NATIONAL RADIO ASTRONOMY OBSERVATORY
POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE ARBOVALE 456-2011

REPORT NO. 5
CONTRACT NO.
PAGE 1 OF
DATE Dec. 28, 1965

PROJECT: LFSP
SUBJECT: Summaries

LFSP Summary of S. von Hoerner

The following serves as a summary to four papers:

1. The Design of Large Steerable Antennas — June 20, 1965
2. Approach to a Structure with Homology Deformation — Aug. 20, 1965
3. Telescope Model for Homology Deformation — Nov. 5, 1965
4. Calculating Method for Homology Solutions — Nov. 17, 1965

These papers are written as contributions to the LFSP group; but most of the results are of a general nature, applicable to antennas of any size.

I. General Survey

There is a growing need for building very large antennas for 10 or 20 cm wavelength, and another need for observing at very short wavelengths with antennas of moderate size. Since both these demands soon run into structural as well as financial limitations, a general survey was undertaken in order to find the basic principles involved and their most economical solutions. This survey deals with the design of tiltable, round reflectors of conventional type.

a) Three Natural Limits

Elementary considerations show that the largest diameter of a conventional antenna must be limited in three ways. First, a maximum height is reached when the own weight of the structure gives a pressure at its bottom which approaches the maximum allowed stress of the material used; this height is proportional to S/ρ, where ρ is the density and S is the maximum stress; we call this the stress limit. A second limit applies to any structure which, while being tilted, shall maintain a given accuracy (1/16 of the wavelength), since the height h of a structure is compressed by the own weight by an amount proportional to $h^2\rho/E$, where E is the modulus of elasticity; we call this the gravitational limit. A third limit holds for any structure which shall maintain a given accuracy while part of it is exposed to sunshine, since a temperatur difference ΔT changes the length ℓ of a member by an amount proportional to $\ell\Delta T$. All three limits are "natural limits", as opposed to technical or financial limits. The second and third limit depend on the shortest wavelength to be used, while the first one does not.

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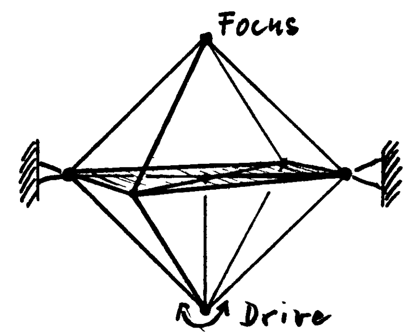
Numerical values for these limits depend on the material used and on the shape of the structure. For all possible materials, the maximum height is much larger than actually needed. The gravitational deformations are the same for normal steel, high stress steel and aluminum (three times larger for concrete). The thermal expansion is for aluminum twice as large as for steel. We thus arrive at normal steel as the best material.

As to the shape of the structure, an almost regular octahedron as its basic part will give the smallest gravitational deformations; the feed supports now are part of the main structure which then becomes as "round" as possible. The lower part of the octahedron will be filled with a frame-work supporting the surface. The surface diameter should be only 1.25 the octahedron diameter for avoiding long cantilevering parts.



Octahedron as the basic structure

An octahedron*) of 100 m diameter will deform under its own weight by 1.7 cm at most. The rms deformation of the surface will be about 0.5 cm. These values hold as long as any additional weight (bracings, surface) can be neglected as compared to the weight of the main chords of the octahedron. Otherwise the deformations are multiplied by a factor K, with

$$K = \frac{\text{total weight}}{\text{weight of main chords}} .$$

The limit, K=1, can be approached with infinite weight only. For an economical design we find about K=1.5 for 100 m diameter, increasing slightly with the diameter. For the type of tiltable antenna as descibed, with antenna diameter D and shortest wavelength λ (16 times the rms deformation), we obtain for steel and with K=1.5 the values:

1. Stress Limit, defined by maximum allowed stress of material: $D \leq 600 \text{ m}$

2. Gravitational Limit, defined by deformations under own weight: $\frac{D}{100 \text{ m}} \leq \left\{ \frac{\lambda}{8 \text{ cm}} \right\}^{1/2}$

