

Interoffice

National Radio Astronomy Observatory
Charlottesville, Virginia

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To: M. Davis and Engineering

From: S. von Hoerner

Subject: Non-Homologous Deformations of the 300-ft.

In his memo of April 2, 1969, M. Davis has plotted the aperture efficiency η of the 300-ft telescope as a function of zenith angle ϕ , as measured at $\lambda = 21.4$ cm. His Fig. 3 shows three curves $\eta(\phi)$: (a) 1962-66, before readjustment; (b) 1967-69, after readjustment; showing the single measured points for both curves; and (c) a prediction for the new surface. By comparison of two wavelengths (21.4 and 40 cm) he also finds the efficiency η_0 for $\lambda \rightarrow \infty$ as (a) $\eta_0 = .67$ and (b) $\eta_0 = .59$ and he adopts (c) $\eta_0 = .63$.

In the following, I make a best-fit of a theoretical formula to (a) and (b) and derive a somewhat different prediction (c), using the same three values η_0 as M. Davis. Let a telescope be adjusted without gravity to a perfect paraboloid. Then, with gravity, call ΔH_1 the rms deviation of the surface from a best-fit paraboloid in zenith position, and ΔH_2 likewise in horizon position. These two parameters fully describe the gravitational effects.

If a telescope is adjusted to a perfect paraboloid at zenith angle θ , and then observes at zenith angle ϕ , the deviation ΔH from a best-fit paraboloid is

$$\Delta H = \sqrt{\Delta H_1^2 (\cos \phi - \cos \theta)^2 + \Delta H_2^2 (\sin \phi - \sin \theta)^2} \quad (1)$$

If the surface itself has an rms error σ_0 , the total rms deviation from a paraboloid then is

$$\sigma = \sqrt{\Delta H^2 + \sigma_0^2} \quad (2)$$

and the aperture efficiency is

$$\eta = \eta_0 e^{-(4\pi\sigma/\lambda)^2} \quad (3)$$

Regarding ΔH_1 and ΔH_2 , it would be interesting to see whether or not there is a difference before and after the readjustment, since this was connected with some strengthening of the back-up structure (mainly the wheel); but the data cover too small a range in ϕ and scatter too much for this purpose. There seems to be a small improvement (about 20%), but in the following we neglect the difference and adopt the same ΔH_1 and ΔH_2 for (a), (b) and (c).

Since the second term in (1) is always much larger than the first one for the range of ϕ covered, we have a large uncertainty for ΔH_1 ; but this does not

effect prediction (c) very much, for the same reason. The best-fitting values and their estimated mean errors are

$$\Delta H_1 = 18.5 \pm 5.0 \text{ mm} \quad (4)$$

$$\Delta H_2 = 7.5 \pm 1.5 \text{ mm} \quad (5)$$

and their ratio is

$$g = \Delta H_2 / \Delta H_1 = 0.405 \quad (6)$$

In a reproduction of M. Davis' Fig. 3, I have entered the points calculated with equation (1), using parameters (4) and (5) and adopting the same values η_0 as M. Davis. The agreement with the measured points is certainly within the scatter of the data.

Next, prediction (c) is plotted for a new surface adopting $\sigma_0 = 4 \text{ mm}$ as M. Davis did. This new prediction gives smaller efficiencies for large ϕ than the old one.

Finally, if the best adjustment angle θ is defined by the demand

$$\Delta H_{-30^\circ} = \Delta H_{+60^\circ} \quad (7)$$

see Fig. 2, then one obtains from (1) and (6) that $\theta = 29.3^\circ$ or roughly

$$\theta = 30^\circ \quad (8)$$

With the available data, the uncertainty of g is rather large. With a probable error range of

$$0.29 \leq g \leq 0.58 \quad (9)$$

we find from Fig. 2

$$22^\circ \leq \theta \leq 36^\circ. \quad (10)$$

Prediction (c) also has a large uncertainty. From (4) and (5) we obtain, for example, the probable error ranges

