REPORT 22

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Telescope Members with Special Design

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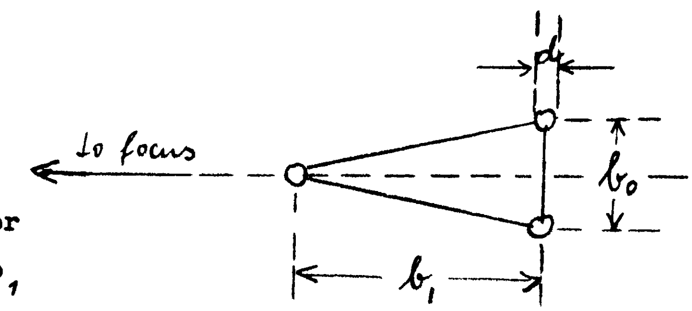
In general, the members of the 300-ft homologous telescope all have one and the same design geometry: they are long, built-up, tapered members of triangular cross section, all of the same shape and with the same internal angles; only length and bar area vary from one member to the other.

This is different for three types of members where a special design is needed: (1) feed support legs, (2) suspension bars which hold the telescope from the elevation bearings, and (3) the central member holding the optical pointing system. In this report we give design details and their derivation only for the feed legs and the focal cabin, whereas for suspension members and central member only general indications are given.

I. Feed Legs and Focal Cabin

1. Dimensions



Cross section of feed leg.

We adopt a triangular cross section for the feed legs, with a large radial width b_1 and a small tangential width b_o. We use lateral guy ropes for keeping b_o as small as possible; b_o must be large enough to prevent tangential buckling over distance l_o between guy points, and b_1 must prevent radial buckling over length l_1, see Fig.1. In the lower third, both widths are equal b_o.

The guy ropes must be attached in a way which does not change the homologous deformation. There are two ways and we will use both: (1) Where the feed leg intersects the surface structure, see Fig.1, we use four guy ropes parallel to the surface, tangential but also radial (for avoiding large radial width and keeping the surface hole small). (2) At 1/2 the upper length, we use two guy ropes attached to the basic square on which the feed legs rest, points 45. The first guy system gives deformations only parallel to the surface which do not hurt; the second system gives no deformations at all.

Buckling occurs if the Euler stress S_e is surpassed. With a slenderness ratio $\Lambda = l/r$ of the member as a whole, where $r = b/\sqrt{6}$ for triangular cross section, with a conventional safety factor of $k = 1.92$ and a modulus of elasticity $E =$ 29 000 ksi, S_e is given by

$$S_e = \frac{\pi^2 E}{k\Lambda^2} = 149\ 000 \text{ ksi} / \Lambda^2 = 24\ 800 \text{ ksi } (b/L)^2 . \qquad (1)$$

Formula (1) holds for prismatic solid bars. For our built-up telescope members, tapered at both ends, our computer analysis yields a correction factor of 1.60; and since the feed legs are tapared at only one end, we adopt a correction factor 1.30, thus the corrected Euler stress is

$$S_{ec} = 115\ 000 \text{ ksi} / \Lambda^2 = 19\ 100 \text{ ksi } (b/L)^2 . \qquad (2)$$

From our computer analysis, the chord stress (in horizontal position) is for equilateral cross section, from dead loads only,

$$S_g = 4.2 \text{ ksi } (L/1000 \text{ inch}) (\Lambda / 50) \qquad (3)$$

which can be applied to the lower chord with $\Lambda = b_1/l_1$ and $L = l_1$. For the two upper chords and tangential forces, an estimate gave a factor 1.5 for S_g because of the increased length and weight of the lacing in radial direction.

If $S_{ec} = S_g$, no axial external load could be taken up by the member. We now demand that, say, $S_{ec} \geq 3S_g$ which leaves at least 2/3 of the chord capacity for taking up external axial loads. Equations (2) and (3) then yield

$$\Lambda \leq 77.5 \ (1000 \text{ inch}/L)^{1/3} . \qquad (4)$$

The 77.5 holds for the general case and b_1, but should be replaced by 66.3 for b_o. Equation (4) shows that the width of very long members must not increase in proportion to the length, but with $L^{4/3}$.

