25 Meter Millimeter Wave Telescope
Memo # 106

TELESCOPE SURFACE MEASUREMENT WITH TWO FEEDS

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Summary

A method is suggested where a reference feed is fixed at the telescope focus, and a second feed has a lateral offset and scans the focal plane. The telescope tracks a celestial radio source, and the receiver measures the phase difference at the two feeds and their (voltage) amplitude ratio. A set of coordinate transformations is applied, after which the surface deviations σ from a paraboloid can be obtained by a Fourier transform. The measuring error is discussed, and the best wavelength is found to be between 14 and 21 σ.

1. Introduction

Accurate measurements of the surface shape of radio telescopes are needed for two cases: adjustments of the surface panels for a selected elevation angle, and compensating the gravitational deformations at other elevations with a deformable subreflector [1]. The best adjustment angle is between 42° and 50° elevation for most telescope structures [2], and one would like to measure the surface, for both cases, at various elevations over the full range from zenith to horizon. Recent accurate measuring methods are mostly confined to zenith pointing [3], and the relative deformations between zenith and other pointings could then be measured by a radar method [4]. But if possible, one would like to have one and the same absolute method for all pointings from zenith to horizon, and the method should also be fast enough to avoid thermal changes in between.

Furthermore, it would be nice to have a method which (a) uses the fact that a telescope surface is almost a paraboloid of revolution and that only small deviations σ from it ought to be measured, and which (b) could use available radio equipment. Since not only amplitude must be measured but also phase (pathlength differences from surface deviations), one needs two receiving feeds. A "holographic method" of this kind was suggested by Bennet et al. [5], using a transmitter at a large distance, a fixed reference feed horn close to the antenna, and scanning the antenna in azimuth and elevation; but because of the needed transmitter, the method is confined to pointing at horizon.

For avoiding this limitation, Scott and Ryle [6] use an unresolved celestial radio source, scanning the antenna in both directions by several beamwidths, and using a second antenna as phase reference which is always centered at the source. If one wants a spatial resolution of n points per diameter, or n^2 measured surface points, one must scan up to n beamwidths off center, or πn^2 scan samples. The surface deviations σ are then obtained, by a simple Fourier transform of ~~a~~ the scanned field, with an rms error of $\Delta\sigma = n\lambda/4\pi R$, where λ is the wavelength, and R is the signal/noise if centered on source and for the same integration time as used for a single scan sample. This method allows measurements at all elevations, but it is confined to cases where a second telescope is available nearby, at less than 1 km distance because of tropospheric irregularities. The spatial resolution is limited by the demand that the gravitational deformations should not change appreciably while the telescope tracks the celestial rotation of sky and source, which limits the integration time for a given pointing. Scott and Ryle [6] obtained a resolution of about 1 m for a 13 m diameter telescope, with an accuracy of $\Delta\sigma \approx 0.1$ mm, for a total duration of 2.5 hours per pointing, at $\lambda = 1.95$ cm.

In the following, we suggest a variation of this method which can be applied to single telescopes. The telescope is always centered at the celestial source; a reference feed stays fixed at the telescope focus, and a movable feed is displaced laterally, scanning the field of the focal plane within a distance $n\lambda$. The spatial resolution is again limited by the integration time. This is no problem for the study of gravitational deformations which mostly will have a long correlation length. But if a higher resolution is wanted, for adjusting many (or internally bumpy) surface panels, one should use in addition the zenith measurement of Payne, Hollis and Findlay [3].

2. One-Dimensional Case

We regard the pathlength $L = L_1 + L_2$ in the one-dimensional example of Fig. 1, with all possible offsets. In the undisplaced case, zero offsets, we have $L_1 = F + z$ and $z = x^2/4F$ for a parabola, $L_2 = F - z$, and $L = 2F$. With all three offsets and a surface displacement, we have a pathlength change $\Delta L = \Delta L_1 + \Delta L_2$, with $\Delta L_2 = -\gamma x - \sigma$, and

$$\Delta L_1 = \zeta \frac{F-z}{F+z} - \xi \frac{x}{F+z} - \sigma(x) \frac{F-z}{F-z} . \tag{1}$$

The phase change then is $\phi = 2\pi \, \Delta L/\lambda$ which we divide into two parts, ϕ_o from offsets and ϕ_s from surface deviations, with $\phi = \phi_o + \phi_s$, where

$$\phi_o = \frac{2\pi}{\lambda} \left\{ \zeta \frac{F-z}{F+z} - \gamma x - \xi \frac{x}{F+z} \right\} , \tag{2}$$

$$\phi_s = - \frac{2\pi}{\lambda} \frac{2F}{F+z} \sigma(x). \tag{3}$$

Calling $a(x)$ the amplitude to be discussed later, and τ any one of the offsets ξ, ζ or γ, the field (A,ψ) at the location of the feed then is described by

$$A(\tau) \, e^{i\psi(\tau)} = \int_{-D/2}^{+D/2} a(x) \, e^{i\phi_s(x)} \, e^{i\phi_o(x,\tau)} \, dx. \tag{4}$$