$$10.08 \leq \sigma (60^\circ) \leq 14.38 \quad (11)$$

and

$$0.324 \leq \eta (60^\circ) \leq 0.465. \quad (12)$$

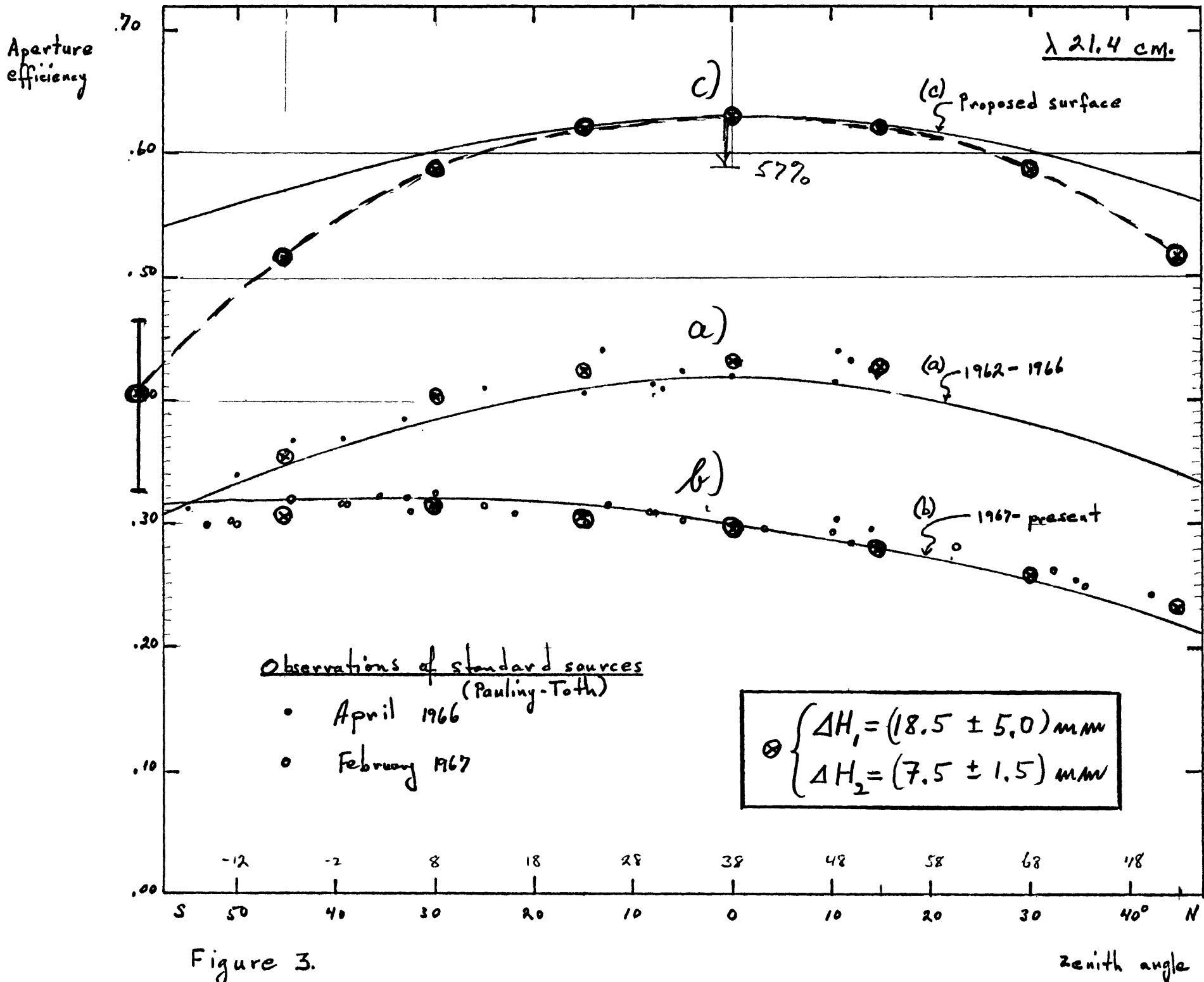


Figure 3.