From Fig. 1 we read l_o= 860 inch and l_1= 1720 inch. With $r = b/\sqrt{6}$ we then obtain for the length of the battens:

$$\text{tangential} \quad b_o \geq 29.6 \text{ inch}, \quad \text{and} \quad \Lambda_o \leq 71.2; \qquad (5)$$

$$\text{radial} \quad b_1 \geq 65.0 \text{ inch}, \quad \text{and} \quad \Lambda_1 \leq 64.7. \qquad (6)$$

For keeping the blocking down, b_1 will actually be taken much larger than (6). And for b_o we adopt

$$b_o = 30 \text{ inch} = 2.5 \text{ feet.} \tag{7}$$

As all telescope members, we divide l_1 into 12 sections, and the length of a single chord then is 1720/12 = 143 inch. The l/r ratio of the chord must be larger than that of (3), and we choose, say, $\lambda = 100$. This yields r = 1.43 inch, and the available standard pipe coming closest has 4 inch nominal (4.5 inch actual) diameter, with r = 1.51 inch, and a bar area of A = 3.174 inch^2. If the lacing is done as in the telescope members, our computer optimization yields 4.5 inch^2 for the 3 pyramid bars (at point 45), 0.78 inch^2 for battens, 0.83 for diagonals, and 0.22 for triangles. The last three values should be increased by about 30 % because of the longer b_1 we will use. The standard pipes coming closest then are:

<u>Table 1</u> : Bar areas for feed legs

	nominal diameter inch	outer diameter inch	bar area inch^2
pyramid	5	5.56	4.300
chord	4	4.50	3.174
batten	2	2.38	1.075
diagonal	2	2.38	1.075
triangle	1/2	.84	.250

Equivalent area and density (of a solid rod of same stiffness and weight) then will be:

$$A_{eq} = 13.0 \text{ inch}^2 , \tag{8}$$

$$\rho_{eq} = .410 \text{ lb/inch}^3. \tag{9}$$

For the maximum allowed external axial load P_m of a member I derived the following formula which in many computer checks was good within ± 4 %:

$$P_m = A_{eq} (S_\lambda - S_g) (1 - S_g/S_{ec}) . \tag{10}$$

S_λ is the maximum allowed stress in a chord as a function of its slenderness ratio and the steel used. With l/r = 143/1.51 = 95 and A36 steel we have S_λ = 13.2 ksi. S_g is found from (3) with the factor 1.5 applied; with $L = l_o$ = 860 inch, and with λ_o = 71.2 from (5) we obtain S_g = 7.73 ksi. And S_g/S_{ec} = 1/3 according to our previous demand. With these values and (8) we find from (10) the maximum allowed axial load as

$$P_m = 47.5 \text{ kip}. \tag{11}$$

From the stress analysis part of our homology program we find that the axial force in one feed leg (due to the weight of the 4 legs) is at most 1.60 times the weight of one leg. The weight of one leg of the present design is

$$W_{leg} = L\, A_{ef}\, \rho_{ef} = 2257 \times 13.0 \times .410 \text{ lb} = 12.0 \text{ kip}, \tag{12}$$

thus the axial force is $1.60 \times 12.0 = 19.2$ kip, which leaves $47.5 - 19.2 = 28.3$ kip per leg for supporting a cabin. The weight of the cabin (with its structure plus receivers and equipment) then can be at most $28.3 \times 2 / 1.6$, or

$$W_{cab} \leq 35.4 \text{ kip} = 16.1 \text{ tons (metric)}. \tag{13}$$

For convenience, the dimension of the cabin should be about

10 ft x 10 ft at floor,
13 ft high, (14)
floor 4 ft above focus.

2. Shadow from Feed Legs

Fig.2 shows the combined shadow of the leg on the dish. Part 1 is a parallel strip of width b_o and is due to the blocking of the incident rays. The outer part 2 is due to the blocking of the feed illumination, it begins at the intersection of the leg with the surface, with width b_o, and it widens up toward the rim with a final width αR, with R = telescope radius = 1800 inch. For the geometry of our present design one can derive, in radians,

$$\alpha = .616\, b_o/h + .000240\, b_o\,, \tag{15}$$

where h is the horizontal distance of the upper leg chords from the focal point. We call f_s the fraction of aperture*) covered by the combined shadow; f_s increases with b_o and decreases with increasing h. We shall determine h if f_s and b_o are given, and for this purpose we demand that

