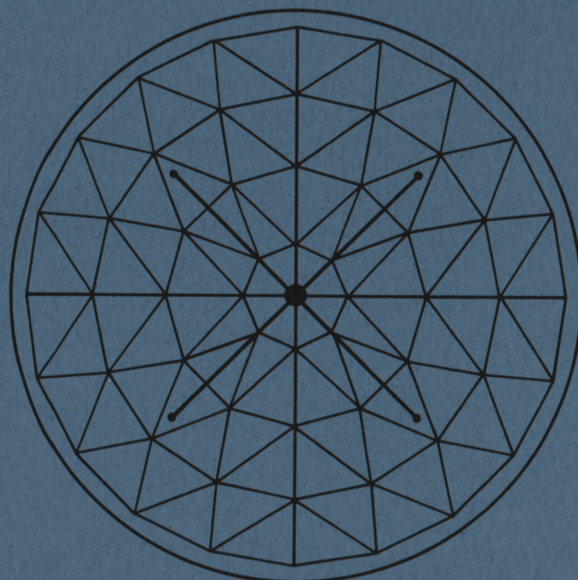

SECTION 5.0

PROJECT: 300 FT. DIA HOMOLOGY RADIO TELESCOPE

SUBJECT: POSITION REFERENCE PLATFORM APPENDIX -I-

5.0 APPENDIX I

ANALYSIS

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PROJECT: 300 FT - HOMOLOGY - RADIO TELESCOPE

SUBJECT: POSITION REFERENCE PLATFORM

APPENDIX I

CALCULATIONS

A. INERTIA

1- AZIMUTH AXIS:

a- POLYGON, QUARTZ AND ALUMINUM
$I = 2600\ OZ\text{-}IN^2$

b- SHAFT, ALUMINUM, HOLLOW
$I = 2700\ OZ\text{-}IN^2$

c- BEARING INNER RACES, STAINLESS
$I = 700\ OZ\text{-}IN^2$ (FOR TWO)

d- TRANSDUCER ROTOR
$I = 180\ OZ\text{-}IN^2$

e- SERVO MOTOR ROTOR
$I = 230\ OZ\text{-}IN^2$ FOR T-5135
$I = 380\ OZ\text{-}IN^2$ FOR T-5730
$I = 1720\ OZ\text{-}IN^2$ FOR T-7203

2- ELEVATION AXIS:

a- SHAFT, STEEL, HOLLOW
$I = 3680\ OZ\text{-}IN^2$

b- POLYGON AND SUPPORT, QUARTZ AND ALUMINUM
$I = 3300\ OZ\text{-}IN^2$

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c- TRANSDUCER AND COUPLING

$I \approx 1200$ OZ-IN^2

d- TRANSDUCER SUPPORT BRACKETS, AL.

$I = 2200$ OZ-IN^2

e- BEARING AND MOTOR BRACKET

$I = 1750$ OZ-IN^2

f- INSULATION, HOUSING, POLYGONE WINDOWS.

$I = 5000$ OZ-IN^2

FOR 1/2" THICK POLYURETHANE INSULATION.

TOTAL ELEVATION AXIS INERTIA:

18,000 OZ-IN^2

$18000 \times 1.3 \times 10^{-5} = \underline{\underline{0.23 \text{ LB-FT-SEC}^2}}$

THIS IS WORST CASE INERTIA FOR SERVO CALCULATIONS.

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B PLATFORM INSULATION
HEAT DISSIPATION

1 CUBIC INCH OF POLYURETHENE DISSIPATES 0.02 WATTS FOR A ΔT OF 130°F WHERE WORST CASE OUTSIDE TEMPERATURE IS 0° TO 120°F AND THE PLATFORM IS KEPT AT 130°F.

$2400 \text{ IN}^2 \times 0.02 \text{ WATTS} = 48 \text{ WATTS}$

IF 1 INCH INSULATION THICKNESS IS USED. 1/2 INCH INSULATION WILL GIVE 96 WATTS. UNDER THIS CONDITION, THE AVERAGE DISSIPATION FOR AN AVERAGE OF 60°F WILL BE 52 WATTS.

C- PLATFORM WEIGHT

1- AZIMUTH AXIS:

a – POLYGON – 9 LBS

b – SHAFT 10.8 LBS

c – BEARINGS 1.8 LBS (FOR TWO)

d – TRANSDUCER 21.0 LBS

e – SERVO MOTOR

T – 5730 (7.3 LBS)
T – 7203 (18.3 LBS)
T – 5135 (6.4 LBS)

WORST CASE AZIMUTH WEIGHT 60 LBS

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SUBJECT: POSITION REFERENCE PLATFORM APPENDIX - I -

2- ELEVATION AXIS

a-	SHAFT - - - -	37 LBS
b-	TRANSDUCER SUPPORT BRACKETS	3 "
c-	TRANSDUCER BASE PLATES	3 "
d-	BEARING AND MOTOR BRACKET	10 "
e-	THERMAL INSULATION, HOUSING, WINDOWS	7 "
f-	BEARINGS	1 "
g-	MOTOR	18 "

TOTAL ELEVATION AXIS WEIGHT 69 LBS

TOTAL WEIGHT THE ELEVATION AXIS SERVO DRIVE HAS TO POSITION IS 60 LBS FOR AZIMUTH PLUS 69 LBS FOR ELEVATION.

USE 140 LBS IN CALCULATIONS FOR TOTAL WEIGHT.

D- FRICTION TORQUE

ASSUMING TORQUE ARM LENGTH AT BEARINGS OF 3" AND 1% OF THE WEIGHT FOR FRICTION

140 LBS × 1% = 1.4 LBS = 23 OZ

EL. 23 OZ × 3" = 69 OZ-IN

AZ. 0.6 LBS = 9.5 OZ

= 30 OZ-IN

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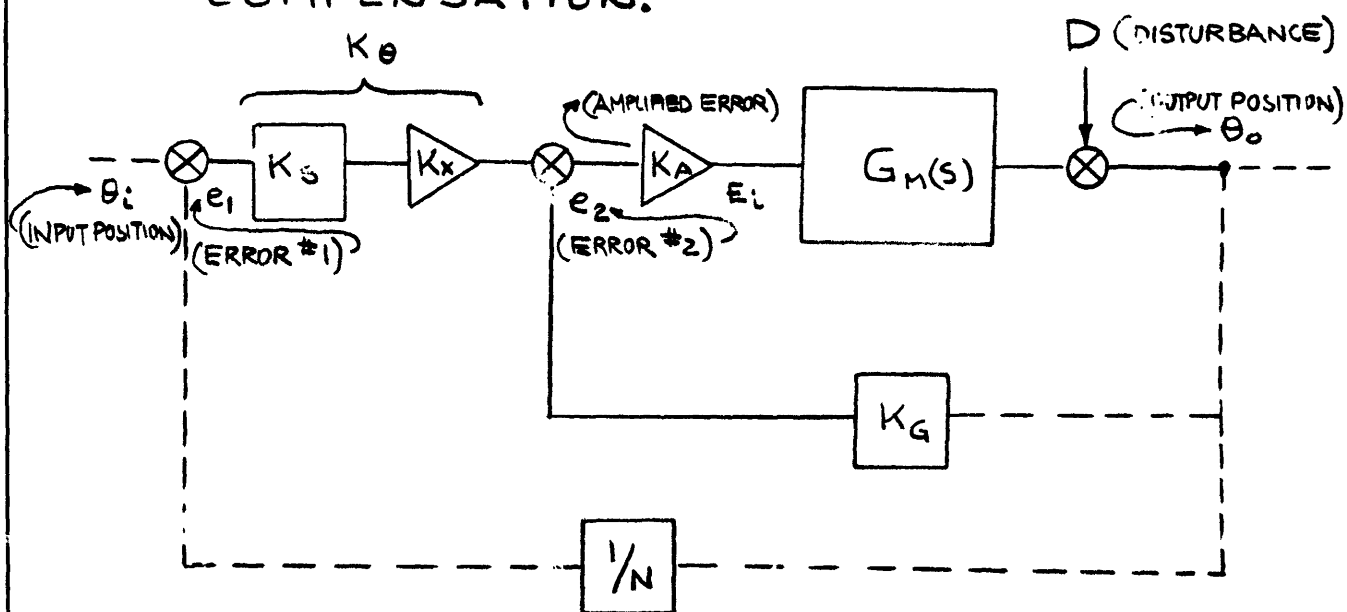
FOR SERVO CALCULATIONS USE 140 OZ-IN (VS. 69 OZ-IN) 140 OZ-IN = 0.74 LB-FT

E - SERVO ANALYSIS

1 - USE SERVO MOTOR T-5730 AS DIRECT DRIVE (N=1) WITH TACHOMETER COMPENSATION.



