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## SURFACE PLATES FOR A WAVELENGTH OF 1.2 MILLIMETER

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I. Introduction and Summary

It seems that commercial firms cannot meet at present the accuracy of about  $10^{-3}$  inch as needed for 1.2 mm wavelength. This may improve in the future and it may well become good enough when actually needed. Meanwhile, however, we definitely need some design of surface plates, if necessary of our own, fulfilling the demands:

- (a) the design must guarantee the accuracy desired,
- (b) should not be too expensive,
- (c) need not be the best possible, and may look awkward.

This was exactly the same situation when we started five years ago with the 65-m design. No firm could then meet the accuracy of  $3 \times 10^{-3}$  inch desired for  $\lambda = 3$  mm wavelength (which they do meet now). We thus developed our own plate design, and we fabricated and measured three such plates at our NRAO work shop at Green Bank, with good results.

The same should be done at present. The easiest solution seems to be just taking our old plate design, and to improve it until the desired accuracy is met. If successful, this will allow us to proceed further-on with the telescope design and the site selection, without being hampered by the objection that we finally may get stuck with having no good surface.

A similar situation holds for the final accuracy of measuring the whole telescope and of setting the plates. Again, present commercial measuring techniques are not good enough, and we must develop our own, which is being done by John Findlay and John Payne.

A discussion of the desired accuracy shows that our present plates must be improved by 55%. This demand is met if we scale down the plate size from 72 to 55 inch, and if the temperature differences between skin and ribs are kept  $\leq 1.5^\circ\text{F}$ . The measuring accuracy of Findlay and Payne needs an improvement of only 25%.

## II. The Accuracy Needed

Defining  $\Delta z_t$ , the total allowed rms surface deviation from the best-fit paraboloid, as 1/16 of the shortest wavelength to be used, we have

$$\Delta z_t = 1.2 \text{ mm}/16 = .075 \text{ mm} = 2.95 \times 10^{-3} \text{ inch.} \quad (1)$$

In a well-balanced design, all major contributions to  $\Delta z_t$  will be about equal. For  $n$  such contributions we then must demand, for the single item,

$$\Delta z_i = \Delta z_t / \sqrt{n}. \quad (2)$$

In our case, the major contributions are (grouping all gravitational deformations together in one single item, since they are  $\sim D^2$  and thus relatively small for a smaller telescope):

1. Plate manufacturing accuracy
  2. Telescope measurement and adjustment
  3. Plates
  4. Panel structures
  5. Backup structure
  6. All gravitational deformations
- } thermal deformations

Thus, with  $n = 6$ , our demand for each item is

$$\Delta z_i = \Delta z_t / \sqrt{6} = .0306 \text{ mm} = 1.204 \times 10^{-3} \text{ inch.} \quad (3)$$

The gravitational deformation is again a result from three contributions (plates, panels, backup), thus the gravitational deformation of the plates must be only

$$\Delta z_{pg} = \Delta z_i / \sqrt{3} = .69 \times 10^{-3} \text{ inch.} \quad (4)$$

The total contribution of the surface plates, from their manufacturing accuracy, their thermal deformations from temperature differences between skin and ribs, and their gravitational deformations when the telescope is tilted,

shall not surpass

$$\Delta z_p = (1.2^2 + 1.2^2 + .69^2)^{1/2} = 1.89 \times 10^{-3} \text{ inch} = .0480 \text{ mm.} \quad (5)$$

We have neglected the wind deformations, because they are already very small for the present plate size and scale down with its third power. They would be omitted altogether in case of a radome or astrodome.

The measuring accuracy for the whole telescope (including the setting accuracy of the plate corners, which should be made negligible) shall not surpass (3), or

$$\Delta z_{mT} = .0306 \text{ mm} = 1.204 \times 10^{-3} \text{ inch.} \quad (6)$$

### III. The NRAO Plate for the 65-m Telescope

The adjustable NRAO plate is summarized in our book (Findlay and von Hoerner: "A 65-m Telescope for Millimeter Wavelength", 1972) and details of its design and performance are given in Reports No. 36 (Jan. 20, 1971) and No. 38 (April 12, 1971).

The plate is made from off-the-shelf aluminum pieces. A skin is riveted to a system of upper ribs, which are connected to lower ribs by 36 adjustment screws which provide the accuracy. The plate size is 72 x 29 inch, the depth between skin and lower ribs is 5 inch at the center. The average distance between neighboring adjustment screws is  $d_0 = 7.9$  inch, and  $12^\circ$  of turn at a screw gives  $10^{-3}$  inch of adjustment. Adjusting one plate took 3 hours for 2 men at Green Bank, with a scale and a very accurate level; but it should not take more than 3/4 hour for one man in a factory, with a jig with 36 dial indicators. The 36-m telescope needed  $N = 3072$  such plates. The cost, including manufacturing, shipping, installing and adjusting on the telescope, was estimated as  $P = 40\$/\text{ft}^2$  for the beginning of 1972.