$$f_s = 5\,\% \text{ (of telescope aperture)}. \tag{16}$$

*) weighted with the illumination function

For the illumination we assume a parabolic (voltage) taper with pedestal p:

$$I(r) = 1 - (1-p)\, r^2 = \begin{cases} 1 & \text{at center, } r=0, \\ p & \text{at rim, } r=1, \end{cases} \qquad (17)$$

with a (power) edge taper of $T = -20 \log p$. For our estimate we adopt $T = 13$ db, which yields $p = .224$ and

$$I(r) = 1 - .776\, r^2. \qquad (18)$$

The shadow in the outer part is actually curved, see Fig.2, but we replace it by a straight line (which is on the safe side). The intersection of leg and surface is at $r = .444$ (800 inch from center) where we neglect the variation of this value with varying h. The weighted surface blocking then can be written as

$$f_s = \frac{\int_0^1 I(r) \frac{b_o}{rR}\, r\, dr + \int_{.444}^1 I(r)\,(\alpha - b_o/R)\, \frac{r - .444}{1 - .444}\, r\, dr}{\frac{\pi}{2} \int_0^1 I(r)\, r\, dr} . \qquad (19)$$

With (18) for I(r) and (15) for α, the evaluation of all integrals yields

$$f_s = .130\, b_o/h + .000787\, b_o. \qquad (20)$$

Or, if f_s and b_o are given, h is found as

$$h = \frac{.130\, b_o}{f_s - .000787\, b_o} . \qquad (21)$$

For b_o we use 30 inch from (7) plus 4.5 inch from Table 1 for the chord diameter, thus $b_o = 34.5$ inch. We demand $f_s = .05$ as in (16) and find $h = 197$ inch. Thus, the vertival distance h of the upper chords must be at least about

$$h = 200 \text{ inch} = 16.7 \text{ feet}. \qquad (22)$$

With this distance we have 5 % area blocking, which gives 10 % gain loss (or .46 db). This holds for longer wavelengths ($\lambda > 1$ m) where the built-up feed leg acts as a closed area. For shorter wavelengths, the holes between the lacing will become transparent, resulting in a smaller gain loss of about 6 % (.27 db). A further increase of h does not help much. For example, if we demand $f_s = 4$ %, we need $h = 350$ inch = 29 feet, certainly a very awkward design.

3. Focal Structure and Cabin

The cabin should have about the dimensions given in (14): 10 x 10 x 13 ft. It will be surrounded by a "focal structure" connecting the feed legs with each other and with the cabin. This structure must be very stiff against compression, shear and torsion in all directions.

The bottom of the cabin must have an open round hole of at least 5.5 ft diameter, for the Sterling mount of feed and receiver box. The top of the cabin must have an open entrance for men and equipment in service position (telescope looking at horizon). One of the cabin walls must provide a horizontal floor in service position, and for convenience there should be no door sill in the entrance. With cabin floor and service tower at same level, equipment then can be rolled in and out on little carts.

Fig.3 shows a design meeting all demends. The focal structure shown, outside the cabin walls, is the most natural way of connecting the feed legs to each other. For complete stiffness, this structure would need diagonals in both top and bottom planes of the cabin. These diagonals, however, must be omitted because of the required openeings; the diagonals thus are replaced by two plane steel plates with large central openings. This focal structure is designed to make it stable even with pin joints, thus giving maximum stiffness in all directions (not relying on bending stiffness).

The cabin should have double walls with heat insulation, and a door at the top. The wall which is the service floor should have double wall thickness and needs stiffening ribs underneath.

I suggest the following sizing:

Plates: A36 steel, 3/16 inch thick.

Structure: A36 steel, standard pipe, 5 inch nominal diameter.

Walls: 6061 T6 aluminum, 1/8 inch thick (service floor 1/4 inch).

The resulting weight then is: 4.14 kip for 24 structure pipes; 1.14 kip for 2 steel plates; 2.42 kip for cabin walls, floor and door; .60 kip for floor ribs. The total weight is

$$8.30 \text{ kip for cabin and focal structure.} \tag{23}$$

Subtracted from the limiting weight of 35.4 kip from (13), we then have a maximum weight of 27.1 kip (12 tons) for all equipment: receivers and mount, heat and air condition. Which is much more than actually needed.

