REPORT 28

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Reducing the Influence of Sunshine by Blowing Ambient Air through the Feed Support Legs

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Summary

The temperature distribution along the feed legs, and the average temperature difference ΔT, between leg and surrounding air, are calculated for the case that the leg is heated by sunshine and is cooled by blowing ambient air through it with velocity v. In order to reduce ΔT from 5 °C (for v = 0) to 2 °C as used in Reports 23 and 25, a speed of v = 56 ft/sec is needed if the legs consist of 4 inch diameter pipes and if a length of 137 feet is exposed to sunshine. The telescope has four legs with three pipes each, and each pipe needs a fan with a motor of 0.5 horsepower.

Compression and friction heat up the air by an amount of $\Delta T_c \sim v^2$. For v = 56 ft/sec, ΔT_c = 1.70 °C which can be neglected since it is the same for all four legs. If $\Delta T < 2$ °C is wanted, ΔT_c would become too large, and a styrofoam insulation is suggested. With 1/2 inch of foam and v = 30 ft/sec, for example, ΔT = 0.90 °C and ΔT_c = 0.48 °C.

1. The Task

The pointing error, resulting from thermal deformations of the feed support legs of the proposed 300-ft homologous telescope, is given in Report 23 (March 1, 1969) as

$$\Delta\vartheta = 2.51\,\Delta T, \tag{1}$$

with $\Delta\vartheta$ in arcsec, and ΔT in $^{\circ}C$. This would be 12.6 arcsec for $\Delta T = 5\ ^{\circ}C$ when one leg is in full sunshine and the opposite leg in full shadow. It thus was suggested to blow ambient air through the main chords of the legs, at about 20 mph, which would reduce ΔT to about 2 $^{\circ}C$ according to a rough estimate.

It should be mentioned that the feed legs then need a special design, with single lacing only, such that the lacing does not contribute to the axial stiffness. The thermal deformation of the legs then depends only on the temperature of the main chords, and only those need to be cooled. According to Report 22 (Febr. 15, 1969), the chords are standard steel pipe with 4.0 inch inner diameter; they are 188 ft long, 137 ft of which are above the telescope surface.

The present report replaces the rough estimate of the cooling effect by proper calculations, and it also includes the heating effect by compression of the air in the blower and by turbulent friction in the pipe. Finally, an experiment at Green Bank is suggested.

2. Derivation of Formulas

Local Equilibrium. Let T_a = ambient air temperature, $T_o = T_a + 5\ ^{\circ}C$ = temperature of pipe in sunshine without cooling, T_1 = temperature of pipe in sunshine with cooling, and T_2 = temperature of air in pipe. Further, let h_o = thermal conductivity between pipe and surrounding, and h_1 between pipe and air flow within pipe, both to be measured in cal/(sec cm^2 $^{\circ}$C). Call

$$k = \frac{h_o}{h_i}, \quad \text{and} \quad q = \frac{1}{1 + k} \tag{2}$$

The temperature of the pipe, in sunshine with cooling, then is found from the equilibrium condition,

$$(T_o - T_1)\, h_o = (T_1 - T_2)\, h_i ,$$

as

$$T_1 = q\,(T_2 + kT_o). \tag{3}$$

Temperature Distribution $\Delta T(x)$. Consider a mass element of air, of length b, moving through the pipe with velocity v. Its mass is $m = (\pi/4)\, D^2\, b\, \rho$, with pipe diameter D and air density ρ . During time dt, it travels a distance $v\, dt$. It receives the heat amount $dW = h_i(T_1 - T_2)\, \pi D\, b\, dt$, and it warms up by $dT_2 = dW/mc_p$, where c_p = specific heat of air. The heat from compression and friction is neglected. This yields

$$\frac{dT_2}{dx} = \frac{4\, h_i}{c_p\, D\, \rho\, v}\,(T_1 - T_2), \tag{4}$$

or, using equation (3),

$$\frac{dT_2}{dx} = \frac{1}{x_o}\,(T_o - T_2), \tag{5}$$

where

$$x_o = \frac{D\, c_p\, \rho\, v}{4\, h_o\, q} . \tag{6}$$

Equation (5) is easily integrated, yielding

$$T_o - T_2(x) = (T_o - T_a)\, e^{-x/x_o} . \tag{7}$$

Let $\Delta T_o = T_o - T_a$ be the temperature difference between ambient air and pipe without cooling, and $\Delta T(x) = T_1(x) - T_a$ be the difference with cooling. Equations (7) and (3) then give

$$\Delta T(x) = \Delta T_o\,(1 - q\, e^{-x/x_o}) . \tag{8}$$

At the beginning of the pipe, where the blower is attached,

$$\Delta T(0) = \Delta T_o(1 - q), \tag{9}$$

whereas the infuence of the cooling decreases exponentially, with scale length x_o as given in equation (6), because cooling the pipe warms up the air. The distribution of equation (8) is shown in Fig.1.