MOTOR PLUS LOAD TRANSFER FUNCTION:

$$\frac{\theta_o}{E_i} = \frac{1/K_B}{s\left[1 + \frac{J_T}{\left(\frac{K_B K_T}{R_T}\right)} s\right]\left(1 + \frac{L_M}{R_T} s\right)}$$

K_B = BACK EMF, FOR T-5730 = 0.72 V/RAD/SEC

J_T = TOTAL MOTOR PLUS LOAD INERTIA = 0.24 LB FT - SEC^2

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SUBJECT: POSITION REFERENCE PLATFORM APPENDIX - I -

KT = TORQUE SENSITIVITY = 0.53 LB FT/AMP
RT = MOTOR TOTAL ROTOR LOOP RESISTANCE, 1.5 OHMS FOR THE ROTOR, PLUS 0.1 OHMS FOR THE 300A SERVO AMPLIFIER OUTPUT IMPEDANCE, = 1.6 OHMS.
LM = MOTOR INDUCTANCE = 0.005 H

$$G_M = \frac{1/.72}{s\left[1 + \dfrac{0.24}{\left(\dfrac{.72 \times .53}{1.6}\right)} s\right]\left(1 + \dfrac{.005}{1.6} s\right)}$$

(MOTOR TRANSFER FUNCTION)

$$= \frac{1.4}{s(1 + 1.01s)(1 + .0031s)}$$

$(T_M)^*$ $(T_E)^*$

TO FIND OPEN LOOP GAIN REQUIREMENT,

FRICTION TORQUE = 0.74 LB FT
VOLTAGE SIGNAL AT MOTOR TERMINALS TO OVERCOME THIS

$$V_H = 19.8 \times \frac{0.74}{7} = 2.1 \text{ VOLTS}$$

WHERE THE RATING OF THE T-5730 MOTOR IS USED I.E. IT PRODUCES 7 LB-FT OF TORQUE FOR 19.8 V EXCITATION.

THE TRANSDUCER SIGNAL IS AMPLIFIED NEAR THE TRANSDUCER (ELEVATION) TO GIVE ± 10V DC FOR ± 30 ARC SEC RANGE

$$K_\theta = \frac{10}{30} \times 60 \times 60 \times 57.5 = 68\,000 \text{ V/RAD}$$

* T_M = MECHANIC. TIME CONSTANT T_E = ELECTRICAL TIME CONSTANT

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TOTAL GAIN NEEDED FOR ± 1 ARC SEC HYSTERESIS IS :

$$\pm 1 \text{ ARC SEC} = 2.1V$$

$$2.1V \times 60 \times 60 \times 57.5 = 435\,000 \text{ V/RAD}$$

THUS, $K_\theta \times K_A = 435\,000$

$$K_A = \frac{435\,000}{68\,000} = 6.4 \text{ VOLTS/VOLT}$$

THE COMBINED MOTOR, TACH. TRANSFER FUNCTION IS :

$$\frac{R}{C} = \frac{G}{1 + GH}$$ WHERE H IS TACH TRAN.

FOR FUNCTION $K_G S$. TO FIND THE VALUE OF K_G FOR STABLE INNER LOOP ASSUME ω_1 AS 1/3 OF THE MOTOR TRANSFER FUNCTION ELECTRICAL TIME CONSTANT AS A RULE OF THUMB.

$$\omega_e = 330 \text{ RAD} \quad (\tau_e = 0.0031)$$

$$\omega_1 = 100 = \frac{\frac{K_A K_G}{K_B}}{T_M} = \frac{6.4 \times K_G}{0.72 \times 1.01} = 8.8 K_G$$

$$K_G = \frac{100}{8.8} = 11.3 \text{ V/RAD/SEC}$$

THUS, TACH TRANSFER FUNCTION $G_T = 11.3S$

$$\frac{\theta_o}{e_2} = \frac{K_A G_M}{1 + K_A G_M G_T}$$

$$= \frac{\frac{1.4 \times 6.4}{S(1 + 1.01S)(1 + .003S)}}{1 + \frac{11.3S \times 1.4 \times 6.4}{S(1 + 1.01S)(1 + .003S)}}$$

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$$= \frac{8.95}{s(102 + 1.013s + .003s^2)}$$

$$\frac{\theta_o}{e_2} = \frac{0.088}{s(1+.01s)(1+.003s)}$$

TRANSFER FUNCTION OF THE POSITION OPEN LOOP IS THE ABOVE FUNCTION TIMES K_θ. GIVING A GAIN OF 6000 OR 20 × LOG 6000 = 75.4 Db

A BODE PLOT IS DRAWN USING 75.4 Db AND CORNER FREQUENCIES OF 100 RAD/SEC AND 330 RAD/SEC

COMPENSATION TECHNIQUES ON THIS PLOT WILL BE DISCUSSED LATER.

IF THE TACHOMETER FEED BACK IS INCREASED TO IMPROVE STABILITY SAY $K_G = 17V$ OR $W_1 = 150$ RAD. TACH LOOP TRANSFER FUNCTION BECOMES

$$\frac{\theta_o}{e_2} = \frac{0.061}{s(1+.0068s+.00002s^2)}$$

THIS EXPRESSION HAS IMAGINARY ROOTS AND DOES NOT HELP STABILITY.

2- CALCULATIONS FOR T-7203 GIVE THE FOLLOWING RESULTS AND DATA:

- $KB = 1.22$ V/RAD/SEC
- $JT = 0.25$ LB-FT-SEC^2
- $KT = .90$ LB-FT/AMP
- $R_T = .98$ OHMS
- $L_M = .005$ H

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CALCULATED $K_A = 2.7$ V/V
K_G 4.0 V/RAD/SEC

$$\frac{\theta_o}{e_2} = \frac{0.225}{s(1+.0065s)(1+.0165s)}$$

OPEN LOOP GAIN 83.6 Db
CORNER FREQUENCIES 61 AND 154
A BODE PLOT OF THIS FUNCTION IS ALSO PREPARED.

3 – CALCULATIONS FOR T-5135 GIVE THE FOLLOWING;

K_B = 1.3 V/RAD/SEC
J_T = 0.25 LB-FT-SEC^2
K_T = 0.94 LB-FT/AMP
R_T = 6.7 OHMS
L_M = 0.020 H

CALCULATED K_A 16.2
K_G 12.2 V/RAD/SEC

$$\frac{\theta_o}{e_2} = \frac{0.76}{s(1+.083s)(1+.003s)}$$

OPEN LOOP GAIN 94.3 Db

CORNER FREQUENCIES 12.0 AND 333
A BODE PLOT OF THIS FUNCTION IS ALSO PREPARED.

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PROJECT: 300 FT. DIA. - HOMOLOGY - RADIO TELESCOPE

SUBJECT: POSITION REFERENCE PLATFORM APPENDIX - I -

4- USING T-5135 WITH N = 100 I.E. AS A GEARED DOWN DRIVE INSTEAD OF DIRECT DRIVE.

K_B = 1.3 V/RAD/SEC
JT = 0.025 LB-FT-SEC^2
(REDUCED BY A FACTOR OF 10)
KT = .94 LB FT/AMP
RT = 6.7 OHMS
LM = .020 H
FRICTION = 0.3 LB-FT

K_A CALCULATED AS 7
K_G " " 2.5
V_H " " 2.3

K_θ NOW HAS TO BE 100 X MORE TO GIVE THE SAME VOLTS/RAD SENSITIVITY

$$\frac{\theta_0}{e_2} = \frac{0.6}{s(1+.0086s)(1+.001s)} = G(s)$$

OPEN LOOP GAIN = 68 000 × 100 × G(s)
= 132.2 Db

CORNER FREQUENCIES 115 AND 1000

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F - COMPENSATION APPROACHES

1- LEAD - LAG NETWORK WITH A TRANS. FUNCTION OF

$$G(s) = \frac{(1 + 0.02s)^2}{(1 + 0.16s)^2}$$

IS TESTED ON THE BODE PLOT OF T- 5730 WHICH GIVES THE BEST PERFORMANCE.