II. Support Members and Central Member

1. Support Members

The three types of support members, holding the telescope from the elevation bearings, are listed in Table 2, with L = length, A_{eq} = equivalent bar area (of a solid rod of same stiffness), S_m = maximum stress in any telescope orientation, l/r = slenderness ratio of single chord if 12 segments were used. The last column mentions the problem which makes a special design neccessary.

Table 2.

Type	points	L inch	A_{eq} si	n	S_m ksi	l/r	problem
a) Side support	45-58	878	150	4	9.4	10	small angle with 45-46 at 45
b) Down support	57-58	1609	360	2	6.0	11	small angle with 47-57 at 57
c) Center supp.	38-58	1206	100	2	11.2	17	must cross 47-57 without touching

All these members have very large bar area and small l/r ratio, which means they should have much less than 12 segments. And unlike the normal members, they should have rectangular cross section (not triangular), and they should be made from heavy combination sections (not pipes). Since the support has only little influence on homology, no exact values of A_{eq} and no special types of lacing are required, thus all details are left to the designing engineer. The bar area should meet the one of Table 2 within $\pm$ 20%, say, and the lacing just should have as little weight as possible. With respect to the problems listed in Table 2, we make the following suggestions, see Fig.4:

a) The side support splits in two parts (up and down) at least toward point 45; such that its upper and lower part leave enough clearance for the end of member 45-46 in between, Fig.4a.

b) The two down supports join in a horizontal piece which holds point 57 from underneath, Fig.4b.

c) The center support is split in two parts (right and left) all the way through, with a clearance of at least 60 inch in between for member 47-57. Fig.4c.

2. Central Member and Pointing Platform

If we could use a pointing system with gyro compass and tilt sensor, the system would be mounted right at the apex of the surface, and no special design would be needed for member 38-57. But at present it is not absolutely clear whether this system would work, and thus an optical system is considered.

The optical system must be located exactly at the crossing of the two telescope axes, which means in the middle of the center support 58-58a. The system needs around it, in the lower hemisphere, as much clearance as possible (blocking of light rays). For this purpose, we divide member 38-57 in two parts. The lower part, 57-59, is a normal built-up member. The upper part is a quadratic pyramid upside-down, with its top at point 59 and its base supplied by the widely split-up center support 38-58. The four sides of the pyramid are single standard pipes, of 10 inch diameter ($A = 11.91$ inch^2, $r = 3.67$ inch). The base of the pyramid has a side length of 110 inch. The shadow of these pipes then coincides with the shadow of four cone members (46-57) and is of the same width, too, thus giving no additional shadow at all.

Point 38 carries a maximum downward load of 340 kip (stow position, 20 bl/ft^2 of snow, plus dead loads). If that is too much to be taken up by the beam action of the center support (38-58), we need an additional point 61 above 38, at the top of another pyramid, connected to all four points 60 and to 38. Structurally, this can be done, since 38 has no upward member along the z-axis, and since all its horizontal members (to 21, 22, 40, 41) leave 38 at azimuth angles of 22.5° and 67.5°, whereas members 60-61 leave 61 at 45°.

If possible structurally, it would be best if the optical platform, with respect to all torsional deformations, could follow the movements of the telescope members (38-21, 22, 40, 41) and not that of the support (38-58). But since the difference will be only small, no emphasis is placed on this demand.

Since the platform must be right on the elevation axis, point 38 must be somewhat above it, giving enough clearance for the elevation movement of the platform. The whole arrangement of central member, platform, and center support is sketched in Fig. 5.