Average Temperature ΔT. The total thermal deformation is given by the average of $\Delta T(x)$ over the length of the pipe:

$$\Delta T = (\Delta T_o / \ell) \int_0^{\ell} (1 - q\, e^{-x/x_o})\, dx, \tag{10}$$

which yields

$$\Delta T = \Delta T_o \left[1 - a\,(1 - e^{-q/a}) \right] , \tag{11}$$

with

$$a = q x_o / \ell = \frac{D\, c_p\, \rho\, v}{4\, h_o\, \ell} . \tag{12}$$

3. Numerical Values

For air, $\rho = 1.22 \times 10^{-3}$ g/cm^3 is used, and $c_p = 0.240$ cal / (g ^{o}C) . The pipe shall have D = 4 inch = 10.2 cm inside diameter, and ℓ = 137 ft = 41.8 m is used for its length above the surface being fully exposed to sunshine. The thermal conductivity of steel pipes with white protective paint was measured at Green Bank (Report 17, Jan. 3, 1967) as

$$h_o = 2.60 \times 10^{-4} \text{ cal/(sec cm}^2\ {}^o\text{C)} \tag{13}$$

with respect to the surrounding. For the conductivity with respect to the internal air flow, O. Heine provided a formula which, transferred to metric units and for D = 10.2 cm, reads

$$h_i = 1.40 \times 10^{-4} \left(\frac{v}{\text{m/sec}} \right)^{0.8} \frac{\text{cal}}{\text{sec cm}^2\ {}^o\text{C}} . \tag{14}$$

Then

$$k = \left(\frac{2.17\ \text{m/sec}}{v}\right)^{0.8} \tag{15}$$

and

$$a = \frac{v}{14.5\ \text{m/sec}} \ . \tag{16}$$

With these values, Fig.2 is calculated from equation (11). It shows that

$$v = 56\ \text{ft/sec} = 17.1\ \text{m/sec} \tag{17}$$

is needed for obtaining $\Delta T = 2\ ^{\circ}C$ as used in Report 23.

If the blowers are used during nights and windy days, too, where $\Delta T_o = 1.5\ ^{\circ}C$, then $v = 56$ ft/sec yields $\Delta T = 0.6\ ^{\circ}C$. The pointing error from the feed legs then is $\Delta\vartheta = 1.51$ arcsec, which is smaller than that of the cone members, see Report 23. The total pointing error at night then is

$$\Delta\vartheta = 3.70\ \text{arcsec}, \tag{18}$$

instead of 4.9 arcsec from Report 23.

4. Heating by Compression and Turbulence

The air flow in the pipe is turbulent, and the dissipation of turbulent energy heats the air. As seen from the blower, the turbulent friction inside the pipe must be overcome by condensing the air to a certain over-pressure which can be obtained from handbook tables. Using the full length of 188 ft, and a diameter of 4 inch, O. Heine found for $v = 100$ ft/sec that an input pressure of 2260 lb/ft^2 is needed. Since the normal atmospheric pressure is 2120 lb/ft^2, the pressure ratio then is 1.065. For adiabatic compression of air, $c_p/c_v = 1.40$, and the temperature ratio then equals the pressure ratio to the power $0.40/1.40 = 0.286$, which is 1.0182. With $T = 20\ ^{\circ}C = 293\ ^{\circ}K$, the air then warms up by $\Delta T_c = 5.34\ ^{\circ}C$ in the blower, and for other air speeds,

$$\Delta T_c = 5.34\ ^{\circ}C \left(\frac{v}{100\ \text{ft/sec}}\right)^2 . \tag{19}$$

When travelling through the pipe, the air will cool by expansion and heat by friction. But since all energy must have been provided by the blower, the air temperature in the pipe cannot excede the value (19). With $v = 56$ ft/sec from (17),

$$\Delta T_c = 1.68\ {}^{\circ}C, \tag{20}$$

which can be tolerated since all four feed legs are warmed up by the same amount, while only temperature differences matter. It thus seems that an air flow of 50 - 60 ft/sec fulfills all requirements. This speed can be obtained with standard fans with forward curved blades, with a motor of only 0.5 horsepower. A total of 12 such blowers is needed, for three main chords of four feed legs.