From equations (2) and (4) we recognize the difference between the three offsets if used for scanning. First, scanning the antenna beam across the source by varying γ (keeping $\zeta = \xi = 0$) yields for equation (4) exactly the form of a Fourier transform as used by Scott and Ryle, where $\sigma(x)$ can be obtained by a reversing Fourier transform of the measured field (A,ψ). Second, scanning the feed along the axis by varying ζ is useless for finding $\sigma(x)$ since ϕ_o depends only on $z = x^2/4F$ which does not distinguish between $-x$ and $+x$. Third, scanning the feed laterally by varying ξ would give a Fourier transform if we had in equation (2) just the product ξx instead of $\xi x/(F+z)$. But this required form can be obtained by a transformation of coordinates, using $q = x/(F+z) = \sin\theta$, instead of x:

$$q = \frac{x}{F + x^2/4F} \tag{5}$$

The integration limit q_o for equation (4) is, with $\Phi = F/D$ and θ_o = rim illumination angle,

$$|q| \leq q_o = \frac{8\Phi}{1 + 16\Phi^2} = \sin\theta_o. \tag{6}$$

If we actually perform a lateral scan, using a fixed reference feed, we do not know the exact position (ζ,ξ) of the reference, nor the exact direction (γ) of the beam in case of a complicated or split-up beam. If we neglect all three, we will obtain the deviations $\sigma(x)$ of the telescope surface from a parabola which has its focus at the reference, and its axis perpendicular to the scan direction (ξ). A least-squares solution for the best-fit parabola, applied to $\sigma(x)$ and having these degrees of freedom, will then yield all three unknowns.

3. Lateral Scan in Two Dimensions

For an application, we must add the second dimension and the amplitudes. As to coordinates, we use for the aperture $x = r \cos \alpha$ and $y = r \sin \alpha$; in the focal plane we use $\xi = \lambda\rho \cos \beta$ and $\eta = \lambda\rho \sin \beta$, where ξ is parallel to x and η to y, where α and β have the same origin and direction, and where the lateral offset ρ is measured in wavelengths. The feed illumination pattern shall be described by $I(\theta,\alpha)$ in terms of voltage, where $G(\theta,\alpha) = I^2$ is the feed gain in terms of power, as a function of the angular distance θ from the axis (Fig. 1), and of the azimuth α in case of an elliptic pattern. We shall normalize $I(o,\alpha) = 1$ on the axis.

We use the coordinate transformation (5), replacing x by r,

$$q = \frac{r}{F + r^2/4F} \tag{7}$$

$$r = 2F \, \frac{1 - \sqrt{1 - q^2}}{q} \tag{8}$$

$$dr = 2F \, \frac{1 - \sqrt{1 - q^2}}{q^2 \sqrt{1 - q^2}} \, dq \tag{9}$$

with the integration limit q_o given by equation (6). In addition, we transform $\sigma(r,\alpha)$ into $s(q,\alpha)$ by

$$s(q,\alpha) = 2\pi \, (1 + \sqrt{1 - q^2}\,) \, \sigma(r,\alpha) \, / \, \lambda. \tag{10}$$

The phase change then is

$$\phi = \phi_s + \phi_o = - s(q,\alpha) - 2\pi q\rho \cos(\alpha - \beta). \tag{11}$$

We use two feeds, with feed 1 at the focus and feed 2 scanning laterally, and the receiver shall measure the phase difference

$$\psi = \psi_2 - \psi_1 \tag{12}$$

and the (voltage) amplitude ratio

$$A = A_2 / A_1 . \tag{13}$$

Regarding the individual ray and its contribution to the measured amplitude, we keep in mind that the voltage decreases as 1/distance = $1/L_1$ of Fig. 1, where $L_1 = F + z$ and $2F/(F+z) = 1 + \cos\theta$, with $q = \sin\theta$. We express r dr in terms of q from equations (8) and (9) and have