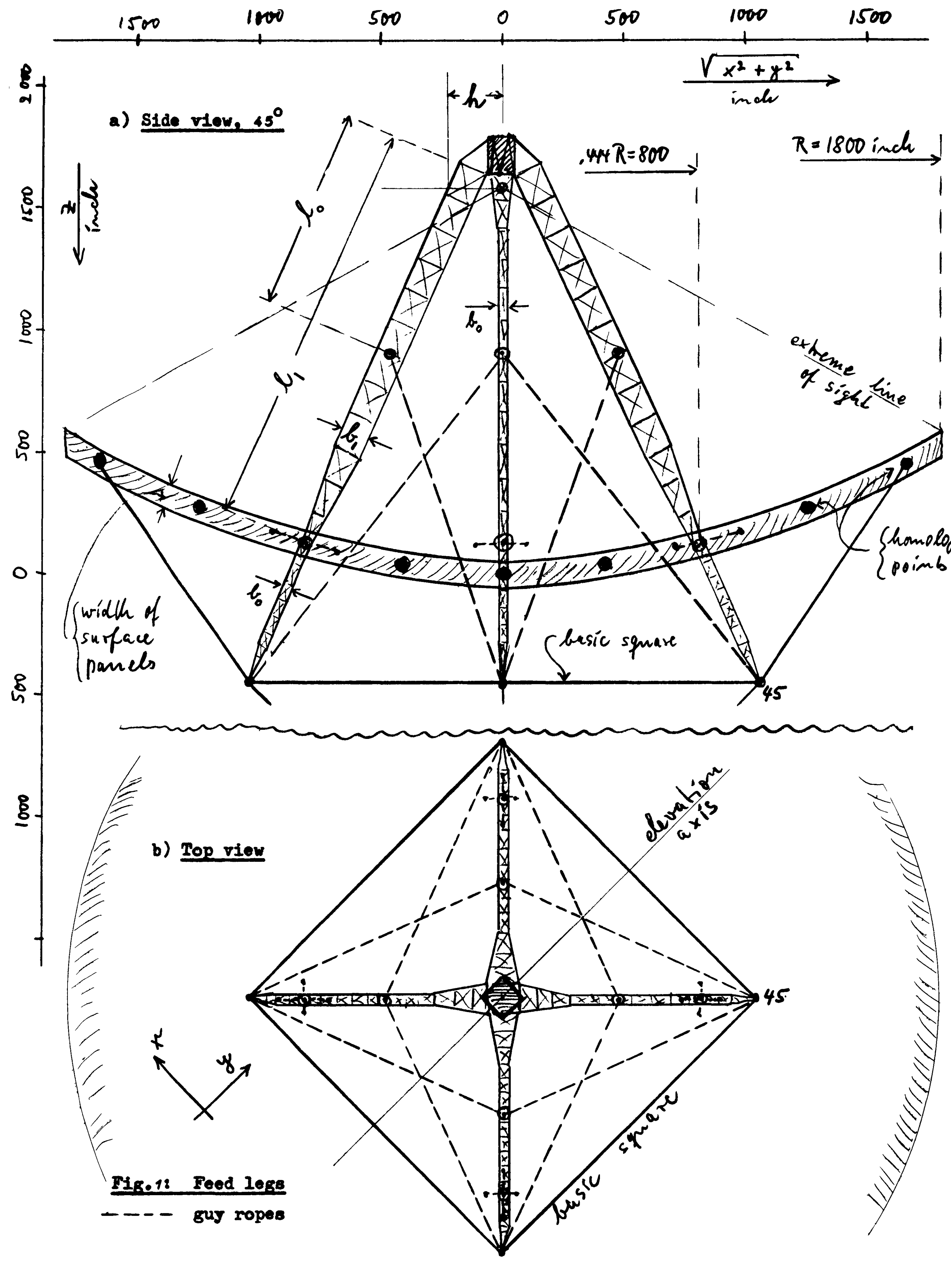


Fig.1: Feed legs

- - - - guy ropes

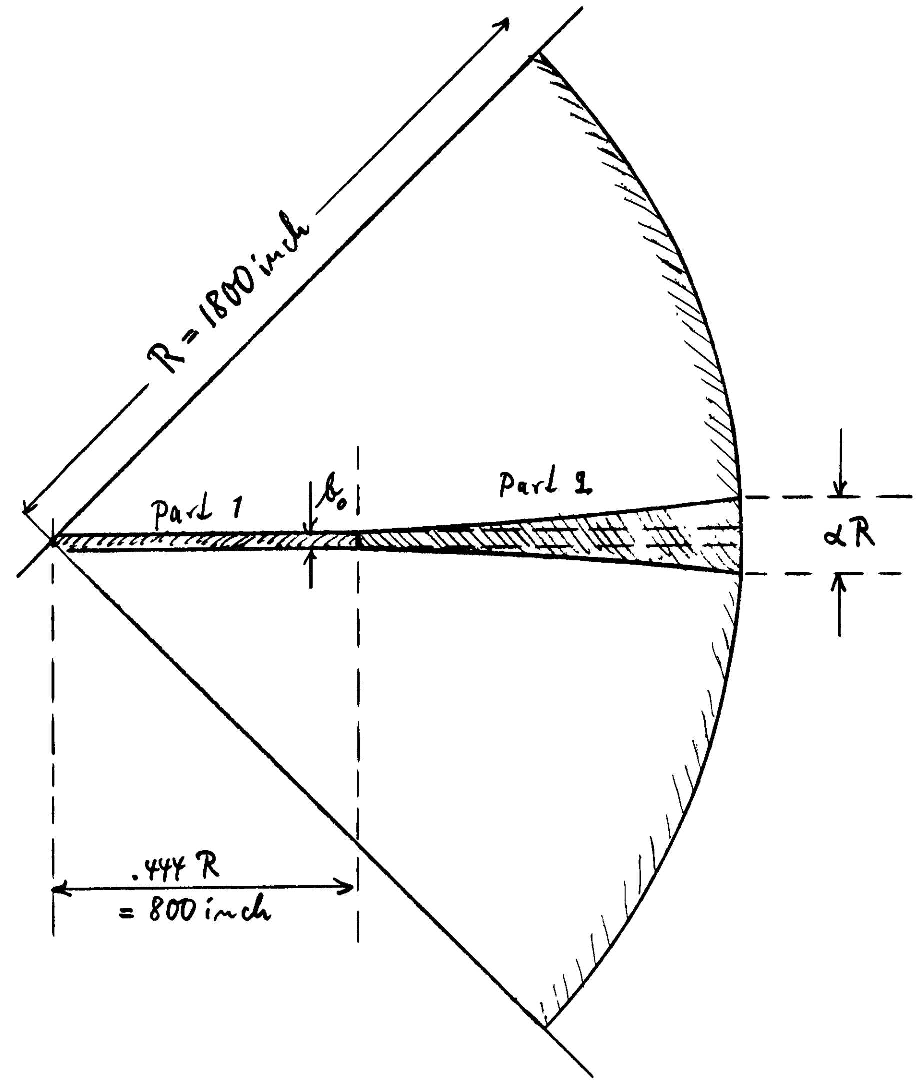


Fig.2: The shadow of a feed leg, and one quadrant of the aperture.
Part 1: shadow for incident rays,
Part 2: shadow for illumination.

Width at rim:

$$\alpha R = .616\ b_o R/h + .432\ b_o$$

Illumination-weighted shadow fraction (of aperture):

