REPORT 1

NATIONAL RADIO ASTRONOMY OBSERVATORY
POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE ARBOVALE 456-2011

REPORT NO. 1
CONTRACT NO.

DATE Oct. 20, 1965

PROJECT: LFSP
SUBJECT: Homology Deformation
TO: R. Jennings
FROM: S. von Hoerner

Approach to a Structure with Homology Deformation

The following is a suggestion of how to approach a most simple, but still useful structure for a tiltable antenna passing the gravitational limit by means of homology. In this first approach, we neglect the curvature and replace the paraboloid by a plane. We thus ask for a tiltable, finite structure, suspended at two points, having a round surface which is an exact plane for all angles of tilt.

1. Suspension

I have given exact solutions for infinite structures, and the main problem left seemed to be the boundary distortions for limited structures. Thus, the present approach starts where this problem seems move severe: at two bearings at the lower end of the structure.

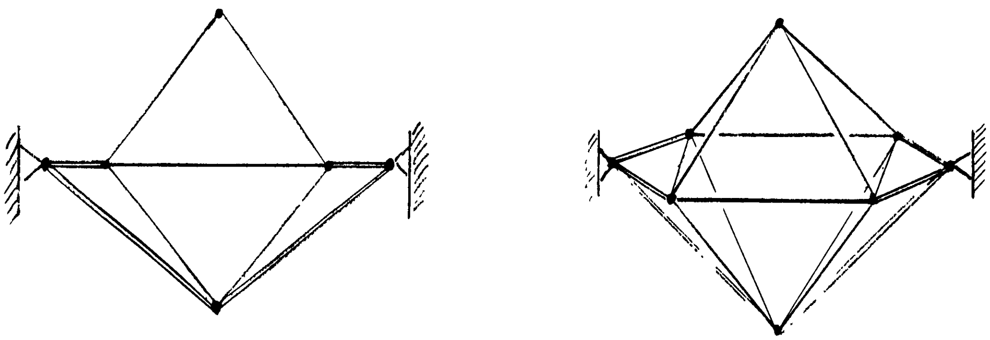
From these two bearings we want to proceed to four basic points (the corners of an octahedron needed later on). Undeformed, these basic points shall form a square. The only condition we impose for gravitational deformations under various tilts is one of homology:

The four basic points shall always be in a plane (1)
(but sides and angles might deform).

A possible and simple solution is the following one:

NATIONAL RADIO ASTRONOMY OBSERVATORY
POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE ARBOVALE 456-2011

PROJECT: LFSP
SUBJECT: Homology Deformation



side view tilted (2)

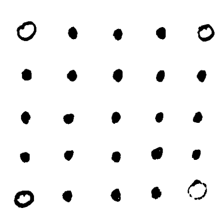
This first step needs no special calculations; condition (1) is fulfilled just by reasons of symmetry.

2. The Surface

In order to learn how much complexity is needed, we go to the upper end of the structure and ask: which type of configuration of the surface points do we want to arrive at?

An estimate showed that a number of about 20 homologous surface points should be plenty for all present goals (with this number, we could pass the gravitational limit by a factor of 9 in wavelength, or a factor of 3 in diameter). A good configuration, then, seems to be the following one with 21 points:

• 21 surface points
O 4 basic points (omitted in the surface)



(3)

It is "round enough" for an antenna surface, and still is "square enough" to be easily derived from the four basic points.

