}^2 \\ \\ t \text{ in inch} \end{array} \right. \tag{24}$$

From this stress, function (22) gives the permanent elongation (and compression) of the extreme fibers. We now ask for the resulting permanent deformation z of the skin perpendicular to its plane. Instead of a rigorous integration over the whole plate size, we assume that the area of radius r (under the load) receives a uniform permanent curvature, its extreme fibers being changed by $\pm \Delta \ell$ of (22), while the remaining part on each side, $b/2 - r$, stays straight after removing the load. Omitting the derivation, this simplification yields, with b = 6 inch and r = 1 inch,

$$\Delta z = p \frac{r(b-r)}{t} = 5.00\ p/t. \tag{25}$$

We now assume various values for t; from (24) we find $S(t)$, from Fig. 3 we get $p(S)$, and (25) finally yields Δz. The result is shown in Fig. 4. The solid part of the curve is obtained from the range $S \geq 28$ ksi of Fig. 3, where scanty data are still available; the broken line in Fig. 4 comes from the extrapolated range $20 \leq S \leq 28$ ksi, while the dotted line is a further extrapolation. We just wanted to show how the method would run if proper information about $p(S)$ could be obtained. Specification (21), with Fig. 4, then yields a skin thickness of $t \geq 0.117$ inch; or, probably being on the safe side,