$$f_s = .130\ b_o/h + .000787\ b_o$$

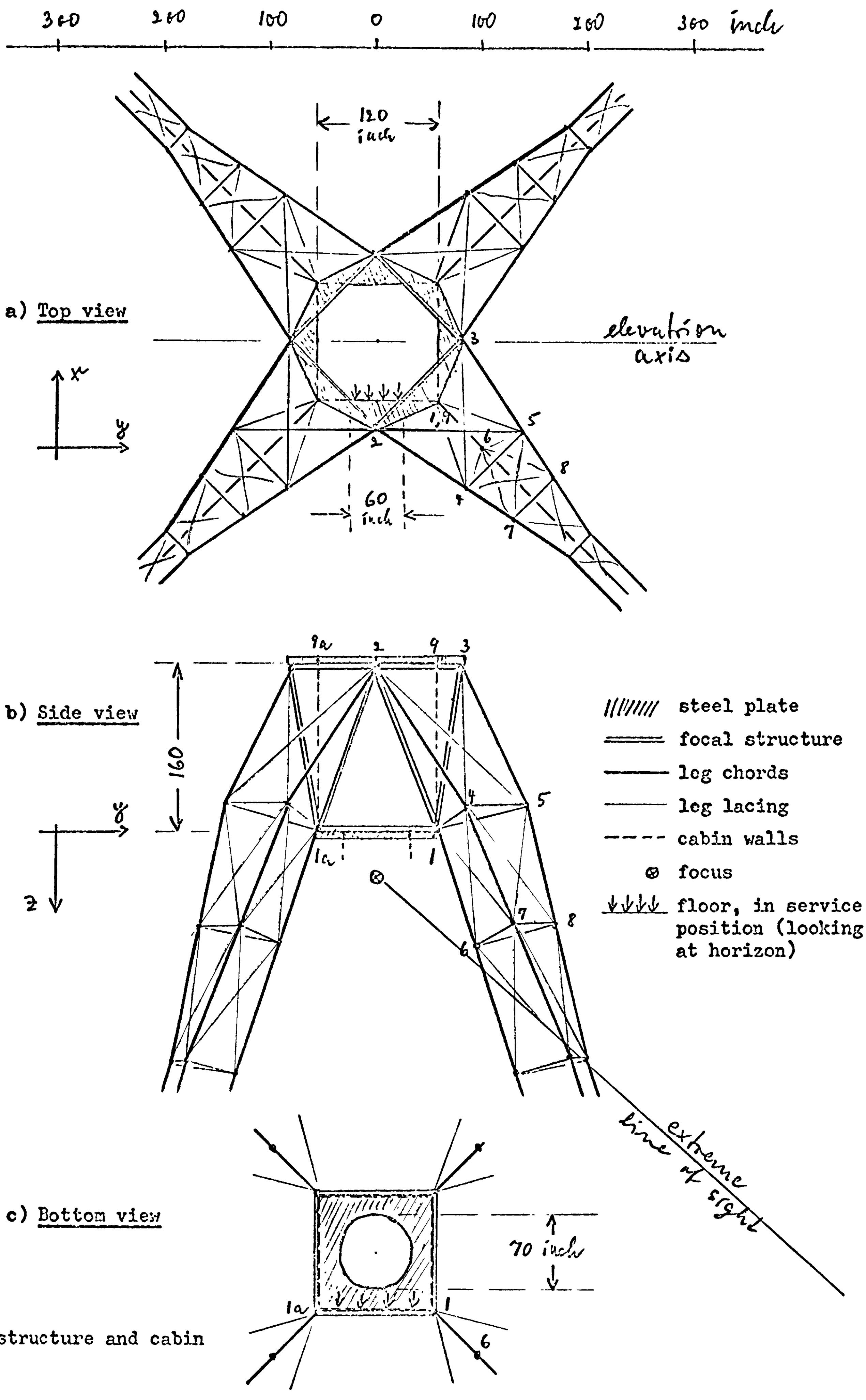
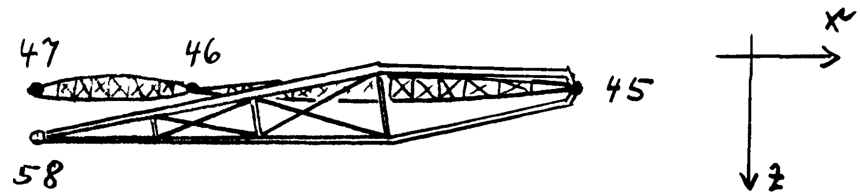


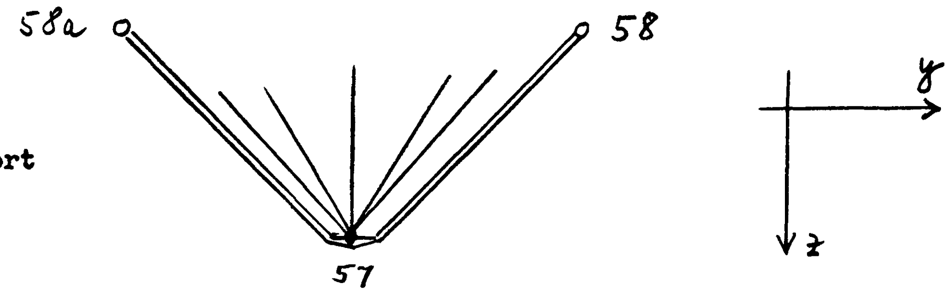
Fig.3:

Focal structure and cabin

a) Side support



b) Down support



c) Center support

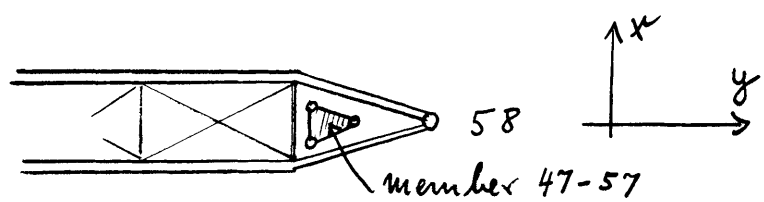


__Fig.4:__ Suggestions for support members.

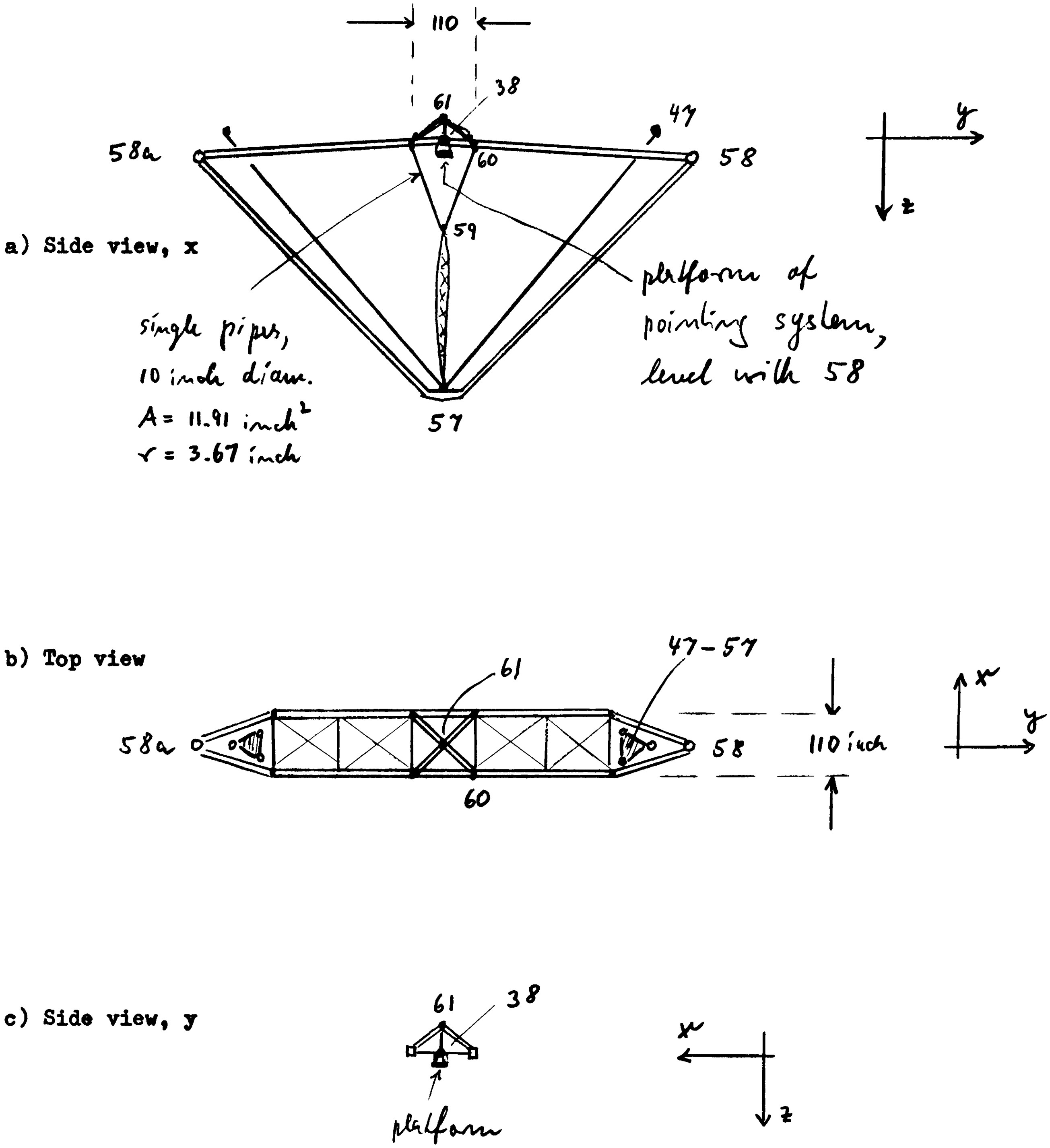


Fig.5: Central member (38-57) and optical pointing system.