3. Thermal Limit, defined by temperature differences of 5°C within the structure: $\frac{D}{100 \text{ m}} \leq \frac{\lambda}{2.4 \text{ cm}}$

*) with diagonals

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At present, the first limit plays no role. Antennas larger than 40 m are gravitationally, smaller ones thermally limited. But the thermal limit is flexible and drops out during nights, cloudy days and within a dome. The gravitational limit is fixed and thus more important; details of the structure do not matter much (as long as they do not make it worse) and even a floating sphere deforms by the same amount as a hanging octahedron. Figure 1 shows the three limits. The 36-foot NRAO telescope passes the thermal limit because it stands in a dome. Some of the existing or designed telescopes come very close to the gravitational limit, but not a single tiltable antenna passes it.

b) The Weight of an Antenna

For investigations of economy and financial limits, formulae are derived for the weight of an antenna as a function of diameter D and shortest wavelength λ. The derivations assume a "near-to-ideal" design of the type described, and the results should be valid within $\pm$ 30%. It is shown that the weight can be defined by four items:

1. Minimum Structure; for any structure, no matter what its purpose, there is a minimum weight for stable self-support, which is proportional to D^2.
2. Survival Conditions (highest winds, snow, ice) define the weight for certain D,λ combinations; the weight then is proportional to D^3/λ, plus the minimum weight.
3. Wind Deformations during observations must be kept below $\lambda/16$. The weight then goes with D^4/λ for closed surface, and with D^4/λ^2 for wire mesh.
4. Gravitational Deformations are decreased by using heavier main chords. The gravitational limit, $\lambda_g = 5.2$ cm $(D/100 \text{ m})^2$, is approached with weight $D^2\lambda/(\lambda-\lambda_g)$.

Figure 2 shows which items dominate in which fields of the D,λ plane. And Figure 3 gives the weight as a function of D and λ. The comparison with some actual telescopes shows that our estimates, although derived on general grounds only, may give some confidence:

Antenna	D	λ	Weight from Fig. 3	Actual Weight
300-ft, Green Bank	92 m	15 cm	300 tons	450 tons
120-ft, Haystack	37	1	80	50
Bonn, design 1	90	7	400	480
Bonn, design 2	80	7	305	350
210-ft, Parkes	64	6	200	270

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c) Economy

The question of getting a good telescope for the lowest possible price is investigated under several aspects. The main results are as follows:

1. There is a most economical wavelength for an antenna of given diameter, any other wavelength leading to a waste of either strength or rigidity.
2. No radomes should be used for antennas above 50 m diameter; but also below 50 m the advantages of radomes are doubtful, at least for radio astronomy.
3. All longer members must be split (like towers) into 3 or 4 chords connected by bracings; very long members even need multiple splitting. Extensive use of splitting reduces the total weight considerably, but the weight of bracings increases the gravitational deformations, and a very careful compromise is needed.
4. The best basic structure seems to be an octahedron which includes the feed supports and yields a "round" shape.
5. The antenna surface should be only 1.25 the octahedron diameter. It should consist of wire mesh for $\lambda \geq 5$ cm (elastic mesh, for preventing permanent deformations).
6. Most economical seems an alt-azimuth mount, holding two corners of the octahedron by bearings at the top of two towers. The towers have three legs each, wide astride, two legs each going on wheels on a circular track, the third leg of each tower going on a strong pintle bearing at the center of the circle, which takes all lateral forces. Uplifting forces are held down by counterweights, leaving only downward forces on the tracks. Larger telescopes need special anchorings in a stow position.
7. As to foundations and tracks, it is shown that standard railroad equipment, with normal roadbed, ties and rails, is fully sufficient for $\lambda \geq 8$ cm. The tower legs go on normal steel gondolas, filled with rock and gravel for counterweight. In this way, a lot of money can be saved in the foundations.
8. A cost estimate is made for a special case. An economical antenna of 150 m diameter (500 feet) and $\lambda = 20$ cm should cost about 4 million dollars, including foundations and drives, provided that the design really uses optimization for every detail, and that economy in fact is the leading principle.