$$t \geq 1/8 \text{ inch.} \tag{26}$$

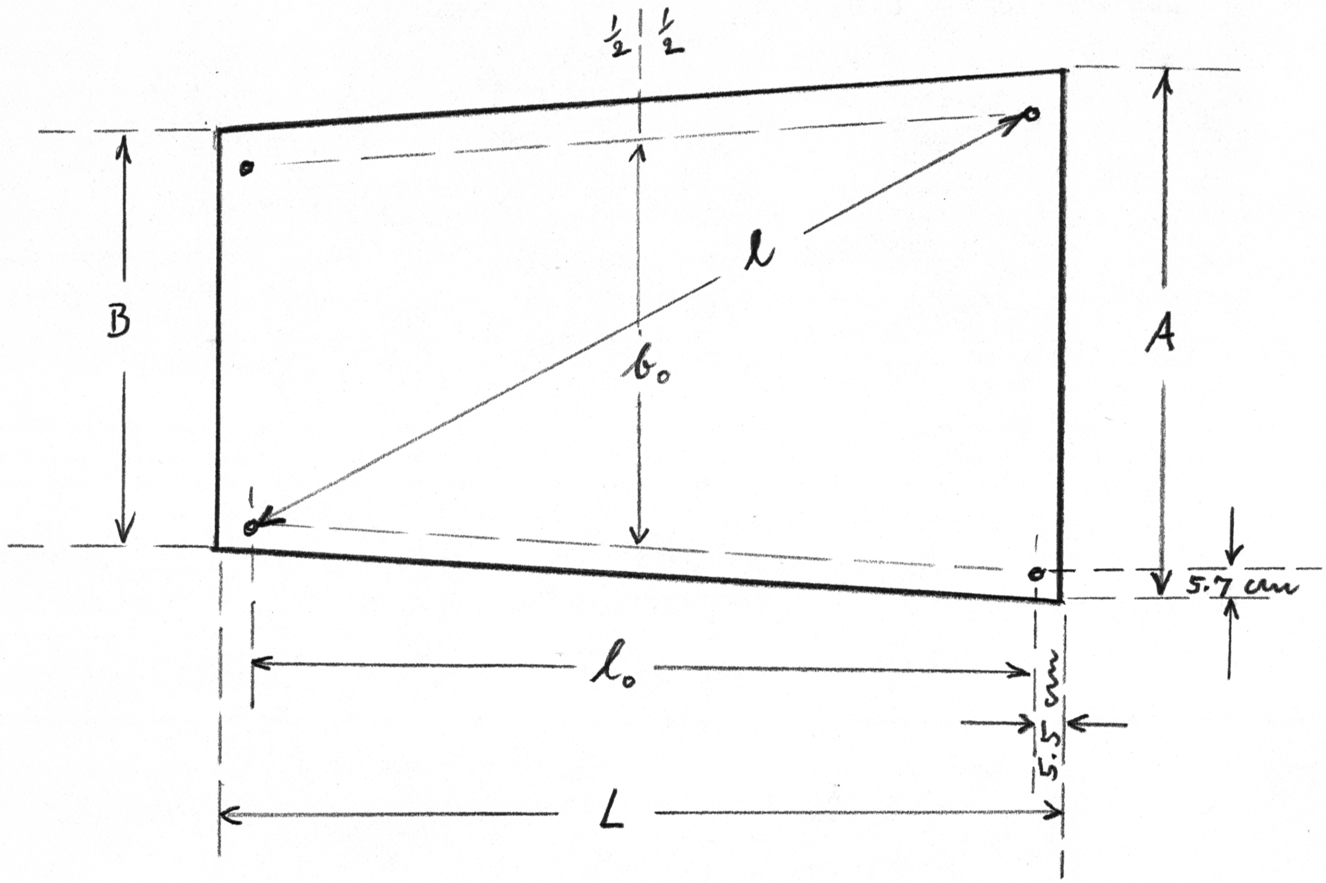


Fig. 1. Dimensions of surface plate (in centimeter).

L = 151.30
A = 91.69
B = 72.70

ℓ_o = 140.30
b_o = 70.80

Length of effective diagonal:

ℓ (gravity) = 142.52
ℓ (thermal) = 157.15

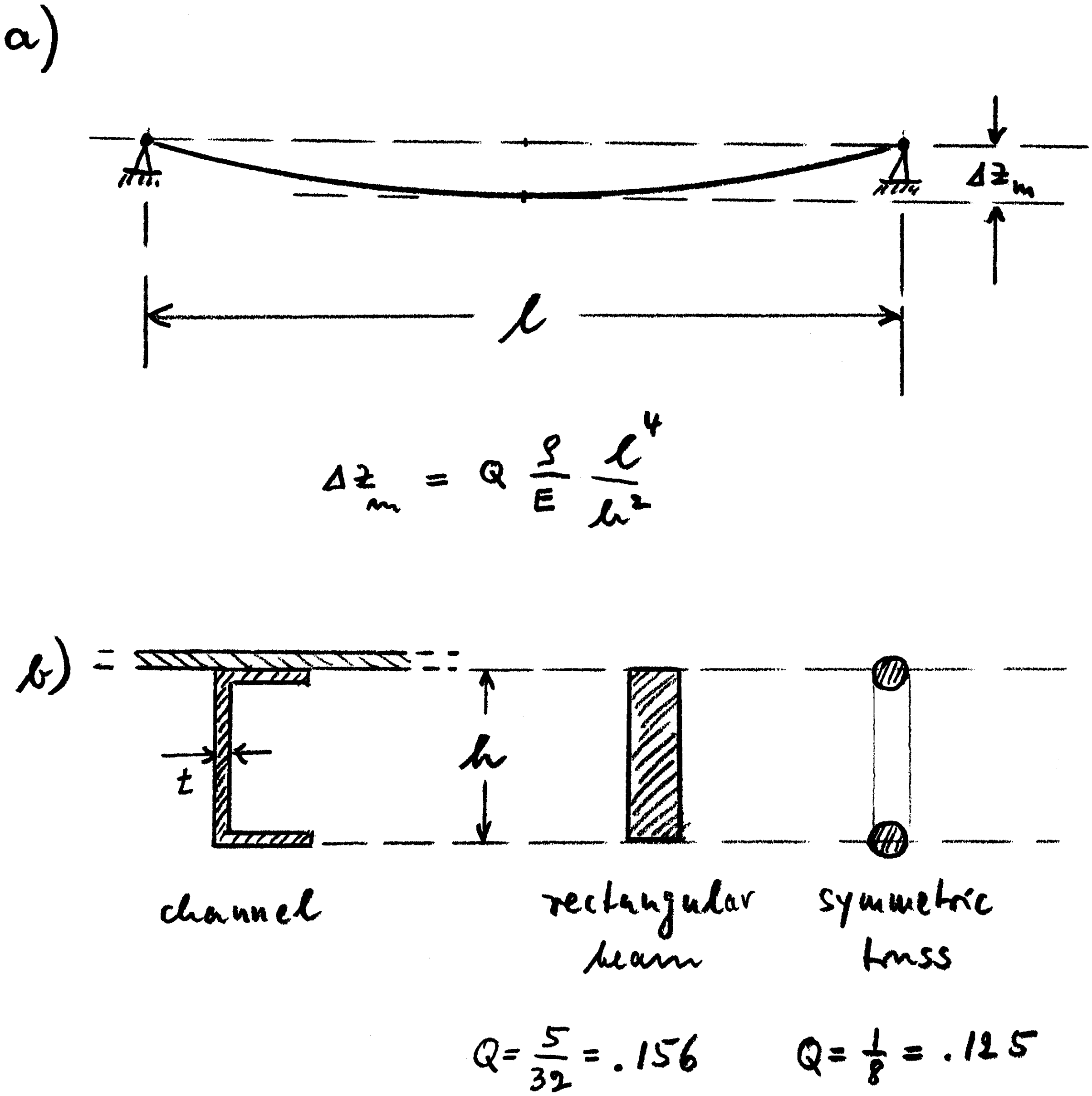
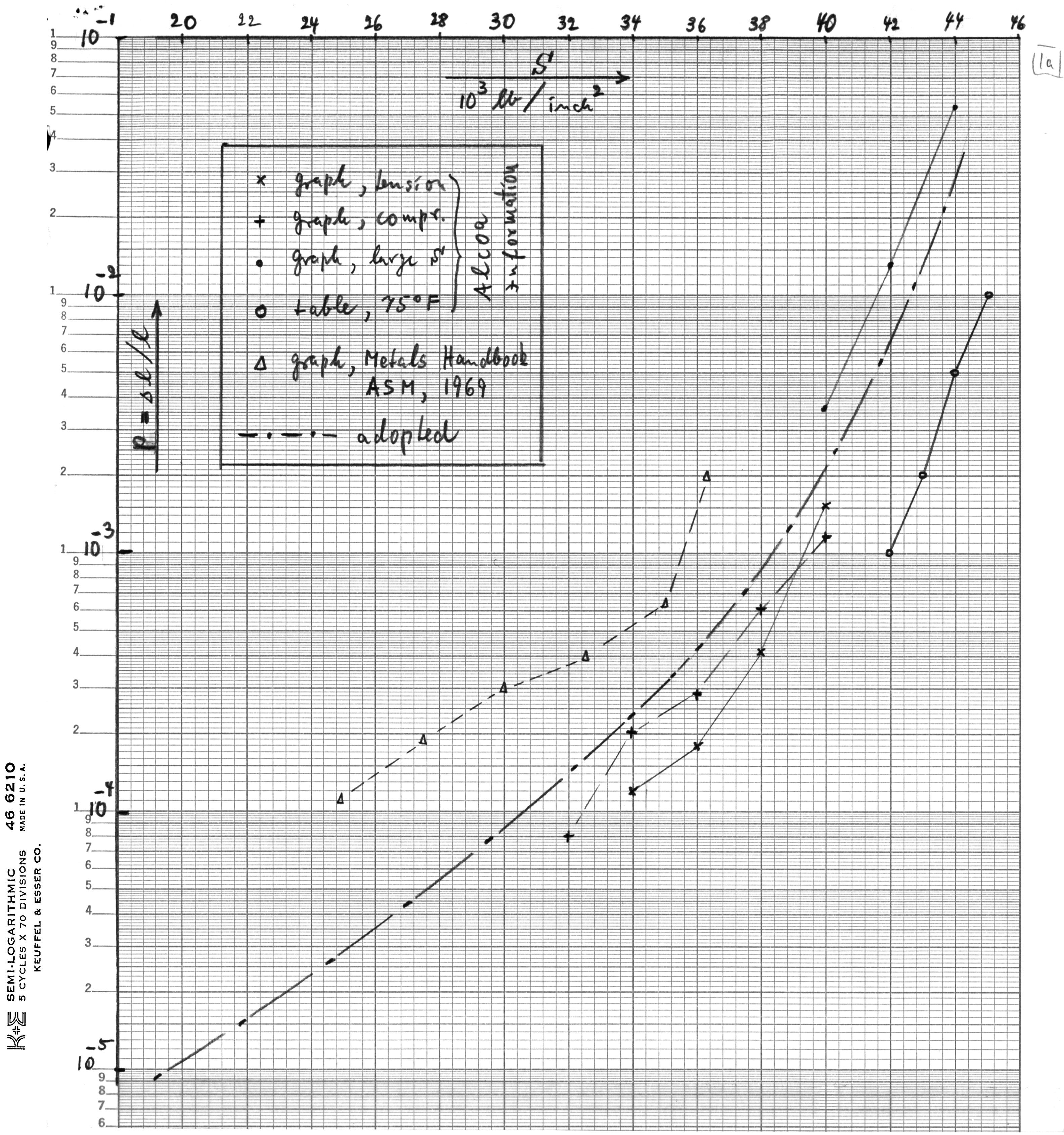