5. Heat Insulation

If a still greater cooling were required, higher velocities would be needed which could result in too much heating by compression. This gives pointing errors if the motors at different legs run with slightly different speed. In this case the pipes could be provided with a thin heat-insulating layer of foam on the outside, which reduces the air speed needed.

According to O. Heine, one inch of styrofoam has a heat conductivity of

$$h_f = 0.22\ \text{BTU/(hr ft}^2\ {}^{\circ}\text{F)} = 0.302 \times 10^{-4}\ \text{cal/(sec cm}^2\ {}^{\circ}\text{C)}, \tag{21}$$

and h_o in equations (2) and (12) should be replaced by $(h_o^{-1} + h_f^{-1})^{-1}$. For layers of thickness δ, $h_f \sim \delta^{-1}$. In this way, Fig.3 is calculated for an air flow of $v = 30$ ft/sec, where the compression results in only $\Delta T_c = 0.48\ {}^{\circ}C$. Fig.3 shows that $\delta = \frac{1}{2}$ inch of insulation is enough for reducing the average pipe temperature from $\Delta T = 2.8\ {}^{\circ}C$ (for $\delta = 0$) to $\Delta T = 0.90\ {}^{\circ}C$.

Some experiments at Green Bank are planned, using three pipes with white paint; a long one where air is blown through, and a short one without blower, both in full sunshine; and a third short one in shadow without blower.

