REPORT 9

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REPORT NO. 9
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PROJECT: LFST

SUBJECT:

Weight Increase by Slenderness

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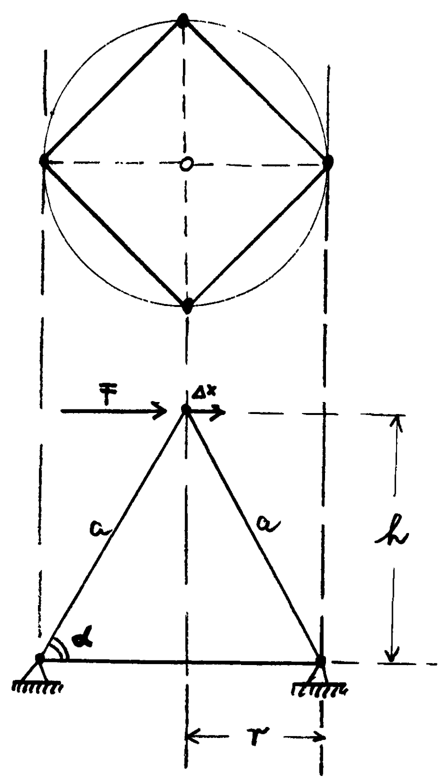
If a structure shall have small deflections from lateral forces, it must not be too slender. This problem arises for all structures whose weight is defined by keeping the wind deflections during observation below a specified value, since we want to achieve the specification, if possible, with a structure of low total weight.

As an example we choose a quadratic pyramid of a given height h; the 4 basic points are fixed on a circle of radius r, to be considered a free parameter of the structure. We assume 8 members of equal cross section Q, density ρ and elasticity E: 4 legs and 4 sides of the basic square. A given lateral force F works at the top, and the deflection Δx of the top is specified; this defines Q.
Question: What is the resulting total weight W of this structure, as a function of the basic radius r ?



With $a^2 = r^2 + h^2$ the total weight is

$$W = 4 \rho Q \left(a + r\sqrt{2}\right)$$

and the deflection is

$$\Delta x = \frac{F}{2QE} \, \frac{a^3}{r^2} \quad .$$

Both equations together yield

$$W = \frac{2 \rho F a^3}{E \, \Delta x \, r^2} \left(a + r\sqrt{2}\right)$$

which we write in the form

PROJECT:

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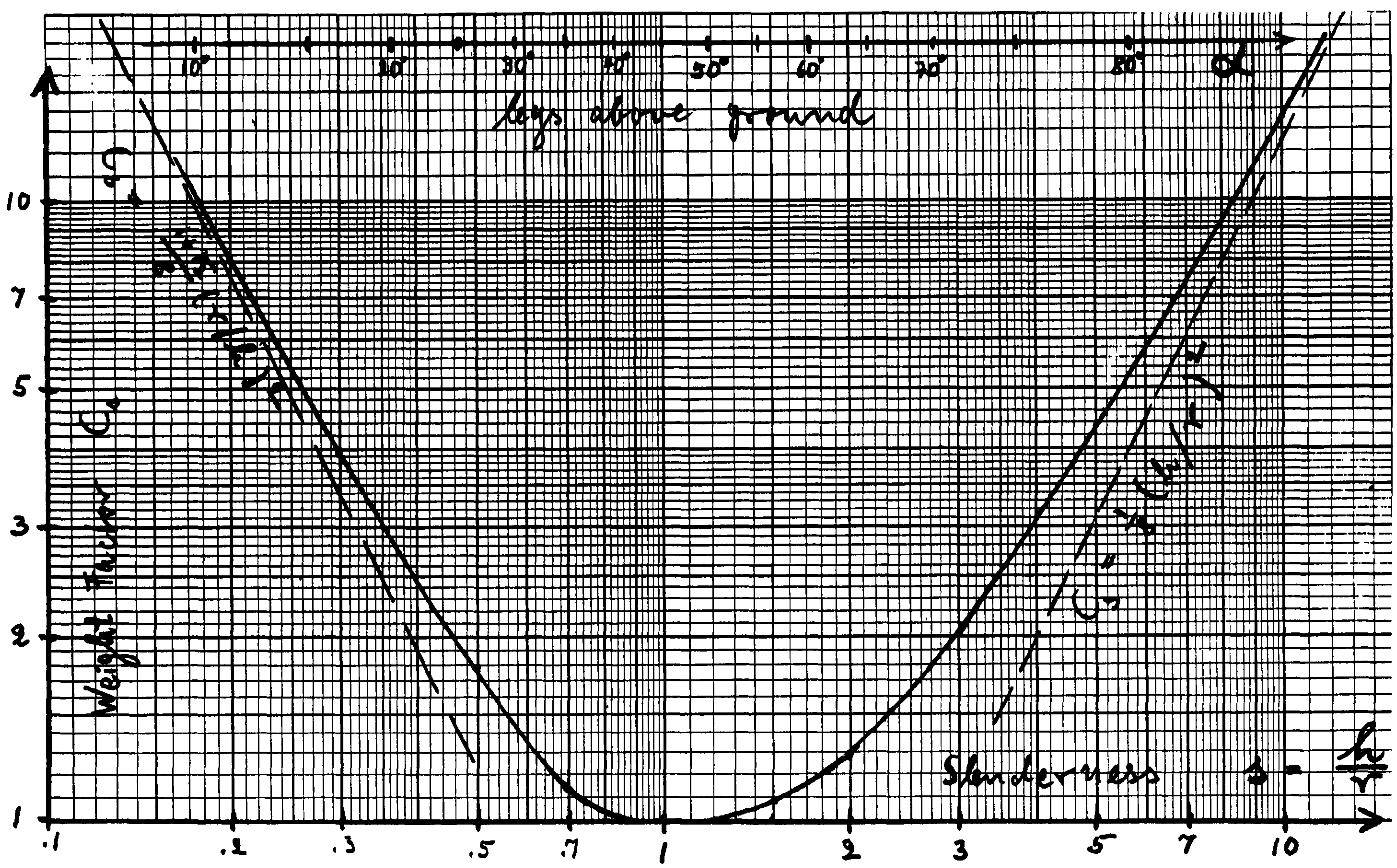
$$W = 16 \frac{\rho}{E} \frac{F h^2}{\Delta x} C_s$$

where we defined a "slenderness weight-factor" C_s as follows (with $s = h/r$ = slenderness)

$$C_s = \frac{1}{8} \left\{ (s + s^{-1})^2 + \sqrt{2} \, (s^{2/3} + s^{-4/3})^{3/2} \right\}.$$



We see that $C_s = 1$ for $r=h$ is very close to the minimum, which means that the legs are 45^0 from the ground. Small deviations from the optimum size do not matter much; but with a slenderness of <u>$h/r = 3$</u> we need already <u>double</u> the weight. Although this result is derived for a special structure, the pyramid, it will hold, with slight modifications, for almost <u>any</u> type of structure.