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II. A Transit Telescope, Flat on the Ground

a) Passing the Gravitational Limit

As we see from Figure 1, one of the most urgent questions is "How can we pass the gravitational limit?" As it seems, this can only be done in three ways:

1. Avoiding the deformations by not moving in elevation angle;
2. Fighting the deformations with strong servo motors in the structure;
3. Guiding the deformations so they do not hurt the performance.

The first way leads to a limited steerability, but this is acceptable for many types of observation. The second way was tried in Sugar Grove; it will always be very complicated and expensive and is omitted in the present investigation. The third way leads to the concept of homologous deformations and will be discussed in the last section.

An extreme example of the first way is the Arecibo antenna which does not move at all, but moving the feed moves the antenna beam by 20° from the zenith in any direction. This is a very economical design which could be extended at least to 10 cm wavelength. A radio source can be followed during two hours (integration time), and the complete sky could be covered if four similar antennas were located at different geographical latitudes; this possibility deserves more attention than it has received up to now. A second and more flexible example was suggested by J.Findlay and is being worked on by E. Faelten (see their LFSP reports). Here, a spherical dish is mounted with a fixed elevation angle and is driven on circular tracks by 360° in azimuth, while moving the feed allows 1 or 2 hours integration time. This telescope gives transit observations of a considerable part of the sky.

The price of this last type of telescope is defined by wind deformations. Suppose we pass the gravitational limit in some way or the other. The next natural limit then is the thermal one, but it holds only in sunshine. In Figure 4 we show the weight of a telescope if gravitational deflections are omitted; these weights are valid for telescopes where the height is comparable to the diameter (as is the case in the Faelten design). This weight increases very steeply for shorter wavelengths in the wind deflection region, especially steep for larger telescopes. We might try a radome, place the antenna in a shielded valley, limit short-wavelength observations to lower wind velocities, or, finally, design a telescope which sits flat on the ground.

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b) A Large Parabolic Mirror Flat on the Ground

A possible solution is sketched in Figure 5, using a parabola at 45° elevation angle with its focus in F. In a more conventional design, we would use part AB, where a large surface is high above ground. Now, we use part CD; it is 40% larger, but is never more than 40 m above ground, and at no place more than 26 m in stow position. This parabolic mirror P (282 m long and 200 m wide) gives a round apperture of 200 m diameter and is mounted on wheels in a flat, cylindrical trough GHI (343 by 200 m) which has its center line through point M. Moving the mirror in the trough by 10° around M yields about one hour of integration time. The trough sits on wheels on horizontal, circular tracks on the ground, giving 360° movement in azimuth around center point Z. The feed is mounted gliding along track T, which is 50 m long and is 10° of a circle around M. This track can be rotated by 360° around a vertical axis which is mounted on a tower 200 m high.

The feed cannot be at the primary focus. In order to illuminate the antenna beam symmetrically, the feed would need an asymmetric pattern; this could be done, but then the feed could not be rotated for polarization measurments. This problem is resolved by using a small secondary reflector of Gregorian type; feed and Gregorian are moved together in one package along track T. If the secondary mirror has the right tilt, it just counteracts the asymmetries introduced by the primary mirror, and the feed illumination becomes symmetrical again. But for isotropic feed illumination, the apperture illumination becomes somewhat tapered.

Recent calculations have shown that there is a one-parameter family of symmetrical solutions for the secondary mirror. If we decide to have a feed illumination angle of 100° (best for simultaneous multy-frequency observations), we get a taper of only 0.8 db (17 %), and the Gregorian ellipse has an eccentricity of 0.46.

I think that this type of design will be the most economical one for telescopes above a certain critical diameter, if a large sky coverage is wanted with a single transit telescope. But whether this critical diameter is below or above 200 m cannot be decided without an actual design and cost estimate. The shortest wavelength λ is not defined by natural limits but will depend on the money available. A rough estimate showed that λ=5 cm could be obtained without serious complications.

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III. Homology Deformation

The transit telescope is satisfactory if a large number of objects is to be observed the same way. But full steerability is wanted or needed for all observations of a few, special objects, for lunar occultations, and for line studies. We thus try to pass the gravitational limit with full steerability. If the concept developed here is of practical use, then a new generation of fully steerable large antennas becomes possible.

a) General Concept

Gravitation lets a structure deform into a state of minimum energy, it must move down in the average; and the material constants ρ and E tell us the amount it must move. But no law of nature tells us that a paraboloid must deform into something different from a paraboloid. We thus look for a structure which deforms down whatever it must, but still gives some exact paraboloid of revolution for any angle of tilt, at least for a number N of surface points. A deformation of this kind, deforming one surface of given type into another surface of same type, we call "homology deformation".