$$\frac{F}{F + z} \; r \, dr = 2 \, F^2 \, Q(q) \, q \, dq \tag{14}$$

with

$$Q(q) = \frac{1 - \sqrt{1 - q^2}}{q^2 \sqrt{1 - q^2}} . \tag{15}$$

Calling

$$B(q,\alpha) = I(q,\alpha) \, Q(q) \, a(q,\alpha) \tag{16}$$

and summing up all contributions, we then have

$$A(\rho,\beta) \, e^{i\psi(\rho,\beta)} = \int_0^\infty \int_0^{2\pi} B(q,\alpha) \, e^{-i \, s(q,\alpha)} \, e^{i \, 2\pi \, q\rho \, \cos(\alpha-\beta)} \, q \, dq \, d\alpha \tag{17}$$

where the last exponential term has the form required for a Fourier transform, and where the phase is normalized by $\psi_1 = 0$. The individual amplitude $a(q,\alpha)$ in the aperture contains all normalizing factors; it is proportional to the flux of the radio source, times the telescope area, divided by the reference amplitude A_1. As a check for the validity of the measuring method, we use the knowledge that the amplitudes must be, except for the shadows of receiver box and support legs, with q_o from equation (6):

$$a(q,\alpha) = \begin{cases} \text{constant}, & \text{for } q < q_o \,, \\ 0 & q > q_o \,. \end{cases} \tag{18}$$

Written in real terms, the reversing Fourier transform yields $B(q,\alpha)$ and $s(q,\alpha)$ in the aperture, from the measured amplitude ratio $A(\rho,\beta)$ and phase difference $\psi(\rho,\beta)$ in the focal plane, as

$$\begin{aligned} B \cos s &= +\int_0^{\infty}\int_0^{2\pi} A \cos[\psi + 2\pi\, q\rho\, \cos(\alpha-\beta)]\ \rho\, d\rho\, d\beta, \\ B \sin s &= -\int_0^{\infty}\int_0^{2\pi} A \sin[\psi + 2\pi\, q\rho\, \cos(\alpha-\beta)]\ \rho\, d\rho\, d\beta. \end{aligned} \tag{19}$$

From the values $B(q,\alpha)$ we check with equations (15) and (16), and with the known feed pattern $I(q,\alpha)$, whether equation (18) is fulfilled. Finally, the wanted surface deviations $\sigma(r,\alpha)$ are obtained as

$$\sigma(r,\alpha) = \frac{\lambda}{2\pi}\ \frac{s(q,\alpha)}{1+\sqrt{1-q^2}} \tag{20}$$

with $r(q)$ from equation (8). These are the deviations from a paraboloid which has its focus at the phase center of the reference feed, and its axis perpendicular to the plane of scanning. The deviations are defined parallel to the telescope axis, and positive is up, see Fig. 1.

Up to now, we have assumed that the measurements are performed in the plane of the prime focus, scanning off axis up to a distance of n beamwidths, for a resolution of n points per diameter. It is also possible to apply the method in the plane of a Cassegrain or Gregorian focus, scanning up to a distance increased by the magnification factor m. In this case, the resulting σ from equation (20) is a combination of the surface deviations of both primary and secondary mirror, and of all misalignments of the secondary which also will show gravitational deformations depending on elevation angle.

4. The Measuring Error

The rms error $\Delta\sigma$ of the deviations σ are for the suggested two-feed method the same as for the method of Scott and Ryle [6] who derive

$$\Delta\sigma = \frac{n\lambda}{4\pi R} \tag{21}$$

if n^2 surface points are to be measured, and where λ is the wavelength, R is the signal-to-noise ratio at the reference feed, for an integration time t equal to the duration of a scan sample, or $1/n^2$ of the total observing time for a given telescope pointing which is limited to one or two hours because of the changing gravitational deformations. The signal/noise is proportional to

$$R \sim \frac{S\,\eta\,\sqrt{bt}}{T} \tag{22}$$

where S is the flux of the source, $\eta \sim \exp\left[-(4\pi\sigma/\lambda)^2\right]$ is the aperture efficiency, b the bandwidth, $t \sim n^{-2}$ the sample integration time, and T the system noise temperature. Thus

$$\Delta\sigma \sim n^2\,\frac{T\,\lambda}{S\,\sqrt{b}}\;e^{(4\pi\sigma/\lambda)^2}\;. \tag{23}$$

We see that $\Delta\sigma \sim n^2$ for a limited total time. The best wavelength λ should be chosen such that (23) is minimized, which depends on the available receivers (T and b) and radio sources (S). For many cases, observations at the strong water line may be best, at $\lambda = 1.35$ cm.