THE PHASE ANGLES AT 50, 75 AND 100 RAD ARE RESPECTIVELY AS FOLLOWS:

$$\angle 50 = \frac{(1+j1)^2}{j(1+j.5)(1+j.15)(1+j8)^2}$$

$$= \frac{\angle 45^\circ + \angle 45^\circ}{\angle 90^\circ + \angle 26^\circ + \angle 8^\circ + \angle 82^\circ + \angle 82^\circ} = -198^\circ$$

$$\angle 75 = -195^\circ$$

$$\angle 100 = -196^\circ$$

THUS NO PHASE MARGIN IS OBTAINED FROM THIS COMPENSATION NETWORK.

2- USE 70 OZ - IN FRICTION WHICH REDUCES OPEN LOOP GAIN TO 69.5 DB VS. 75.4

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BODE PLOT FOR COMPENSATION #2 GIVES THE VALUES FOR THE NETWORK TRANSFER FUNCTION:

$$G_s = \frac{(1 + 0.029s)^2}{(1 + 0.2s)^2}$$

THE PHASE ANGLE AT 0Db CROSSING (65 RAD/SEC) IS: $-179°$ WHICH IS QUITE AN IMPROVEMENT.

3 - COMPENSATION NO. 3 USES 35 OZ-IN OF FRICTIONAL TORQUE, GIVING 63.2 Db. THE COMPENSATION NETWORK TRANSFER FUNCTION NOW IS:

$$G_s = \frac{(1 + 0.04s)^2}{(1 + .2s)^2}$$

THE 0Db CROSSING IS AT 50 RAD/SEC THIS GIVES A PHASE ANGLE OF $-164°$ WHICH IS SATISFACTORY.

G CONCLUSIONS

1- WEIGHT, INERTIA AND FRICTION OF ELEVATION AXIS IS CHRITICAL. 140 LB TOTAL WEIGHT, 35 OZ-IN TORQUE 0.23 LB-FT-SEC^2 INERTIA CAN BE TOLERATED.

2- 1 ARC SEC DEAD BAND IS POSSIBLE

3- TACHOMETER COMPENSATION IS NEEDED

4- T-5730 IS THE BEST SERVO MOTOR CHOICE

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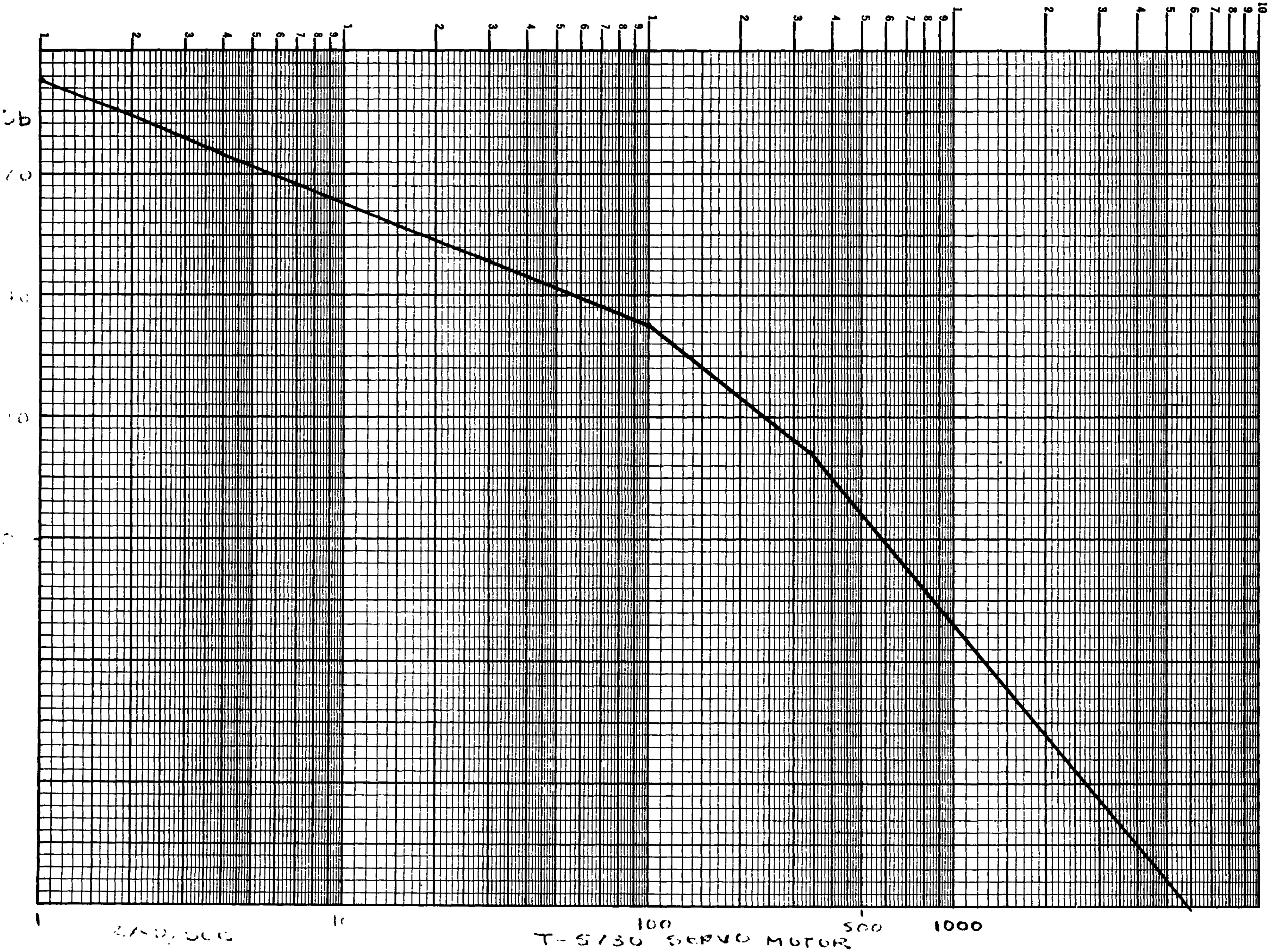
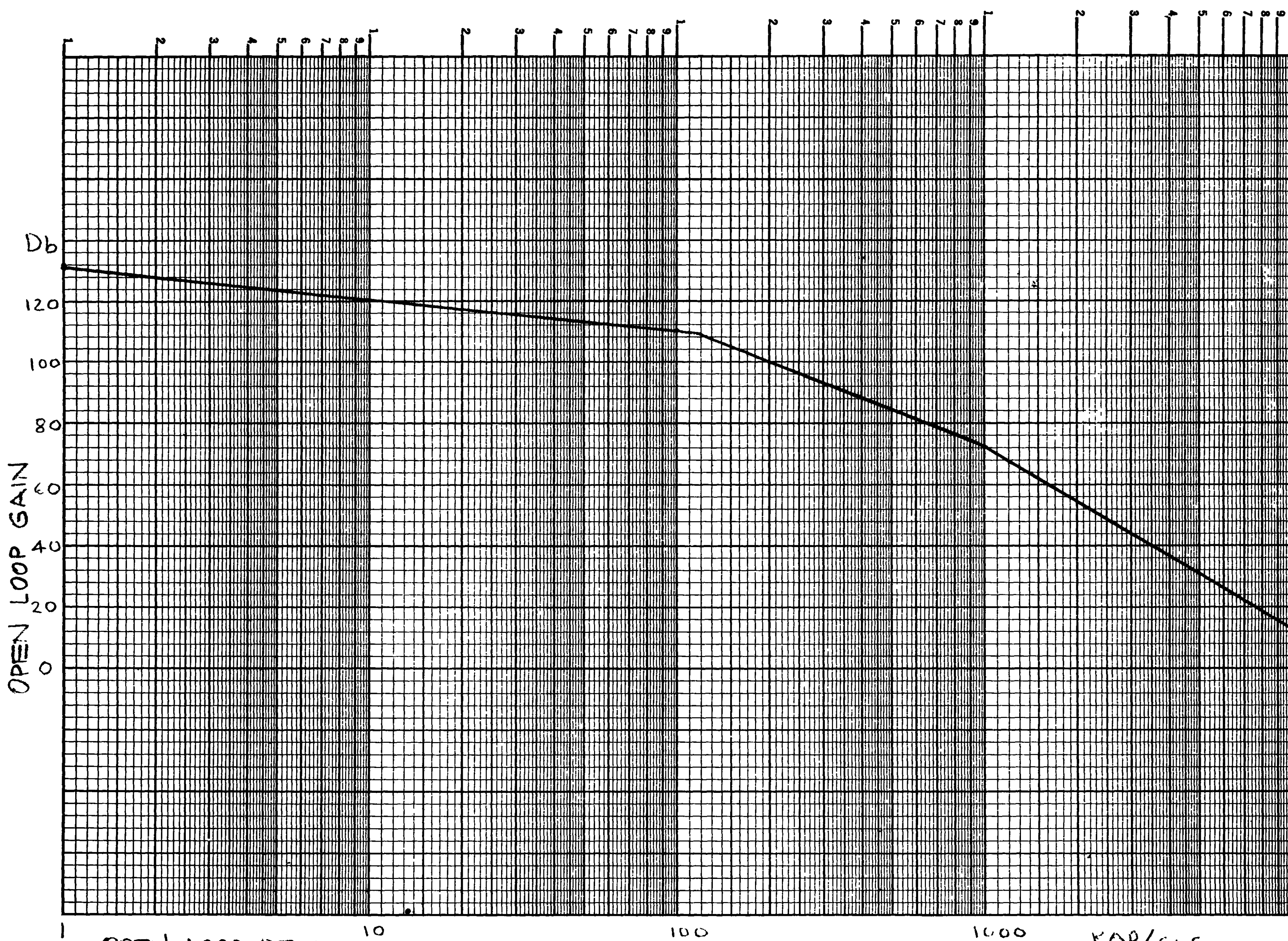
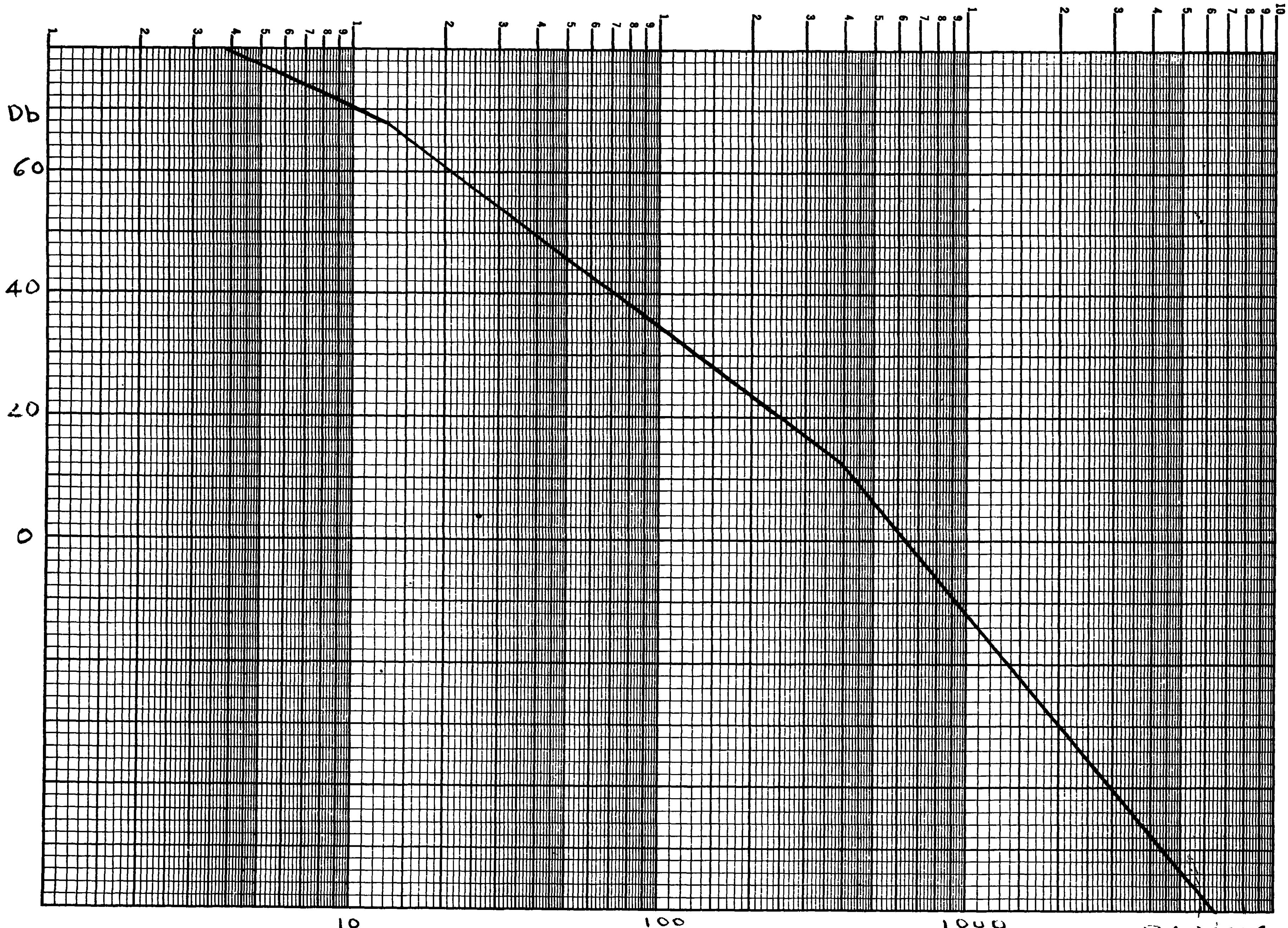