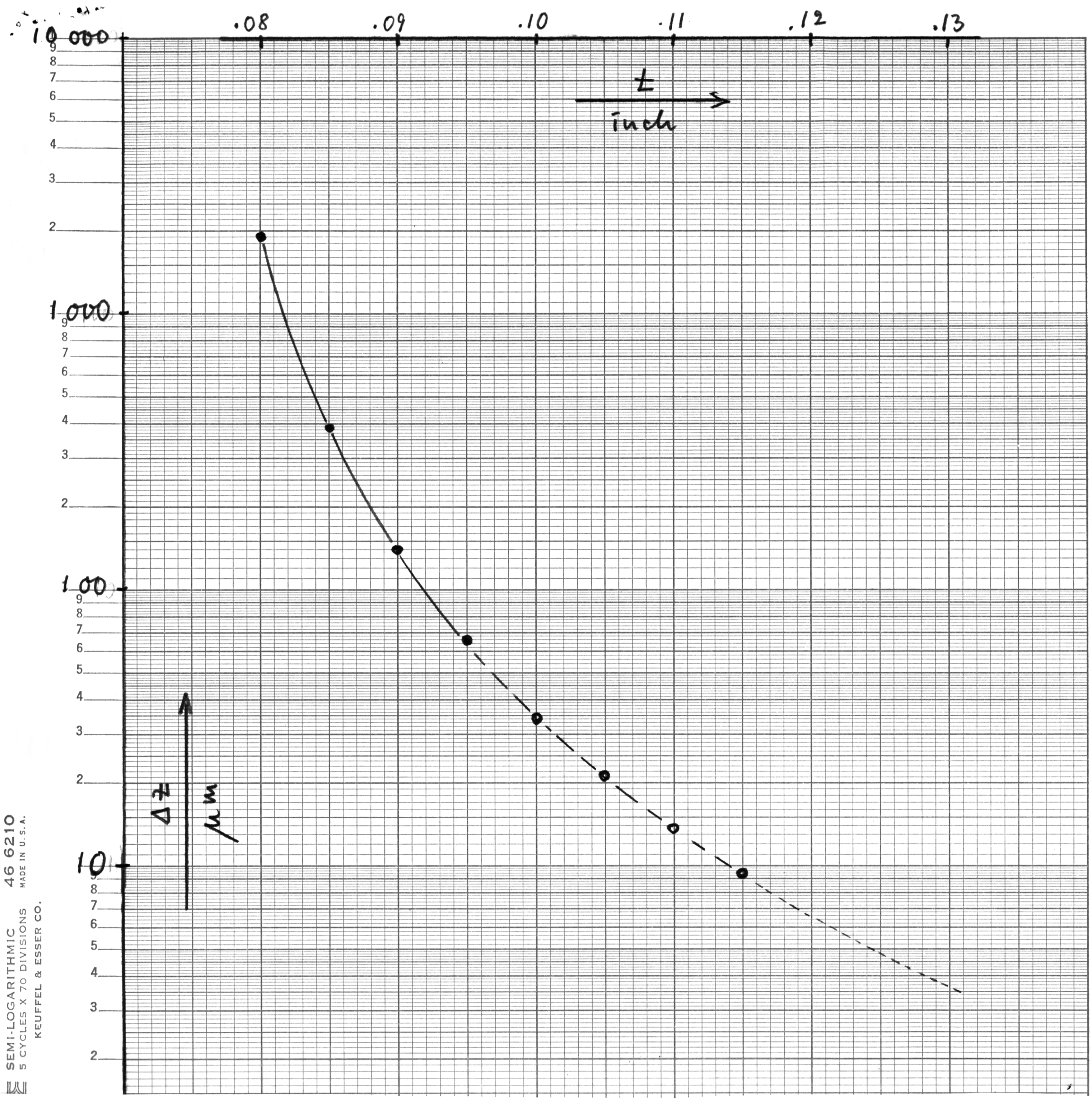
Fig. 2. Gravitational deformation of beam under dead load.

a) maximum deformation at center;

b) cross section, and simplifications.



<u>Fig. 3.</u> Permanent deformation of aluminum, as a function of stress.



<u>Fig. 4.</u> Permanent deformation of skin, Δz, after walking on surface, as a function of the skin thickness, t.