Our accuracy of measuring and adjusting the 36 points was  $.95 \times 10^{-3}$  inch (average over 3 plates made at NRAO), and the internal bumpiness of the skin

(after walking all over the plate) was  $2.00 \times 10^{-3}$  inch, both adding up to a manufacturing accuracy of  $2.21 \times 10^{-3}$  inch.

The thermal deformation contributed  $.90 \times 10^{-3}$  inch/°F (with  $\Delta T = 2^\circ\text{F}$  adopted for clear nights in the open, from a long series of measurements). The gravitational deformation gave  $1.40 \times 10^{-3}$  inch if adjusted for zenith and tilted to horizon, or  $.70 \times 10^{-3}$  inch for a reasonable short-wavelength limit of  $30^\circ$  above horizon. The wind deformation was  $.60 \times 10^{-3}$  inch for 18 mph wind (3/4 of all time at Green Bank).

The total rms error of our present NRAO plates, from manufacturing accuracy, thermal deformations ( $\Delta T = 2^\circ\text{F}$  between skin and ribs), and gravitation deformations (from zenith to  $30^\circ$  above horizon), then is

$$\Delta z_p = (2.21^2 + 1.80^2 + 0.70^2)^{1/2} = 2.93 \times 10^{-3} \text{ inch} = 0.0745 \text{ mm}, \quad (7)$$

which is too high, as compared to our demand of equation (5), by a factor of 1.55.

#### IV. Improvements and Scaling

The accuracy of measuring and adjusting the 36 points should immediately improve when this is done in a factory on a jig with 36 simultaneously and easily visible dial indicators. Setting the 36 points should be at least as good as  $\Delta z = 0.50 \times 10^{-3}$  inch.

The largest single contribution of the present plate is the internal bumpiness of the skin surface. It may be improved by taking a thicker skin (1/8 inch at present), but an estimate would need new plates to be made and measured. Another improvement can be achieved by choosing a smaller distance  $d$  between neighboring adjustment screws, to be most easily obtained by just scaling

the present plate design down to smaller sizes (length  $\ell_0 = 72$  inch at present). The resulting improvement can be predicted from already available measurements of the autocorrelation of the surface bumpiness. This was done (Report No. 30, Fig. 6) on larger pieces of skin, with the result  $\Delta z \sim d^{1.3}$ , and a second time on the present NRAO plate between the screws, with  $\Delta z \sim d^{1.8}$ . For further use we adopt  $\Delta z \sim d^{1.5}$ . Together with the adjustment accuracy of  $0.50 \times 10^{-3}$  inch, and from the present bumpiness of  $2.00 \times 10^{-3}$  inch, we obtain the manufacturing accuracy as

$$\Delta z_{pm} = \left\{ 0.25 + 4.00(\ell/\ell_0)^3 \right\}^{1/2}. \quad (8)$$

The thermal deformations go with  $\Delta z \sim \ell$ , and the gravitational ones with  $\ell^2$ . Wind deformations go with  $\ell^3$ , if the same rib type is used, but shall be neglected.

Regarding other plate designs, or offers from firms, it should be mentioned that the slenderness ratio  $s$ , between plate length and center depth, must not be larger than our present one ( $72/5 = 14.4$ ). Thermal deformations increase with  $s$ , gravitational ones with  $s^2$ . The following assumes

$$s = 14.4. \quad (9)$$

We call  $S = \ell/\ell_0$  the scaling factor. The total contribution of the plates (manufacturing, thermal and gravity) then is

$$\Delta z_p = \left\{ 0.25 + 0.81 S^2 (\Delta T)^2 + 4.00 S^3 + 0.49 S^4 \right\}^{1/2}. \quad (10)$$

#### V. Application to a Telescope of 25 Meter Diameter

In a recent paper ("Radio Telescopes for Millimeter Wavelength", submitted to the Astronomical Journal) I found that scaling our 65-m design down, from a wavelength of  $\lambda = 3.5$  mm to one of  $\lambda = 1.2$  mm, would yield a maximum telescope

diameter of  $D = 22$  m, with a pointing error of 2.16 arcsec (1/6.3 of the beam-width), and for a total cost of about 4.0 M\$. But since some small improvements of this design seem possible, we use  $D = 25$  m in the following.

The scaling factor for the telescope as a whole then is  $S_t = 25/65 = 0.385$ . Scaling the surface plates by the same factor would be much more than needed, and would result in a design very awkward for the manufacturing. On the other side, a complete redesign of our panel structure (which is held by the backup, and holds the plates), or even a redesign of our backup structure, as needed for fitting some arbitrary plate length  $\ell$ , would be an undesirable complication. The easiest change, then, is to let the panels hold only 2 rings of plates (instead of the present 4). This means a scaling factor for the surface plates of

$$S = 2S_t = 0.769. \quad (11)$$

These plates then have a size of 55 by 22 inch, and  $N = 768$  of such plates are needed, mounted in 9 rings with an average of 80 plates (of identical shape) in each ring.