db
100
500
1000
T-5730 SERVO MOTOR

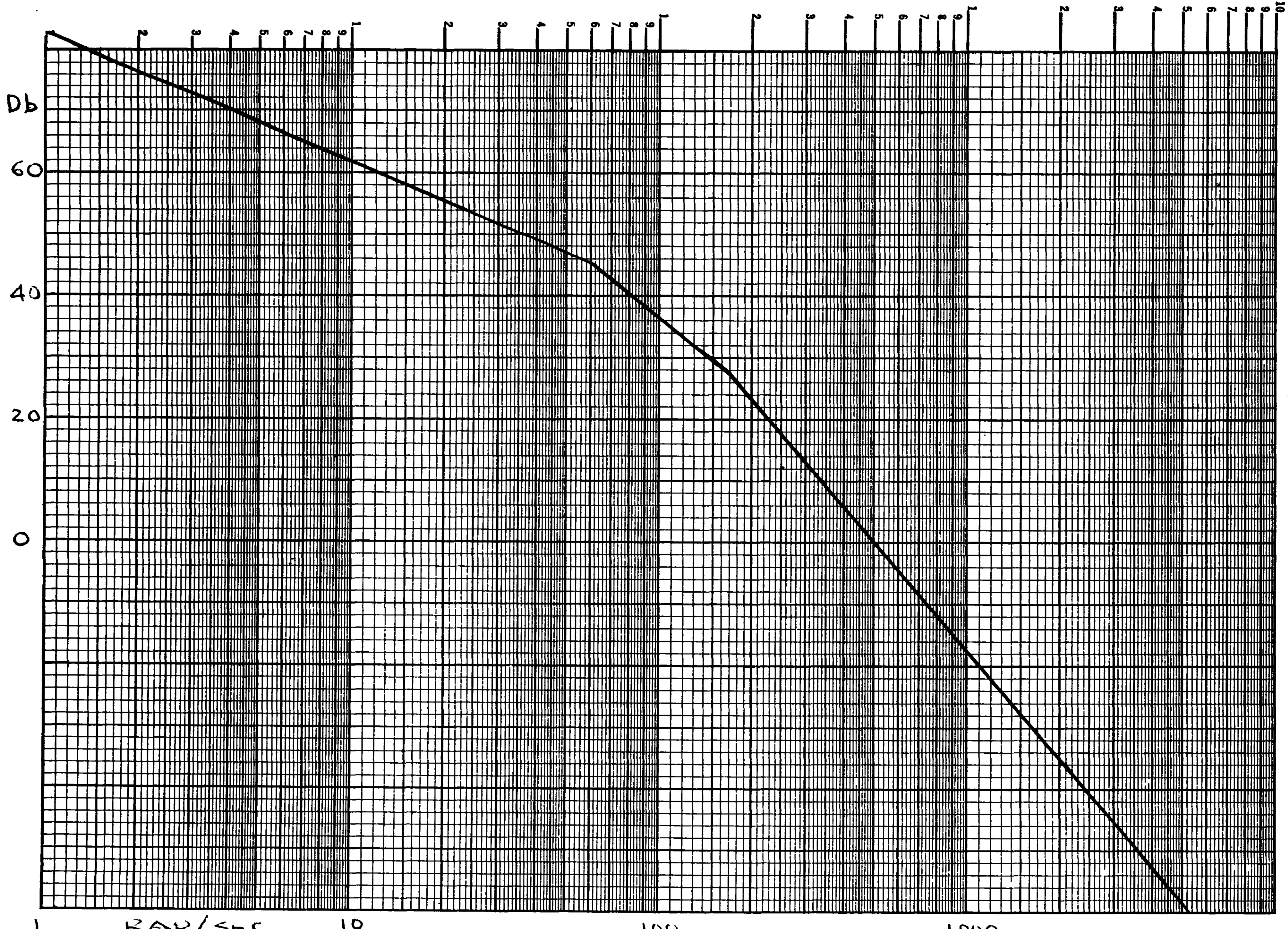


OPEN LOOP RESPONSE

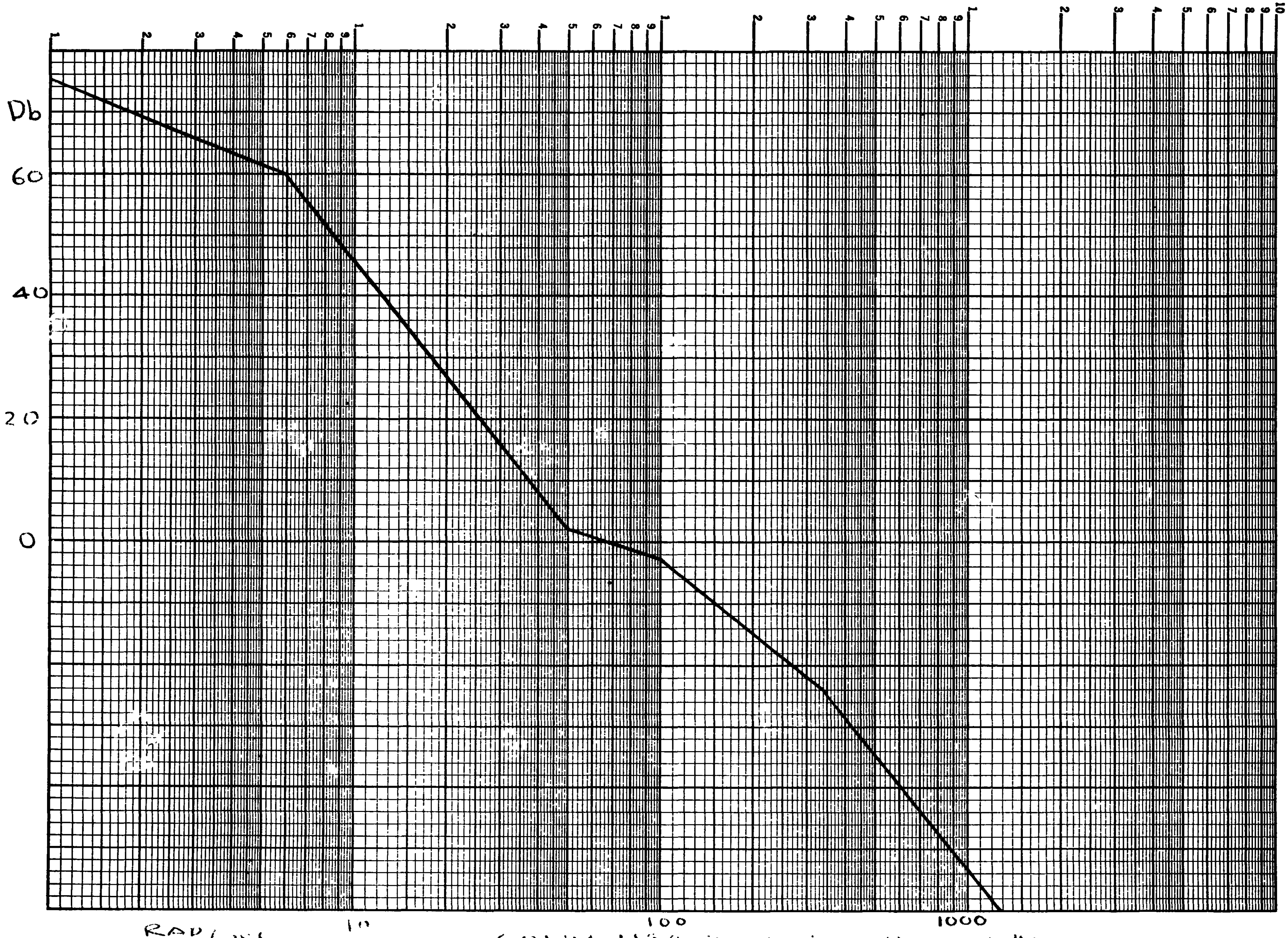
T-5135 SERVO MOTOR
WITH D-100



T-5135 SERVO MOTOR

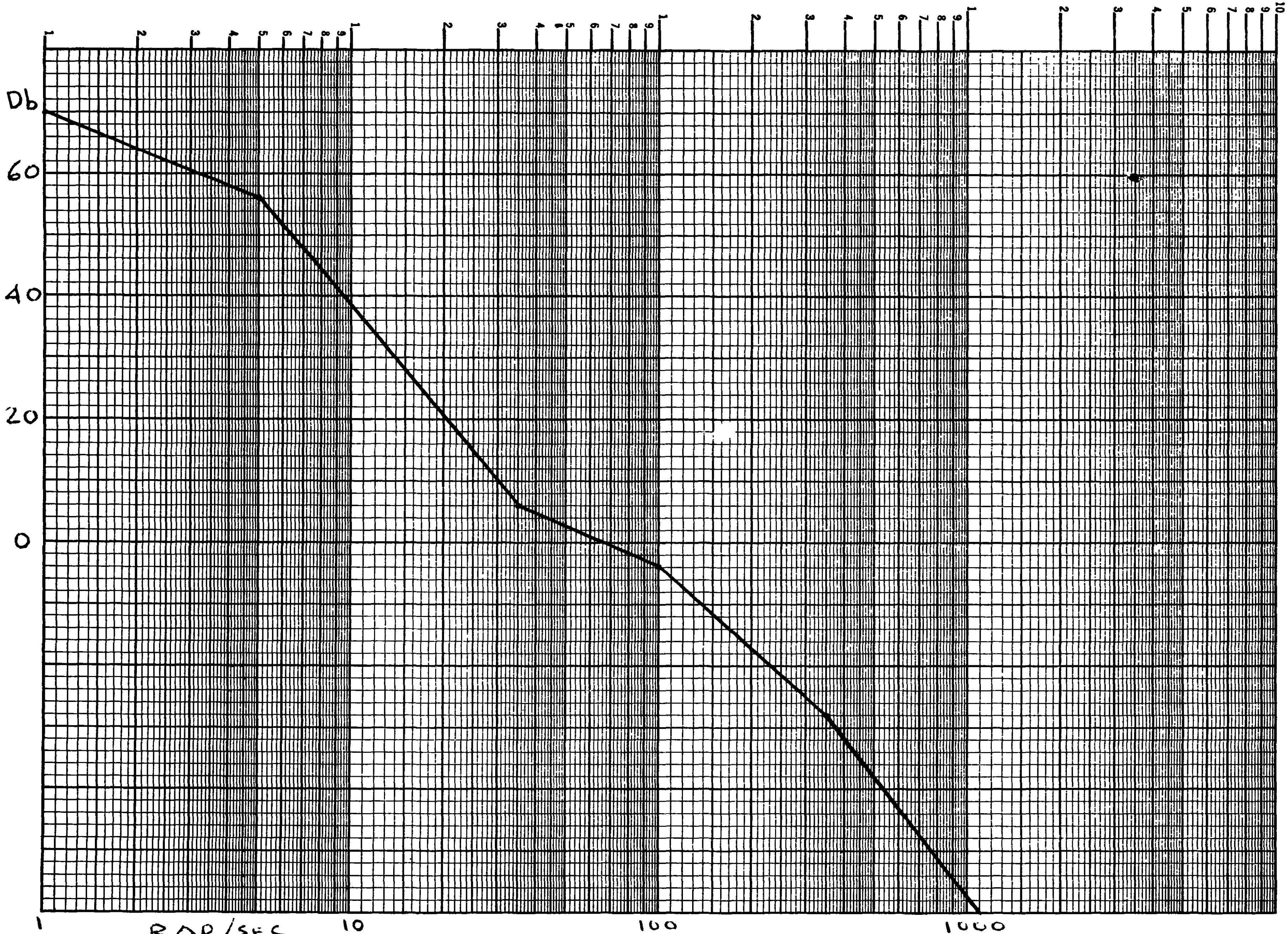


T-7203 SERVO MOTOR

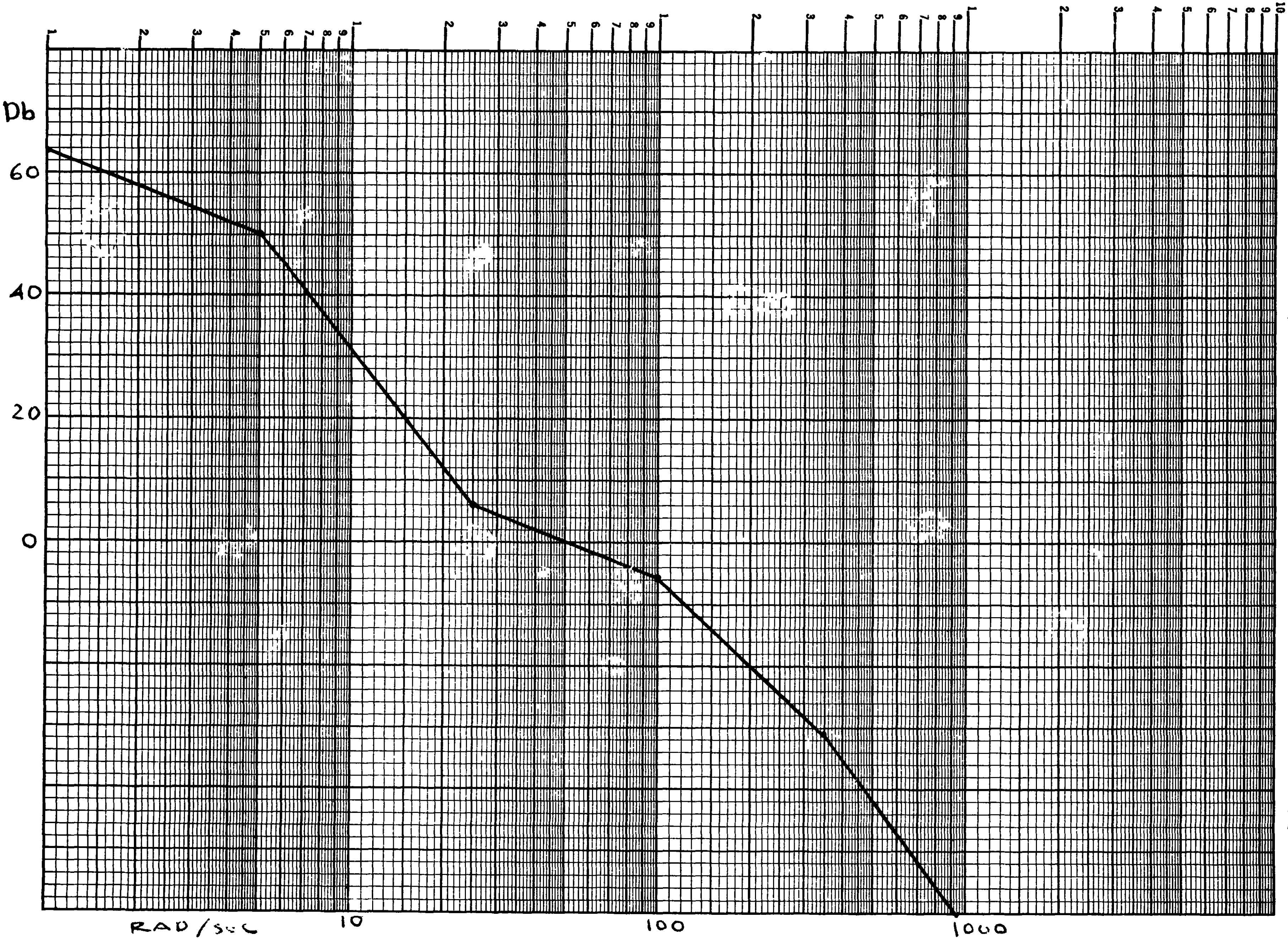


COMPENSATION APPROACH #1

H 79-7 APPENDIX-I



COMPENSATION #2



COMPENSATION #3

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DATE Nov. 1968

PROJECT: 300 FT. DIA. HOMOLOGY RADIO TELESCOPE

SUBJECT: POSITION REFERENCE PLATFORM

I. ADDENDUM TO APPENDIX I.

EXPLANATION OF APPLIED SERVO DESIGN METHOD

This step to step explanation of the applied servo design method contained herein was prepared for the benefit of those reviewers of this concept study who are not thoroughly familiar with this type of engineering discipline.

The analysis contained in section 5.0 demonstrates that attainments of both servo stability and desired frequency response is possible with the envisioned system through careful matching of suitable servo components and through shaping of the transfer function by application of lead-lag networks.

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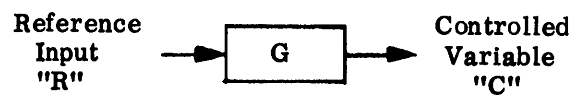
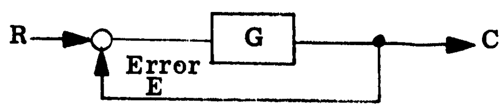
A - TYPES OF SERVO SYSTEMS

Open Loop Control System



Feedback Control System

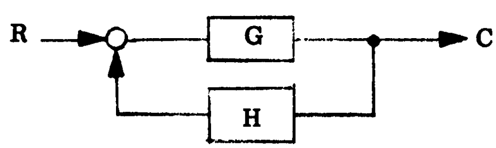


where $E \times G = C$

$E = R - C$

$\therefore \frac{C}{R} = \frac{G}{1+G}$

Feedback Control System with Feedback Element

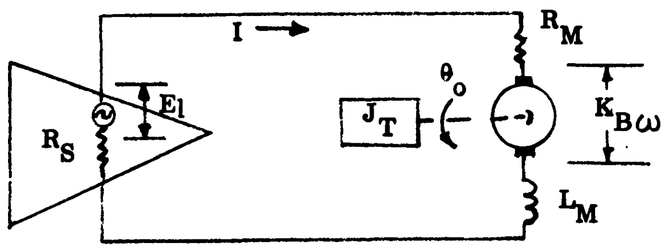


where $C = G(R - HC)$

$\frac{C}{R} = \frac{G}{1+GH}$

For a tachometer feedback element $H = K_G S$. For a gear reduction feedback element $H = 1/N$ where "N" is the reduction ratio.

B - DC SERVO MOTOR TRANSFER FUNCTION



K_B = Back-electromotive force (volts/rad/sec)
K_T = Torque sensitivity (lb-ft/amp)
ω = Angular velocity $d\theta/dt$ (rad/sec)
R_M = Armature resistance (ohms)
L_M = Armature inductance (h)