It can be shown that mathematical solutions exist, there is a family of solutions with 7N free parameters. But all cross sections must be positive and finite for a physical solution, and a useful solution should have a convenient shape, not too much weight, and so on. Mathematical solutions exist; but are there useful solutions? And how to get them?

b) The Homologous Cell

We cannot play at random with 7N free parameters until hitting something useful; we need some logical principle to guide us, and a possible way is the following. We divide the space between bearings and surface into layers of decreasing thickness with an increasing number of joints, each layer being divided into cells by the joints, and all cells being topologically identical (same basic structure, but different sizes and cross sections). Figure 6 shows an example with three layers.

The basic idea is to let the single cell fulfill a set of conditions such that the whole structure deforms homologously. Solving the problem for one cell will give a solution for the whole telescope, and the number of free parameters is reduced to the few ones of a single cell. Even if this principle would not yield exact solutions for the whole telescope, it still should yield good approximations.

For two dimensions, two types of cells were investigated analytically, and both gave exact solutions. But a structure composed of these cells would be exactly homologous only if it were indefinitely long; otherwise we get "boundary distortions" consisting of pressure and torsion. The pressure term is removed by making the second type of cell pressure-stable (Fig.6c). This cell is a very useful solution, it needs only 37% aditional weight for fulfilling the homology conditions. Three problems then are left: boundary torsion, extension to three dimensions, and constructing a telescope from cells.

c) Model of a Homologous Telescope

An attempt to solve these three problems led to the structure of Figure 7. It starts at two bearings with a suspension, which holds the octahedron and gives 5 homologous points in the basic subsurface. Using two types of cells alternately, we reach 21 surface points with a single layer. With 21 homologous points, the gravitational limit can be passed at least by a factor 10 in wavelength (or a factor 3 in diameter). The boundary distortions are removed, first, by using pressure-stable cells. Second, boundary torsions can only come from an upper layer, deforming the subsurface under it into an S-type shape; but our basic subsurface has not enough points for an S-shape, and the upper surface of the only layer cannot be deformed because there is no layer above it. The result is a uniform tilt, which again is a homologous deformation.

Dr. Jennings has started on the quite involved calculations. When he has numerical results, a model about 10 m large should be built, fairly crude, even without any surface. Its deformations during rotation in elevation angle may be measured with an optical interferometer. The model should show whether a numerically obtained homology solution can be realized in practice. If the model gives good results, it can be applied to telescopes of any size (which might be most important for millimeter wavelengths). If we have a given homology solution and multiply each length with one and the same but arbitrary factor, the result again is a homology solution. And the same holds, independently, for multiplying the cross sections. The factor for the cross sections is obtained from two conditions: first, that the wind deformations are never more than $\lambda/16$; second, that the structure can withstand the survival load. The first condition will dominate, if we pass the gravitational limit by a large factor.

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d) Calculating Method for Complex Structures

As soon as a good solution is found for the single cell, one designs a telescope consisting of these cells as in Figure 7. Most probably, this structure will not be an exact solution but only a first approximation, to be changed by some steps of an iterative method until the approach to a true solution is close enough. The mathematical procedure for these iterations is worked out in detail, and in such a way that it can be applied to any type of complex structure.

The conditions of homology lead to a system of algebraic equations of very high order. The method suggested regards the geometrical shape as being given, and it is linearized in the changes wanted for the cross sections of members. It uses the means and notations of linear algebra, for which good subroutines are available at large computers.

The task of changing a first approximation into a homology solution is not defined because of the free parameters. In order to make it uniquely defined, the missing number of equations is derived from the following demand:

> From all possible homology solutions, choose the one which is most similar to the first approximation.

This definition gives the additional advantage of making all changes as small as possible, which makes our hope as large as possible that the linearized method will converge, and that the result again will be a useful solution if the first approximation is. The method also contains several checks, giving failure indications if no solution exists for the geometrical shape used.