PROJECT: LFSP
SUBJECT: Homology Deformation

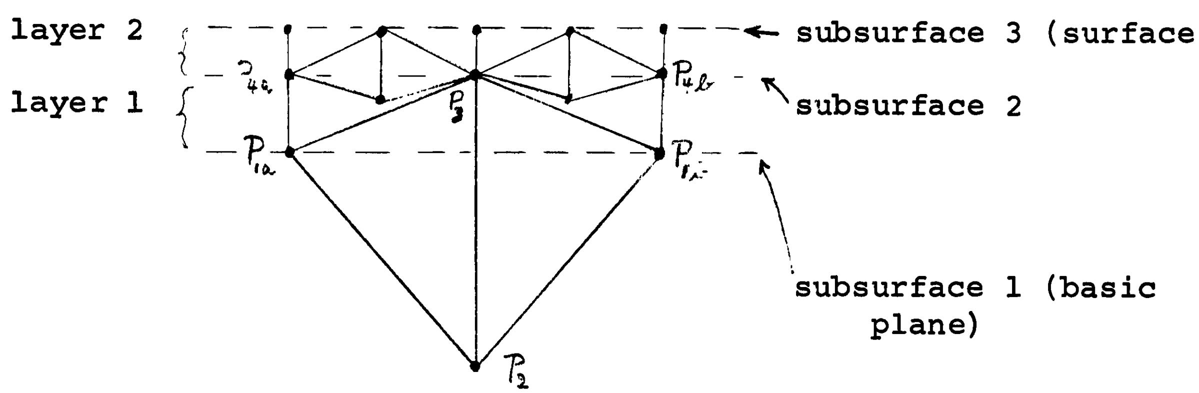
3. Layers

Accepting surface configuration (3), we see that we can reach it from the basic square with only

2 layers. (4)

Since the square will be deformed within its plane, we must choose a pressure-stable type of cell, and we take the one from my antenna paper, leading to the following general picture:

(5)



P_{1a} and P_{1b} represent two of the four basic points; P_2 should be, if possible, the "antifocus point" of the octahedron.

4. Boundary Distortions

In my antenna paper, I found that the pressure part of the boundary distortions can be completely removed by using pressure-stable cells, only the torque part of the distortions remaining. With the antenna looking at horizon, the torque from all upper layers distorts the upper subsurface of the preceding layer, giving it an S-type shape. But in case of only two layers, we have no boundary distortions at all, since the first and second subsurface

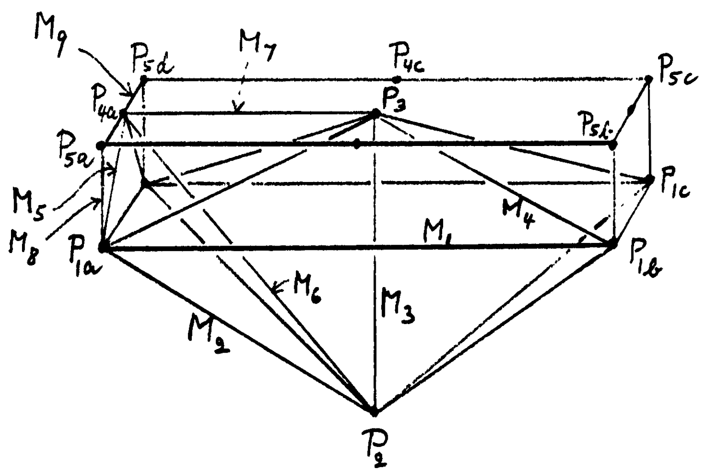
PROJECT: LFSP
SUBJECT: Homology Deformation

have not enough points for an S-type shape (the result being only a tilt, which is a homologous deformation), and since the second layer cannot be distorted because there is no third one above it.

5. The Single Cell in Three Dimensions

There are several ways of extending the single cell of (5) into a third dimension, and the easiest I can think of is the following:

members M_5, M_6, and M_7 are drawn in only one quadrant



(7)

Calling p the number of structural points and m the number of members, m must be within the limits (in general, for a stable structure):

$$m_1 = 3(p-2) \le m \le \frac{1}{2} p(p-1) = m_u. \tag{8}$$

In our case, we have

$$p = 14 \text{ and } m = 41, \tag{9}$$

while $M_1 = 36$ and $m_u = 91$. With 41 members and a minimum of 36,

PROJECT: LFSP
SUBJECT: Homology Deformation

we have a fair degree of redundancy which looks alright. No redundancy could give a structure which collapses completely if one member breaks, and too high a redundancy would give a very complicated analysis.