In case of continuum observations, we use $S \sim \nu^{-k} \sim \lambda^k$, where ν is the frequency and k the spectrum index; we shall assume $b \sim \nu \sim \lambda^{-1}$ and T = constant for simplicity. Then

$$\Delta\sigma \sim n^2 \, \lambda^{1.5-k} \, e^{(4\pi\sigma/\lambda)^2} \tag{24}$$

which has its minimum at

$$\lambda = \frac{8\pi}{\sqrt{3 - 2k}} \, \sigma. \tag{25}$$

For example, $\lambda = 14\,\sigma$ for $k = 0$ (flat spectrum), and $\lambda = 21\,\sigma$ for $k = 0.8$ (normal spectrum). We see that the best wavelength is about the same as the shortest wavelength of observation, usually adopted as $16\,\sigma$, where the efficiency is degraded by a factor of two.

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References

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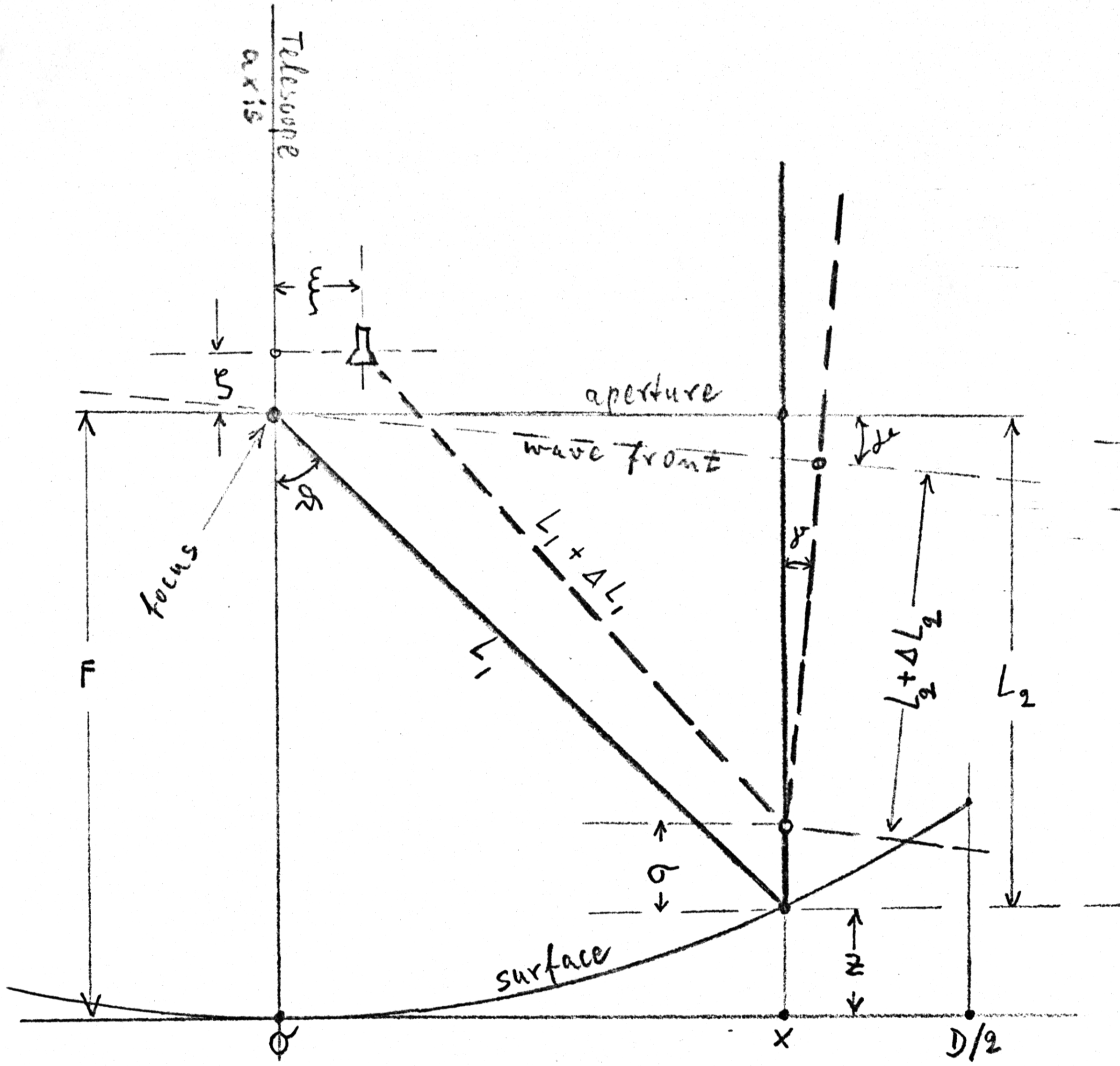
Figure Caption

Fig. 1. Geometry of parabolic reflector.

Heavy lines are ray paths, full line for undisturbed case. Broken line with beam tilt γ, axial feed displacement ζ, lateral displacement ξ, and surface deviation σ.



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