zenith angle

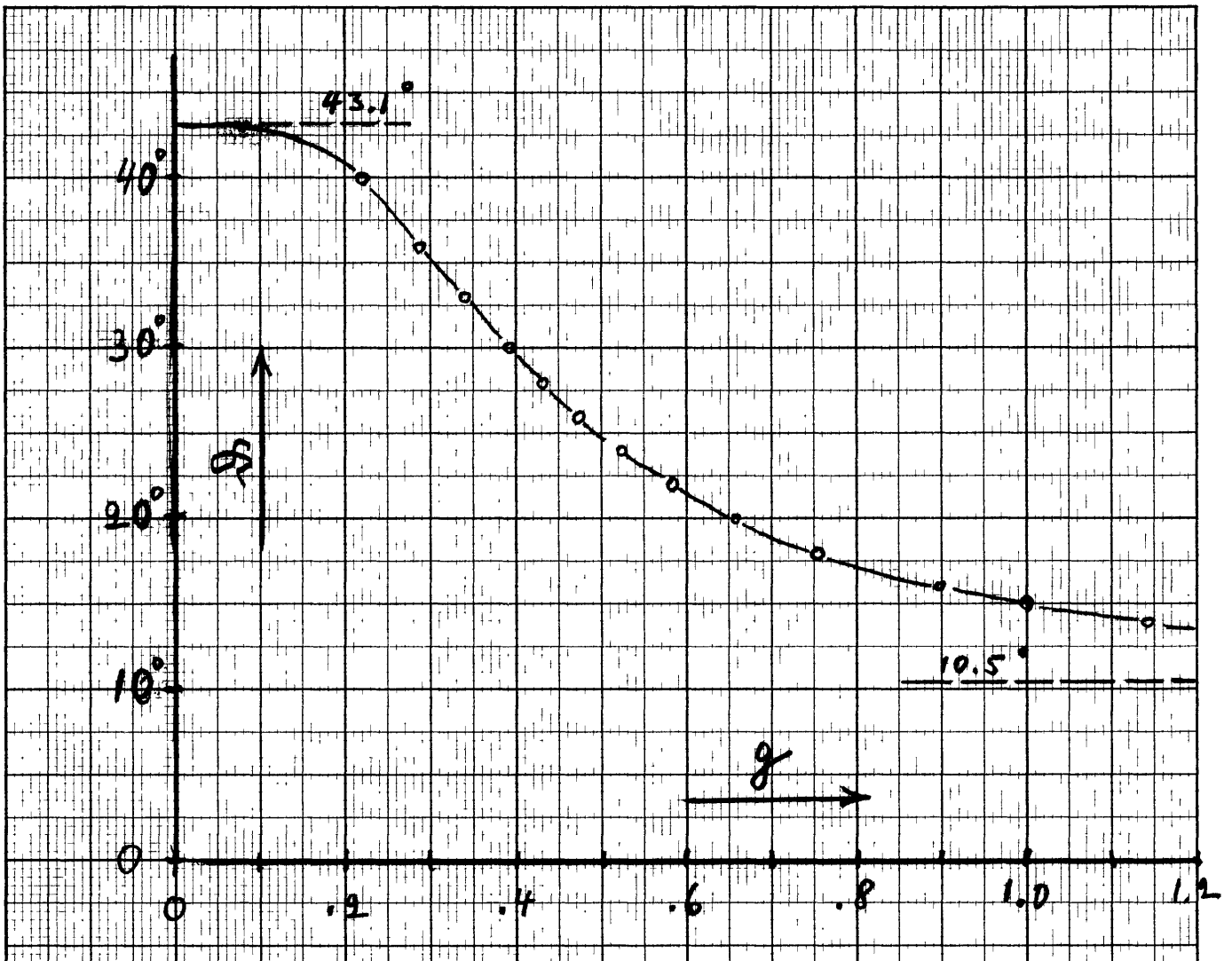


Fig. 2. Adjustment angle δ as a function of $g = \Delta H_2 / \Delta H_1$, if one demands the same efficiency for zenith angles $\gamma = -30^\circ$ and $\gamma = +60^\circ$.