With this scaling factor, the resulting single contributions of the plates are now

$$\left. \begin{array}{ll} \text{manufacturing accuracy} & \Delta z_{pm} = 1.44 \\ \text{thermal deformations} & \Delta z_{pt} = 0.69 \Delta T \\ \text{gravitational deform.} & \Delta z_{pg} = 0.41 \end{array} \right\} \times 10^{-3} \text{ inch} \quad (12)$$

For  $\Delta T = 2^\circ\text{F}$ , as used for the 65-m design during clear nights in the open, this would add up to a total of  $\Delta z_p = 2.04 \times 10^{-3}$  inch, just a little high as compared to our demand of  $1.89 \times 10^{-3}$  inch from equation (5). For structural reasons, we do not want to diminish the plate size nor its slenderness ratio. The manufacturing accuracy thus cannot easily be improved, unless some firm will come up with a

different and better design. On the other side, a radome or astrodome seems very much desired by the astronomers, for improving the performance during the day by shadowing against sunshine. If we are optimistic and expect at least a slight improvement also during nights, we may turn this into a specification (regarding the dome and its ventilation) for the maximum temperature difference between skin and ribs of

$$\Delta T = 1.5^\circ\text{F}. \quad (13)$$

If this specification is met, we obtain  $\Delta z_{pt} = 1.04 \times 10^{-3}$  inch, and a total plate contribution (manufacturing, thermal, gravity) of

$$\Delta z_p = 1.82 \times 10^{-3} \text{ inch} = 0.046 \text{ mm}. \quad (14)$$

This shows that our present plate design, scaled to a size of 55 by 22 inch, is able to meet the accuracy as demanded by equation (5) for a wavelength of  $\lambda = 1.2$  mm.

The cost of the old plate was  $P = 40$  \$/ft<sup>2</sup>, including manufacturing, shipping, erection and adjustment on the telescope. We assume  $P \sim \ell^n$  for different items. Then,  $n = -2$  for the labor on screws, rivets, and adjustment (both internal and on telescope);  $n = -1$  for measuring, cutting and handling all pieces, for shipping and erection; while  $n = 0$  for the cost of all material. (Note: this is cost per ft<sup>2</sup>, not per plate!) In the average, we adopt  $n = -1.7$ , or  $P = 40 \text{ S}^{-1.7} = 62.6$  \$/ft<sup>2</sup>. In addition, we apply an inflation rate of 7% per year, for three years = 22.5%, and we obtain for the beginning of 1975:

$$P = 77 \text{ $/ft}^2. \quad (15)$$

The single plate has  $8.40 \text{ ft}^2$  and costs 647 \$. The whole 25-m telescope has 768 plates, and its surface costs

$$P_t = 500,000 \text{ \$}, \quad (16)$$

after erection and adjustment.

VI. The Measuring Accuracy on the Telescope

Dr. Findlay and John Payne have worked out a measuring method combining two procedures: (a) the surface details are measured by moving a "mouse" along many radial paths; this is a 20 inch long bar on wheels at both ends, with a digital distance indicator at its center, measuring the skin curvature; two integrations then yield the surface shape approximately, or inverting a simple triangular matrix would yield it exactly. (b) Several points along the rim are directly measured with a modulated lazer beam. This method was tested on the 36-foot telescope, and after some improvements and more tests it should give an accuracy (at distance  $r$  from the dish center, with  $R = 25$  m) of

$$\Delta z_{mT} = \left\{ (1.00)^2 + (2.00 r/R)^2 \right\}^{1/2} \times 10^{-3} \text{ inch.} \quad (17)$$

For getting the rms accuracy of the whole dish, we must weigh (17) with the illumination pattern. I have taken a "parabola with pedestal", with 15 db down at the rim. The weight function consists of three terms: (a) the area per radius, increasing as  $r/R$ ; (b) the parabolic tapering; (c) the surface slope, increasing to the rim. In total we have the weight function, with  $F =$  focal length,

$$w = \frac{r}{R} \left\{ 1 - 0.968(r/R)^2 \right\} \left\{ 1 + (r/2F)^2 \right\}^{-1/2} \quad (18)$$

We multiply (17) by (18), integrate over  $dr$ , and divide by the total weight  $\int w dr$ . The resulting total rms measuring accuracy for the whole telescope then is

$$\Delta z_{mT} = 1.50 \times 10^{-3} \text{ inch} = 0.038 \text{ mm,} \quad (19)$$

which is almost good enough as compared to the demand of (6). It needs a further improvement of only 25 per cent.