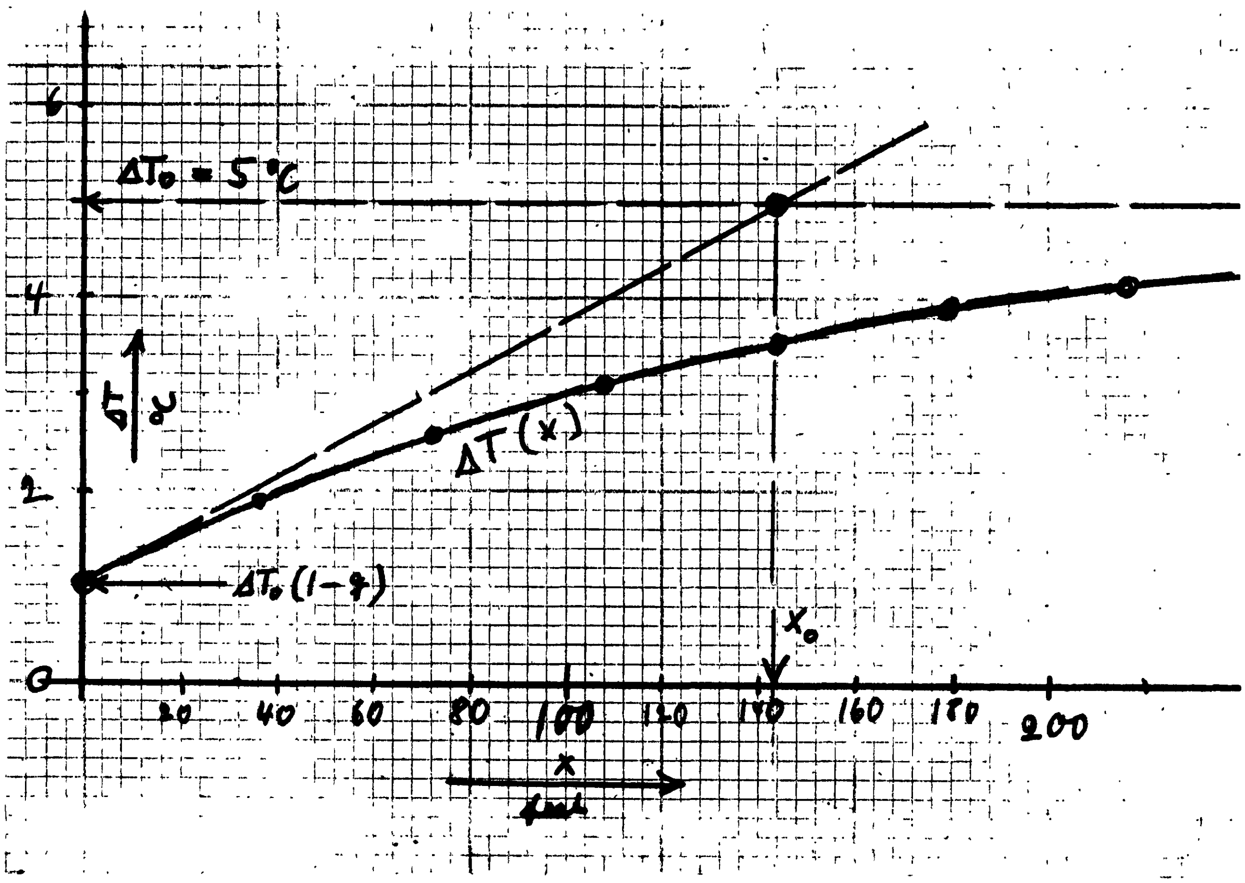

Fig. 1. Temperature distribution $\Delta T(x)$ along a pipe in sunshine, cooled by blowing ambient air through it. Without cooling, the pipe would be $\Delta T = 5\ ^{o}C$ warmer than the ambient air. The distribution is calculated for a pipe of 4 inch inside diameter, and an air flow of 40 ft/sec = 27.3 mph.

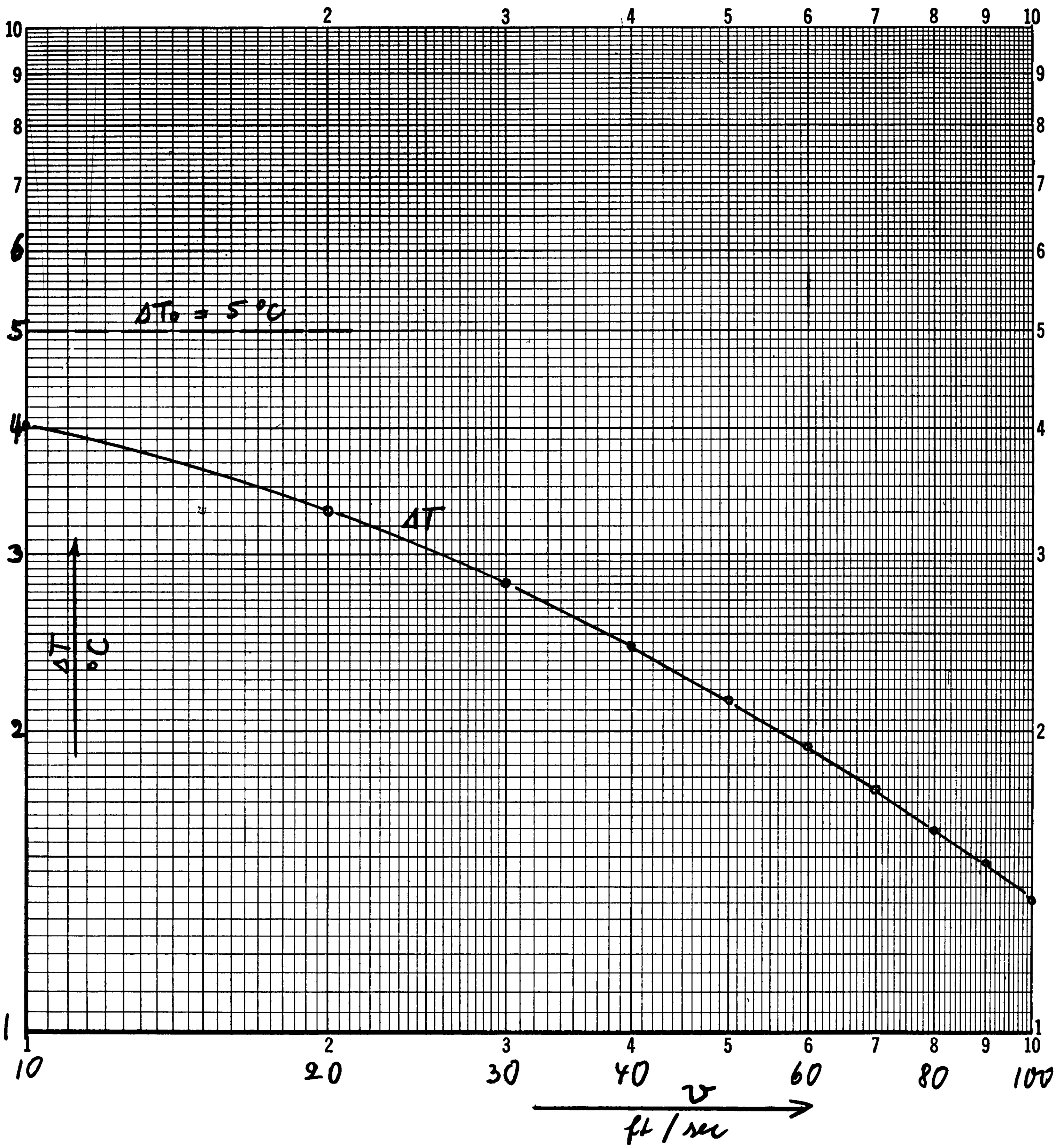

Fig. 2. The average temperature ~~of~~ a pipe of 137 feet length and 4 inch inside diameter, heated by sunshine and cooled by ambient air of speed v.