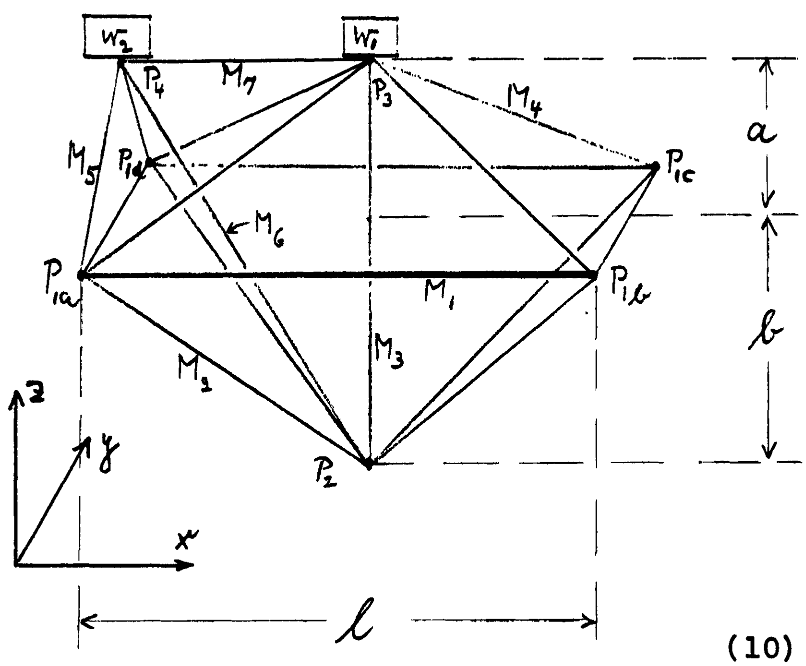
6. Structure to be Analyzed

Members M_9 are used for lateral stability only and have nothing to do with our homology problem. A condition imposed on P_5 will influence cross section M_8 but nothing else, and thus is separable from the other conditions. Therefore we confine the following discussion to a slightly reduced structure:

members M_5, M_6 and M_7 are drawn in only one quadrant

w_1 and w_2 represent the weight of upper layers and surface

directions:



(10)

Structure (10) has

$$p = 10 \text{ and } m = 29, \tag{11}$$

while the minimum would be $m_l = 24$ and the maximum $m_u = 45$.

7. Degrees of Freedom

Structure (10) has 7 types of members ($M_1 \ldots M_7$) giving 6 degrees of freedom for the ratios of cross sections, say, $q_1 = Q_1/Q_7 \ldots q_6 = Q_6/Q_7$. The geometrical shape yields 2 degrees of freedom for the ratios of lengths, say, $\alpha = a/l$ and $\beta = b/l$. Altogether, we have

$$8 \text{ degrees of freedom } (q_1 \ldots q_6; \alpha, \beta). \tag{12}$$

8. Homology Conditions

We demand that the upper surface should always be a plane, for any pressure within the lower surface, and for any tilt in elevation angle, provided that the lower surface is a plane. In order to prepare the future use of this cell within a curved surface, we further demand that P_3 moves by the same amount for any direction of gravity. This leads to the following set of conditions:

If P_{1a}-P_{1d} is pressed, P_4 shall not move in z-direction. (13)

If P_{1a}-P_{1c} is pressed, P_3 shall not move in z-direction. (14)

If structure looks at zenith (z-direction) and is held at P_2, P_4 shall move in z-direction as much as P_3 does. (15)

If structure looks at horizon (x-direction) and is held at all P_1, P_4 shall not move in x-direction. (16)

P_3 shall move down by the same amount, whether looking at zenith or horizon. (17)

PROJECT: LFSP
SUBJECT: Homology Deformation

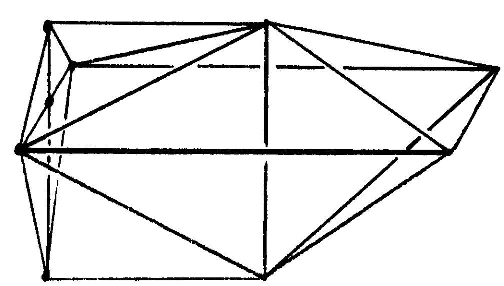
The first question I would like to have investigated is whether or not these 5 conditions are a) complete, and b) compatible with each other. For example, we need two more conditions for complete homology:

If P_{1a}-P_{1d} is pressed, P_3 shall not move in z-direction. (18)

If P_{1a}-P_{1c} is pressed, P_4 shall not move in z-direction. (19)

It seems to me that these conditions are already included in conditions (13) and (14), but I might be wrong. And even if this assumption was right, I cannot tell whether all four pressure conditions can be fulfilled simultaneously.