T_M = Motor torque (lb-ft)
θ = Shaft angle (rad)
J_T = Total moment of inertia referred to the armature (lb-ft-sec^2)
R_S = Source resistance (ohms)
E_1 = Control voltage
I = Armature current (amps)
R_T = $R_S + R_M$, total loop resistance (ohms)
τ_M = Mechanical time constant (sec)
τ_E = Electrical time constant (sec)

$$J_T = \frac{J_L}{N^2} + J_A$$

where J_L is the load moment of inertia in lb-ft-sec^2, J_A is the motor armature, hub and gear inertia on the motor shaft and "N" is the gear reduction ratio between the motor shaft and the load.

$$T_M = J_T \frac{d^2\theta_o}{dt^2}$$

$$E_1 = I(R_M + R_S) = L_M \frac{dI}{dt} + K_B \frac{d\theta_o}{dt}$$

THE LAPLACE TRANSFORMS:

$$E_1 = (R_M + R_S)I + L_M IS + K_B \theta_o S$$

$$K_T I = J_T \theta_o S^2$$

COMBINING THE LAST TWO EXPRESSIONS:

$$\frac{\theta_o}{E_1} = \frac{\frac{1}{K_B}}{S\left[1 + \frac{J_T}{\left(\frac{K_B K_T}{R_T}\right)} S\right]\left(1 + \frac{L_M}{R_T} S\right)}$$

$$\tau_M = \frac{J_T}{\left(\frac{K_B K_T}{R_T}\right)}$$ MECHANICAL TIME CONSTANT

$$\tau_E = \frac{L_M}{R_T}$$ ELECTRICAL TIME CONSTANT

C - DETERMINATION OF PHASE ANGLE AND MAGNITUDE

G_S = Open loop transfer function

G_{MT} = Motor-tachometer transfer function

G_{LL} = Lead-lag compensation network transfer function

K_θ = Transducer plus preamplifier gain (volts/rad)

K_A = Servo preamplifier gain

G_M = Motor transfer function

G_T = Tachometer transfer function

EXAMPLE:

$$G_M = \frac{1.4}{S(1 + 1.01S)(1 + .003S)}$$

$$G_T = 11.3S$$

$$G_{LL} = \frac{(1 + 0.02S)^2}{(1 + 0.16S)^2}$$

$$G_{MT} = \frac{0.09}{S(1 + .01S)(1 + .003S)} = \frac{K_A \times G_M}{1 + K_A \times G_M \times G_T} = \frac{G}{1 + GH}$$

K_θ = 68000 v/rad or 3v/arc sec

K_A = 6.4v/volt

$$G_S = K_\theta \times G_{LL} \times G_{MT}$$

$$G_S = \frac{6000 \times (1 + .02S)^2}{S(1 + .01S)(1 + .003S)(1 + .16S)^2}$$

This expression contains angle and magnitude. The magnitude at S = 1 can be approximated as 6000 or 20 x log 6000 = 75.4 Db.