If all goes well, this method will yield a useful solution, but not necessarily the best one. As the best solution I have defined the one which meets the following conditions with a minimum weight:

1. Exact homology deformation of all surface points;
2. All cross sections positive;
3. All deformations are elastic for winds up to 100 mph;
4. All surface deformations due to wind of 25 mph smaller than $\lambda/16$.

A minimum task cannot be solved with a linearized method. A two-gradient procedure is suggested with which I had good experience in other, similarly complicated cases.

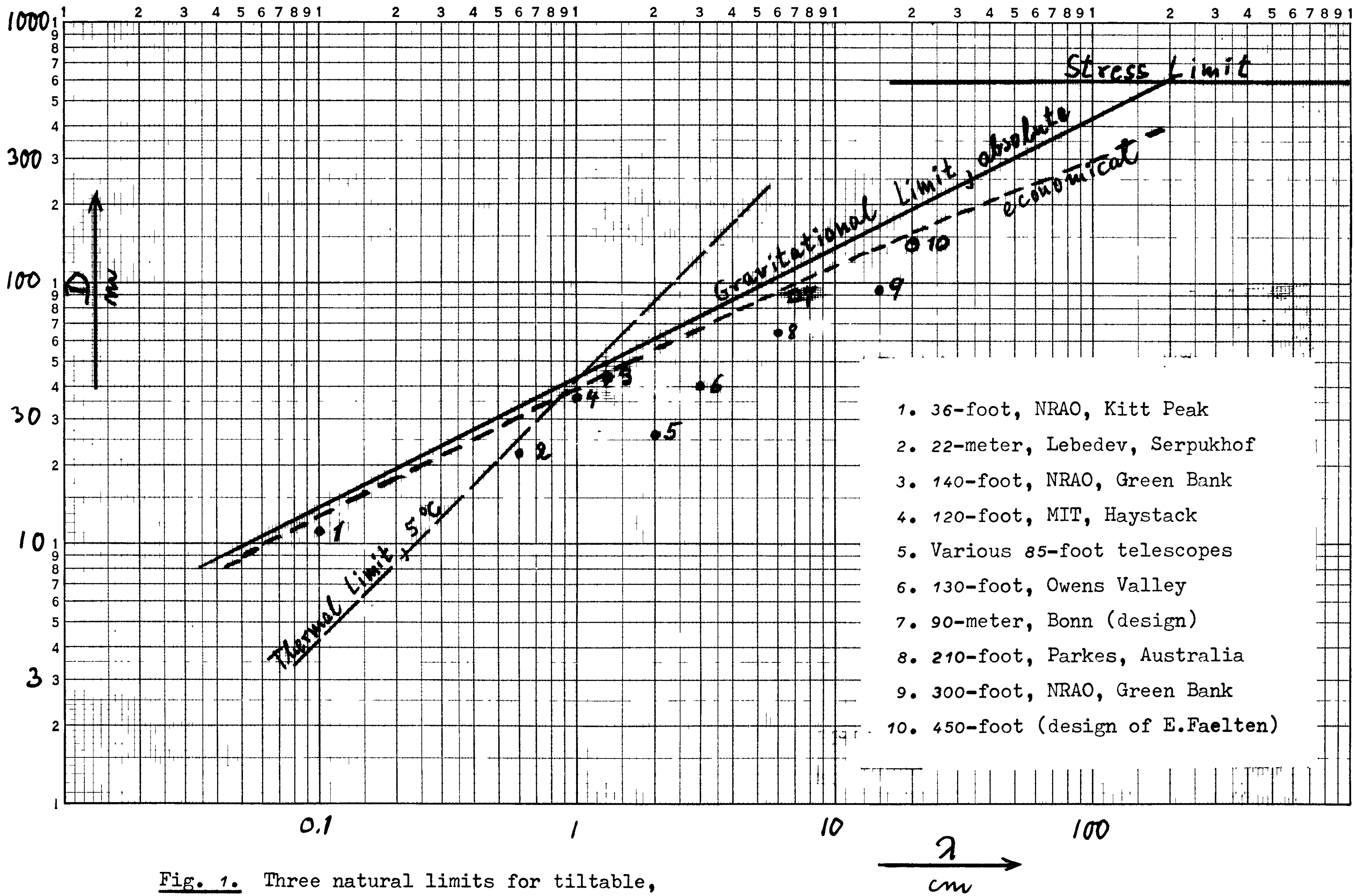


Fig. 1. Three natural limits for tiltable, conventional telescopes.