If this investigation should result in noncompatibility, conditions (13) and (14) can be, structurally, completely separated by a slightly more complicated version of (10):



(20)

For the following, we will assume completeness and compatibility for structure (10) and thus have

5 conditions. (21)

PROJECT: LFSP
SUBJECT: Homology Deformation

9. Free Parameters

Having 8 degrees of freedom and 5 conditions, we are left with

$$\text{3 free parameters.} \tag{22}$$

They should be used for a good compromise between the two further demands:

$$\text{maximum rigidity for given total weight,} \tag{23}$$

$$\text{convenient geometrical shape.} \tag{24}$$

A convenient shape means that β should be large and α small if the cell is used in the first layer, and both should be small if used in the second layer. By "small" we mean about 0.2 ... 0.3, by "large" about 0.6 ... 0.9. From this consideration it seems best to choose α and β as free parameters, and to use the third parameter always for obtaining maximum rigidity. The rigidity might be measured in the following way. Call Δh the deflection of P_3 if a force F acts on P_3 and if the cell is held at all P_1. Call W the total weight of the cell, and let us define a dimensionless quantity

$$R = \frac{\rho}{E} \frac{l^2}{\Delta h} \frac{F}{W} = \text{relative structural rigidity.} \tag{25}$$

R is the rigidity of our structure, divided by the longitudinal rigidity of a straight rod of weight W and of length ℓ. Condition (23) then means

$$R = \max. \tag{26}$$

PROJECT: LFSP
SUBJECT: Homology Deformation

10. Loads From Upper Layers and Surface

Our problem has two more parameters: the loads w_1 and w_2 shown in (10). We express them in a dimensionless way as $\Omega_1 = w_1/W$ and $\Omega_2 = w_2/W$. Since the loads are given, these two parameters are not free, and we call them

$$2 \text{ given parameters } (\Omega_1, \Omega_2). \qquad (27)$$

If P_4 is at the boundary, then $\Omega_1 = 2\,\Omega_2$, while for inner cells $\Omega_1 = \Omega_2$. This holds if looking at zenith and is somewhat more complicated if looking at horizon. Some estimates show that Ω_1 should always be within the range 0.0 ... 0.2. For the present purpose, it might be enough to try only two cases: a) $\Omega_1 = \Omega_2 = 0$, and b) $\Omega_1 = 1/8$, $\Omega_2 = 1/16$.

11. Numerical Treatment

This treatment will only be possible on a fast computer. First, we need a program for structural analysis, which gives the deflections of a given structure under given forces. For the present goal, all joints should be considered as being pin joints, and all members as being simple rods. The program should determine the elements of the deflection matrix and invert it. This matrix is square and has 3p-6 columns, or, using at least one symmetry, (3p-6)/2. The deflections are obtained by multiplying the inverse matrix with 4 sets of forces (zenith gravitation, horizon gravitation, two external pressure cases). Having the deflections, we then calculate the deviations Δ_i (i = 1 ... 5) from our set of homology conditions (13) to (17).

PROJECT: LFSP
SUBJECT: Homology Deformation

Second, we need a method of solving for all $\Delta_i = 0$. Let us define α, β, and q_6 as the 3 free parameters, and $q_1 \ldots q_5$ as 5 unknowns to be obtained by the conditions $\Delta_i = 0$. Starting with a first guess of $q_1 \ldots q_5$, we look for an iteration method converging quickly to the final value of $q_1 \ldots q_5$, and I think we ought to go to a gradient method of first order.