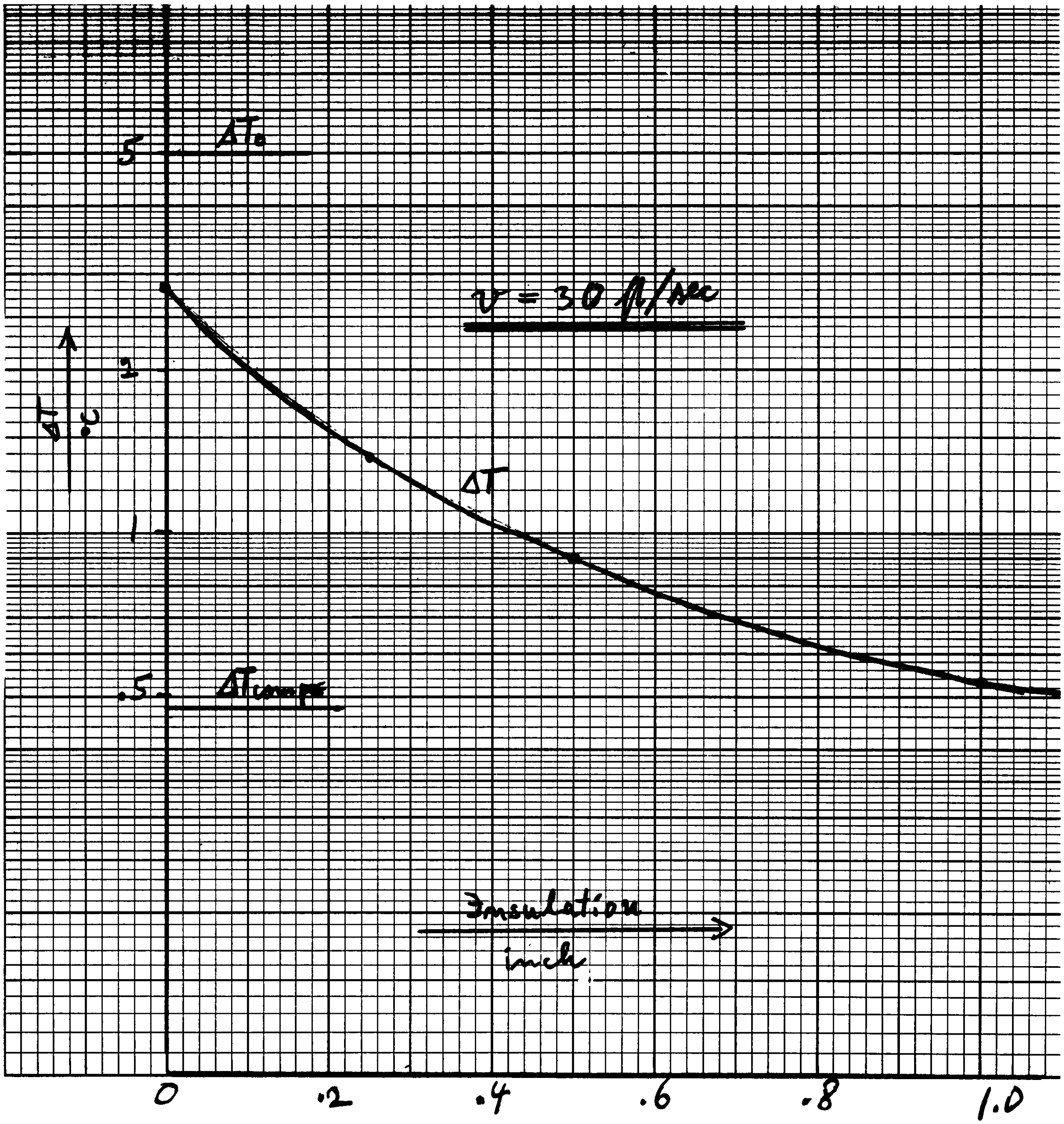

Fig. 3. The average temperature difference ΔT, between one pipe in sunshine and one in shadow, as a function of the thickness of an insulating foam layer around the pipes. Ambient air is blown with 30 ft/sec through the pipes.