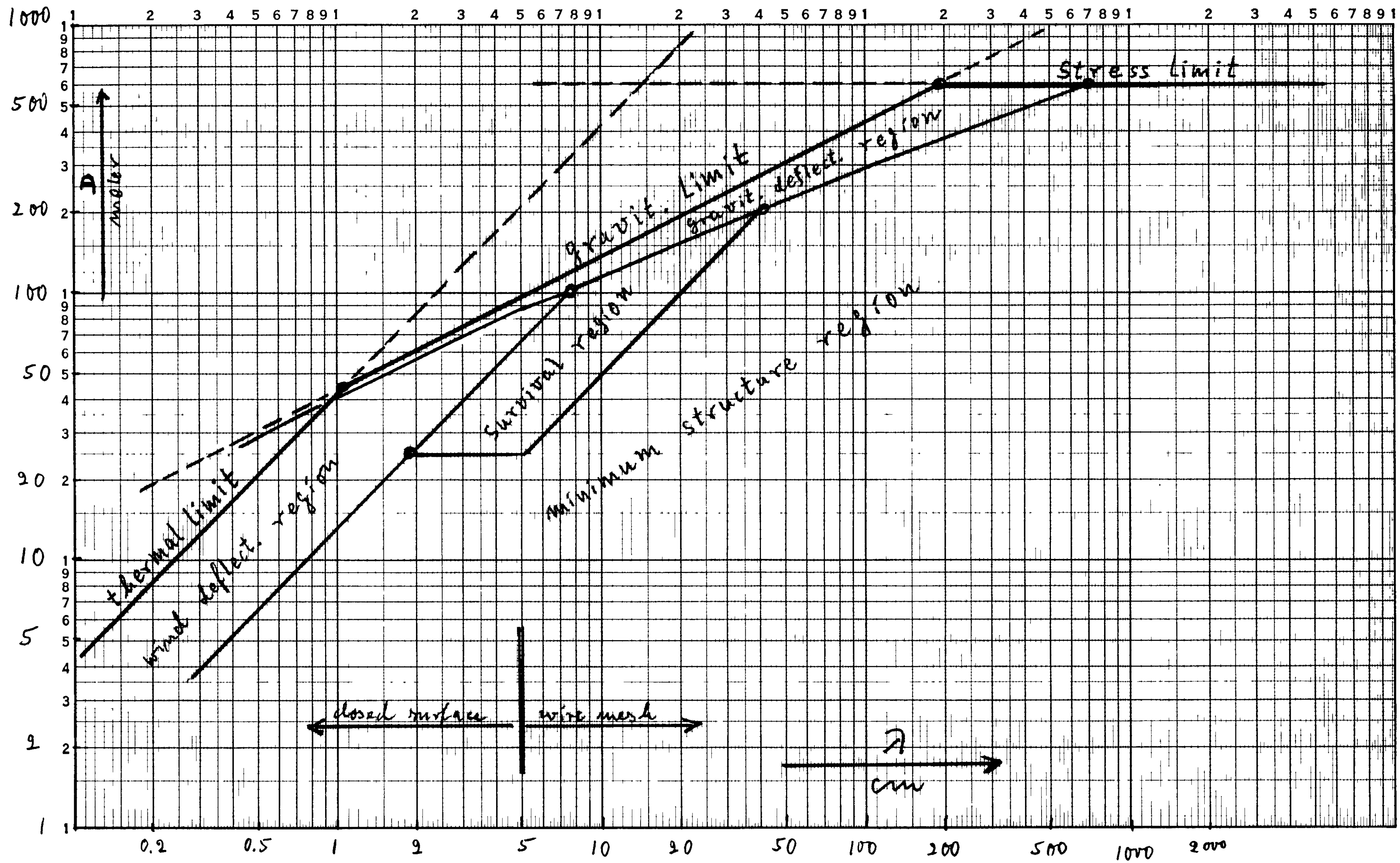


Figure 2. Regions of diameter D and wavelength λ, in which the weight of the structure is defined by different conditions, and the three natural limits of antennas.