We keep 4 unknowns constant and change the fifth, q_j, by a small amount dq_j. We apply our analysis program and find that the 5 deviations Δ_i have changed by $d\Delta_i$. We call

$$D_{ij} = \frac{d\Delta_i}{dq_j} \cdot \tag{28}$$

In this way, we calculate all 25 values D_{ij}. Assuming that the single deviations just add if we change all 5 q_j simultaneously, we have $d\Delta_i = \sum_j D_{ij}\, dq_j$ for a simultaneous change. Since we want all Δ_i to vanish, and assuming that the deviations are linear, we solve the five linear equations

$$\sum_j D_{ij}\, dq_j = -\Delta_i \tag{29}$$

for the five unknowns dq_j. Then, we use the values $q_j + dq_j$ instead of q_j for the next iteration.

This method should always work, if a) a real and regular solution exists, and b) the first guess was not too far off. It should converge in one step under the assumption of linearity. Both the analysis program and the iteration program should be written in a general way, applicable to various and more complex structures.

PROJECT: LFSP
SUBJECT: Homology Deformation

The whole procedure then might run as follows:

1) Choice of Ω_1 and Ω_2 2) Choice of α and β	manually	
3) Choice of q_6 4) Apply iteration program and find $q_1 \ldots q_5$ 5) Calculate total weight W and rigidity R 6) Repeat 3) to 5) until R = max	fully computerized	(30)
7) Vary α and β, repeat 3) to 6) 8) Vary Ω_1 and Ω_2, repeat 2) to 7)	manually	

An estimate of the computing time showed that one run of the computerized part of (30), until we have the maximum rigidity, might take about 8 minutes on an IBM 7040. With two combinations of Ω_1 and Ω_2, and four combinations of α and β, the whole procedure (30) then would take about one hour. For more complex structures, the time would increase with p^3.

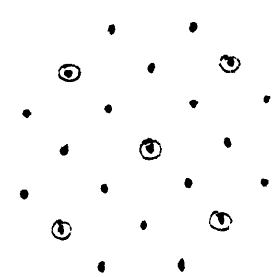
Another question to be investigated is the sensitivity of the deflections due to small changes of all parameters, because we must allow for certain tolerances in all manufacturing and erection procedures. I suggest the following way. After having obtained a homology solution, we change all parameters (cross sections, coordinates) by small amounts, using random numbers; then we calculate again the deviations Δ_i from our set of homology conditions. Repeating this, say, three times with different random numbers should give us a useful measure of sensitivity to inaccuracies. Changes of cross sections and of coordinates might be investigated separately.*)

12. A Different Configuration

The preceding approach tried to be most simple with respect to the analysis. It led to a structure which actually could be used, but still shows two undesirable features. First, the diameter of the antenna surface is smaller than the diameter of the basic plane. Second, having only four basic points and having two layers brings the surface rather high above the basic points (especially if we proceed to a curved surface), which would result in very long feed supports.

The following approach starts from five basic points and reaches the surface (again in 21 points) with only one layer. The surface is twice as large as in the first approach, (for equal size of the basic square), and the surface can now be placed much closer to the basic points than before, especially for a curved surface.

• 21 surface points
◯ 5 basic points



(31)

---

*) Having changed, for example, all q_j into $q_j(1+\delta_j)$, with $\delta = \mathrm{rms}(\delta_j)$, and having obtained the Δ_i, with $\Delta = \mathrm{rms}(\Delta_i)$, we define as the sensitivity of the cell:

$$S = \Delta / (\delta \ell) .$$

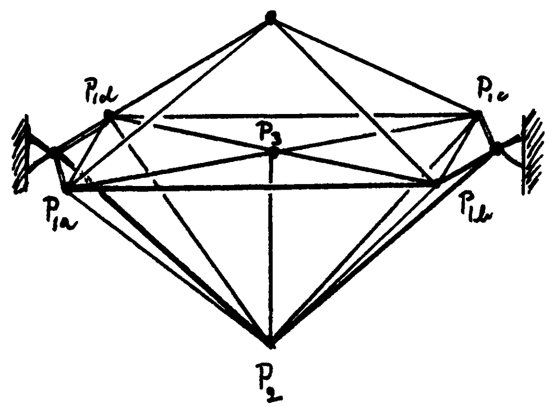
NATIONAL RADIO ASTRONOMY OBSERVATORY
POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE ARBOVALE 456-2011

REPORT NO. 1
CONTRACT NO.