Thus, the bode plot starts at 75.4 Db for one radian. The first term "S" in the denominator causes the magnitude curve to proceed at -20Db/decade of frequency increase, until it hits the frequency corresponding to the next time constant, .16 sec in this case. For every term in the denominator of the G_S expression, the slope of the bode plot changes by -20Db. Thus as the plot beyond 6 rad/sec [corresponding to $(1 + .16S)^2$], the slope changes from -20Db/decade of frequency to -60Db/decade. The next time constant is .02. This being in the numerator of G_S and a square term, the slope changes from -60Db to -20Db, or for every $(1 + T_S)$ term in the numerator the slope changes by +20Db/decade.

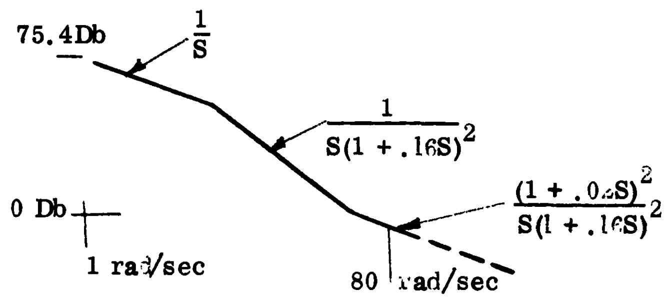


Figure No. 1

From the expression for closed loop transfer function:

$$\frac{C}{R} = \frac{G}{1 + GH},$$

the expression C/R becomes ∞ if the denominator of the transfer function becomes zero. Or,

$$1 = GH = 0$$

$$GH = 1$$

$$GH = 1 \angle -180°$$

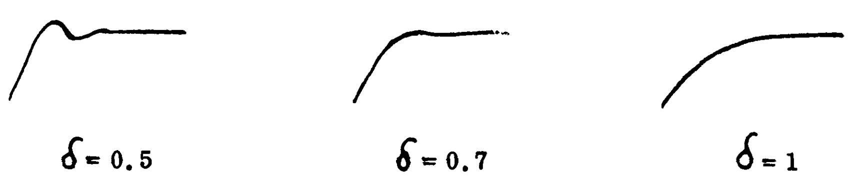
GH represents the open loop transfer function. Magnitude of "1" represents zero Db on the bode plot. -180° is the phase angle.

The angle has to be less than -180° for stable operation. The difference between -180° and actual phase at 0 Db is called phase margin.

$$\varphi_M = -180° - \varphi_o$$

As a rule of thumb, a phase margin of 30° produces only one overshoot for a step input (or a damping ratio δ of 0.5).

A phase margin of 60° represents a damping ratio of 0.7 or a critically damped system with no overshoot. Phase margins over 60° represent overdamped systems without overshoot.



Responses for step input with various values of δ.

Figure No. 2

Calculations for phase angle are performed after the bode plot has been completed and the zero Db crossing has been determined. Thus, if the above bode plot crosses the 0 Db at 75 rad/sec, the phase angle in this area is needed.

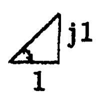
$j\omega$ is substituted for S (inverse transform) since the phase angle is of interest, the constant part of G_S is left out.

$$\angle G_S = \text{angle of } G_S = \frac{(1 + j\,.02\omega)^2}{S(1 + j\,.01\omega)(1 + j\,.003\omega)(1 + j\,.16\omega)^2}$$

Substituting $\omega = 50$ rad/sec

$$\angle G_{50} = \frac{(1 + j1)^2}{j50(1 + j0.5)(1 + j.15)(1 + j8)^2} = \frac{45° + 45°}{90° + 26° + 8° + 82° + 82°} = -198°$$

Every j term represents an angle, Thus,

$(1 + J1) =$  $= 45°$

$(1 + J1)^2 = 2 \times 45°$

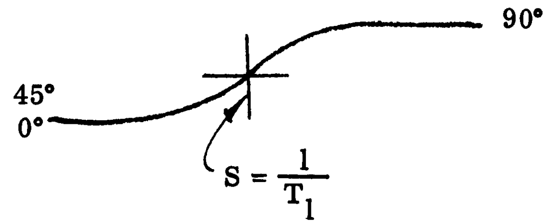
$j50 = 90°$

Angles in the numerator are positive angles. Angles in the denominator are negative angles.

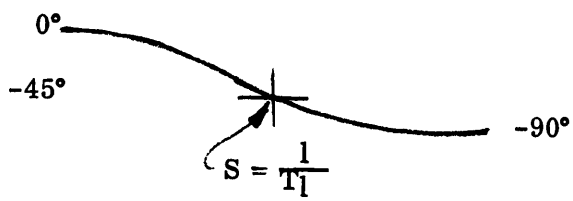
From the above expression for G_S, the entire excursion of the phase angle can be determined. Thus for very small values of ω, $\angle G_S = -90°$. For very large values this becomes $-270°$.

An alternate method of determining phase angle is to plot on the bode plot the phase angle of each individual $(1 + T_S)$ function.

Thus for $(1 + T_1S)$ the phase angle is:



For $\frac{1}{(1 + T_1S)}$ the phase angle is:

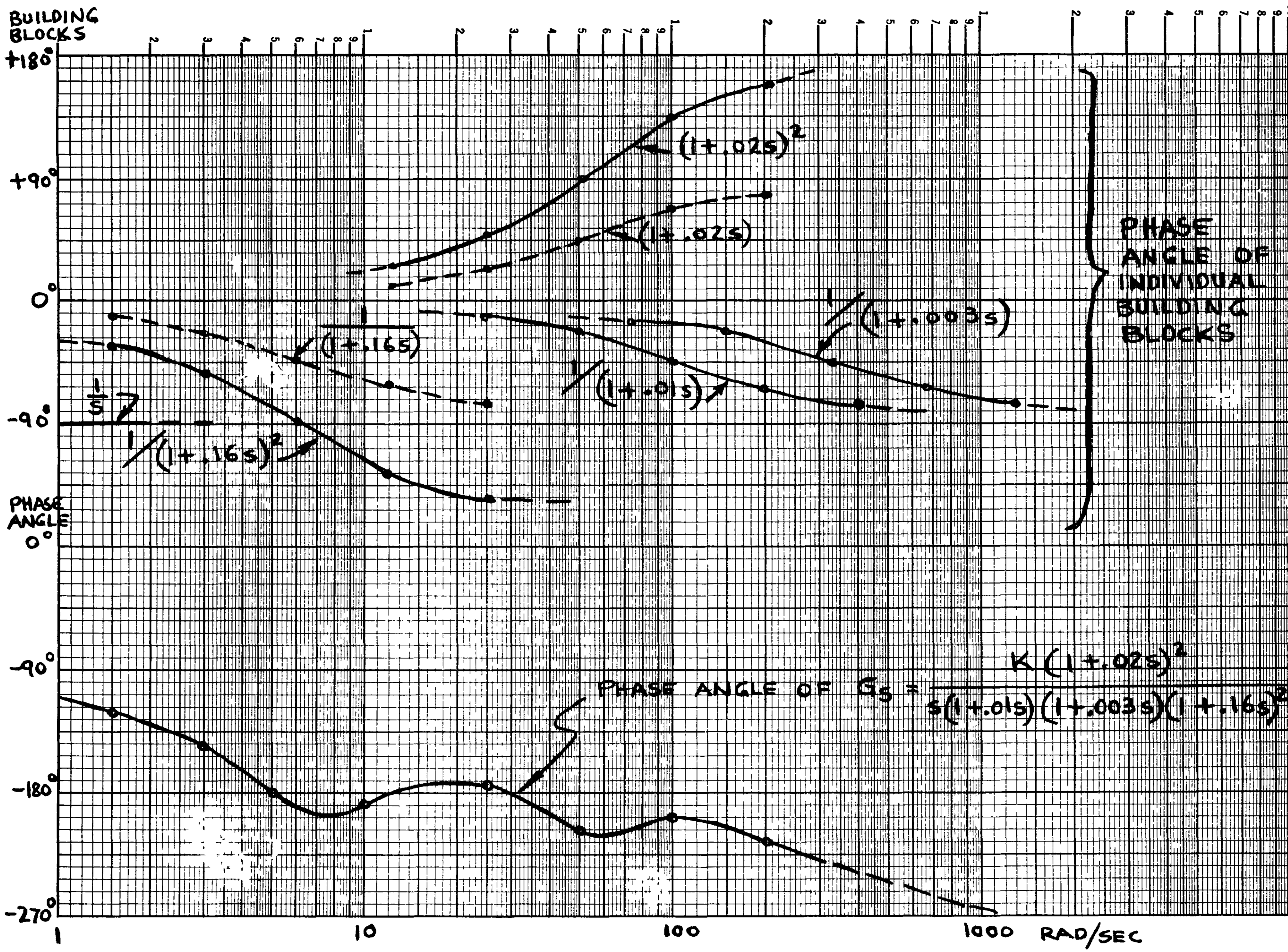


After all angle plots are drawn, their magnitudes can be added to find $\angle G_S$. The mathematical approach on the preceding page and the graphic approach on this page are identical.

Figure No. 3 shows this procedure. On the upper half of the semi-log paper the phase angle of each expression is drawn. Note that the phase angle of 1/S is a constant 90°. The phase angle of each expression is 45° when $T_S = 1$ for that expression or when S = 1/T. It also can be observed from Figure No. 3 that phase angle of an expression squared is equal to twice the phase angle of that expression.

$$\text{i.e.,} \quad \angle(1 + .02S)^2 = 2 \times \angle(1 + .02S)$$

If the expression is in the numerator, the phase angle increases from zero towards +90°. If the expression is in the denominator, the angle increases from zero towards -90°.



BUILDING BLOCKS
+180°
+90°
0°
-90°
PHASE ANGLE 0°
-90°
-180°
-270°
(1+.02s)²
(1+.02s)
1/(1+.16s)
1/(1+.003s)
1/(1+.01s)
1/s
1/(1+.16s)²
PHASE ANGLE OF INDIVIDUAL BUILDING BLOCKS
PHASE ANGLE OF Gs = K(1+.02s)² / s(1+.01s)(1+.003s)(1+.16s)²
1
10
100
1000 RAD/SEC

FIGURE NO. 3

D - CLOSED LOOP TRANSFER FUNCTION

The magnitude of a closed loop system is unity up to its natural frequency, or within its band width.

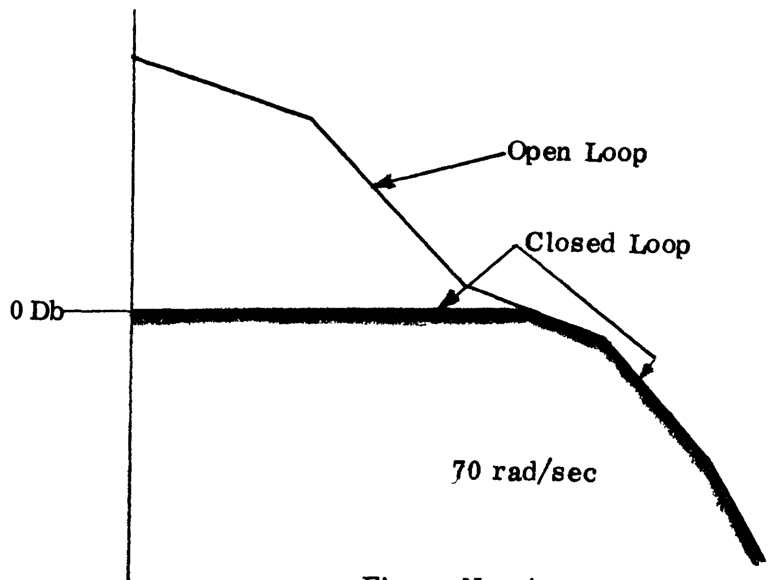


Figure No. 4

Beyond the natural frequency the magnitude will decrease as shown.

The phase angle of the closed loop will start at 0 degrees. By the time the frequency increases to the value of natural frequency, it will be -180°. For frequencies beyond this it will become asymptotic to -270°. In practice the above band width plot reflects also closed loop system damping ratio as shown below:

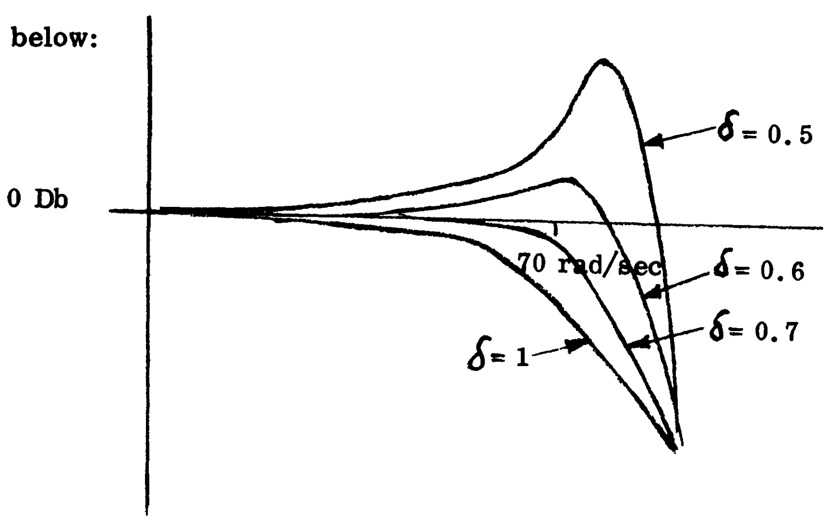


Figure No. 5

E - TYPICAL LEAD-LAG COMPENSATION NETWORKS

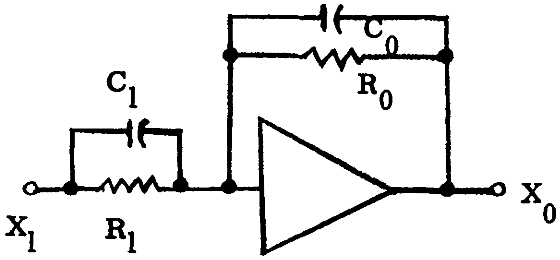


Figure No. 6

The diagram above represents an operational amplifier used as an active lead-lag network. C_0 and R_0 are the feedback components. C_1 and R_1 are the input components. X_1 is the input and X_0 the output signal. The transfer function is:

$$\frac{X_0}{X_1} = -\frac{R_0}{R_1}\left[\frac{1 + R_1C_1}{1 + R_0C_0S}\right] = -K\frac{(1 + T_1\ )}{(1 + T_2S)}$$

The parameters can be selected to give unity gain at steady state. The negative sign indicates polarity inversion. Non-inverting operational amplifiers or operational amplifiers with inverting and non-inverting inputs are also available.

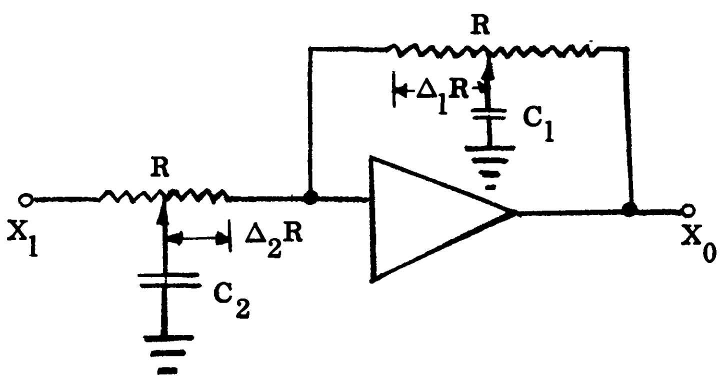


Figure No. 7

$$\frac{X_0}{X_1} = \frac{1 + (\Delta_1 - \Delta_1^2)\ RC_1S}{1 + (\Delta_2 - \Delta_2^2)\ RC_2S} = \frac{(1 + T_1S)}{(1 + T_2S)}$$