DATE Oct. 20, 1965

PROJECT: LFSP
SUBJECT: Homology Deformation

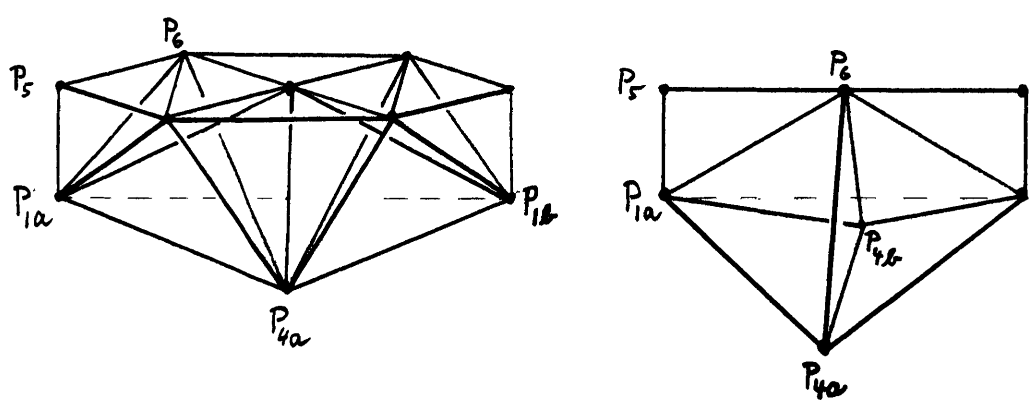
The suspension shown below can be made to yield 5 homologous basic points:



(32)

The framework between basic plane and surface, then, consists of two different types of cells:

(33)



PROJECT: LFSP
SUBJECT: Homology Deformation

As indicated by the lettering in (33), both cells have some members in common, and it is this complication which might make the analysis of a single cell useless and might call for a simultaneous analysis of the complete layer as a whole, which would take about 2 hours on an IBM 7040 for the computerized part of (30). Although configuration (31) looks much more desirable than (3), I suggest postponing this approach until we have learned how to handle the first one.

Addendum

In (29) we have assumed that the deformations are linear in the dq_j. The same assumption applied to Δh makes it possible to solve for R = max simultaneously with (29), without repeating steps 3) to 5) according to 6) in procedure (30). This becomes important for more complex structures with many free parameters. Call f = number of degrees of freedon (from cross sections only), c = number of homology conditions, r = f - c = number of free parameters to be used for R = max. Call ℓ_j = length of member j, ρW_o = present total weight, and Δh_o = present value of Δh. Define $H_j = \partial \Delta h / \partial q_j$. Then

$$R^{-1} \sim W \,\Delta h \sim (W_o + \sum_j \ell_j dq_j)\,(\Delta h_o + \sum_j H_j dq_j). \qquad (34)$$

For maximum rigidity we demand $\partial(R^{-1})/\partial dq_k = 0$, for k = c + 1, ... f. We now have f equations (instead of only c equations in (29)) for f unknowns dq_j:

$$\left.\begin{aligned} &\sum_j D_{ij}\, dq_j = -\Delta_i && \begin{cases} j = 1, \dots, f \\ i = 1, \dots, c \end{cases} \\ &\sum_j (H_i \ell_j + H_j \ell_i)\, dq_j = -(H_i W_o + \Delta h_o \ell_i). && \begin{cases} j = 1, \dots, f \\ i = c+1, \dots f \end{cases} \end{aligned}\right\} \qquad (35)$$

The sensitivity S, too, can be obtained directly, without repeating the analysis with random variations. It can be shown that

$$S = \frac{1}{\ell} \sqrt{\frac{1}{c} \sum_i \sum_j (D_{ij} q_j)^2}\,. \qquad \begin{cases} j = 1, \dots f \\ i = 1, \dots c \end{cases} \qquad (36)$$