Figure 3. Weight of dish structure (surface, framework, octahedron) as function of diameter D and wavelength λ. -------- limits, —·—·—·— boundaries for regions of Figure 2. Economical antennas should be near to the uppermost boundary.

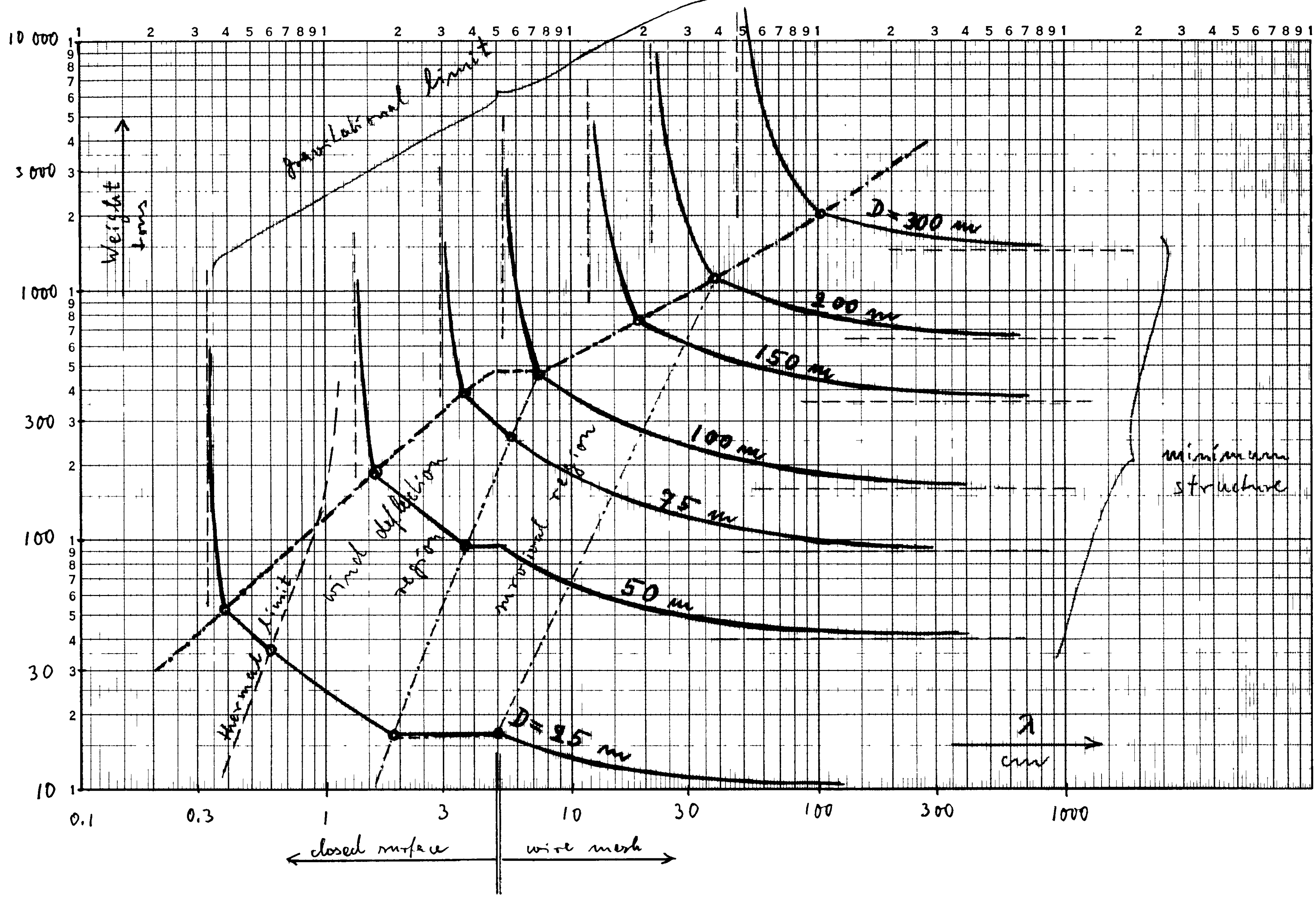
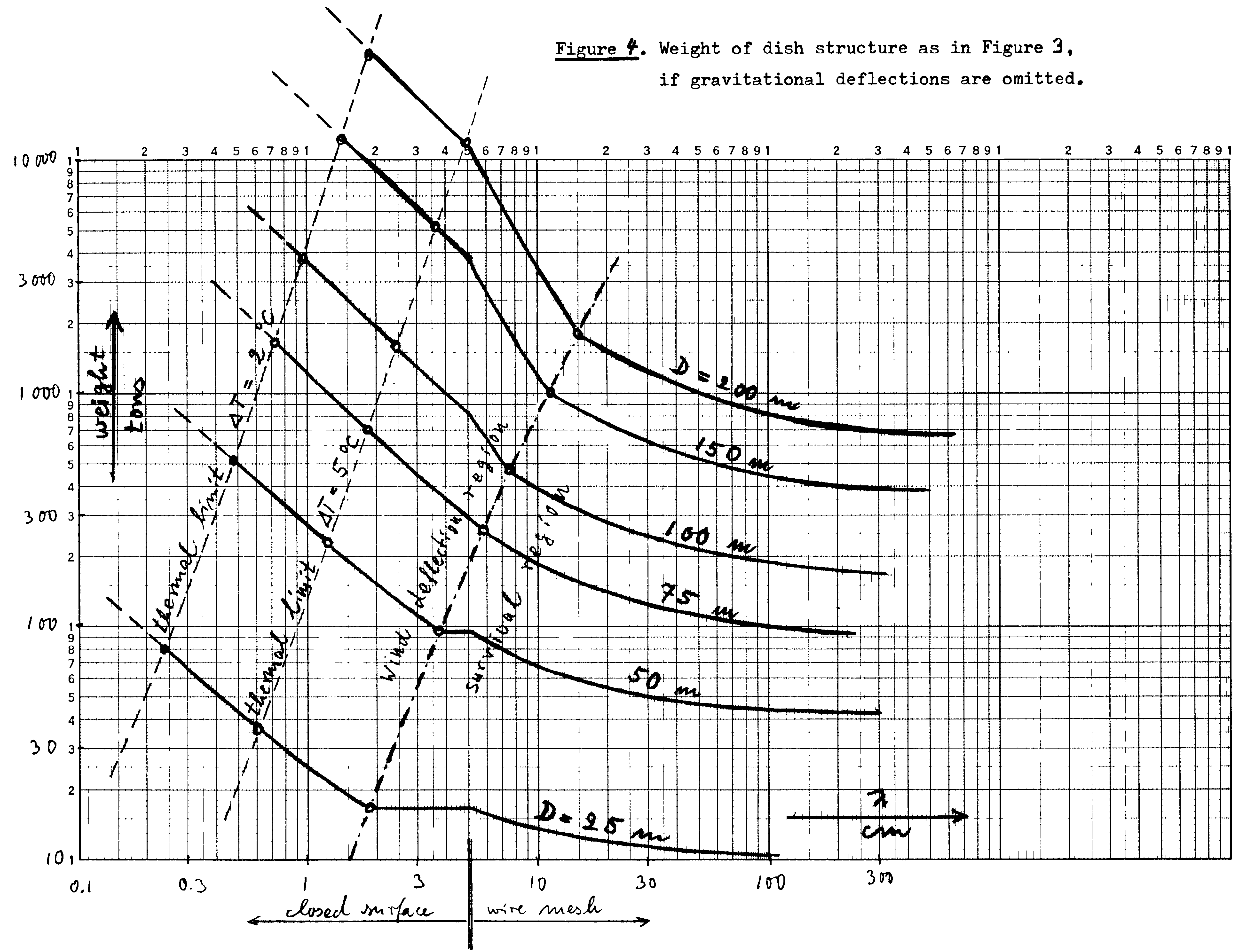


Figure 4. Weight of dish structure as in Figure 3, if gravitational deflections are omitted.



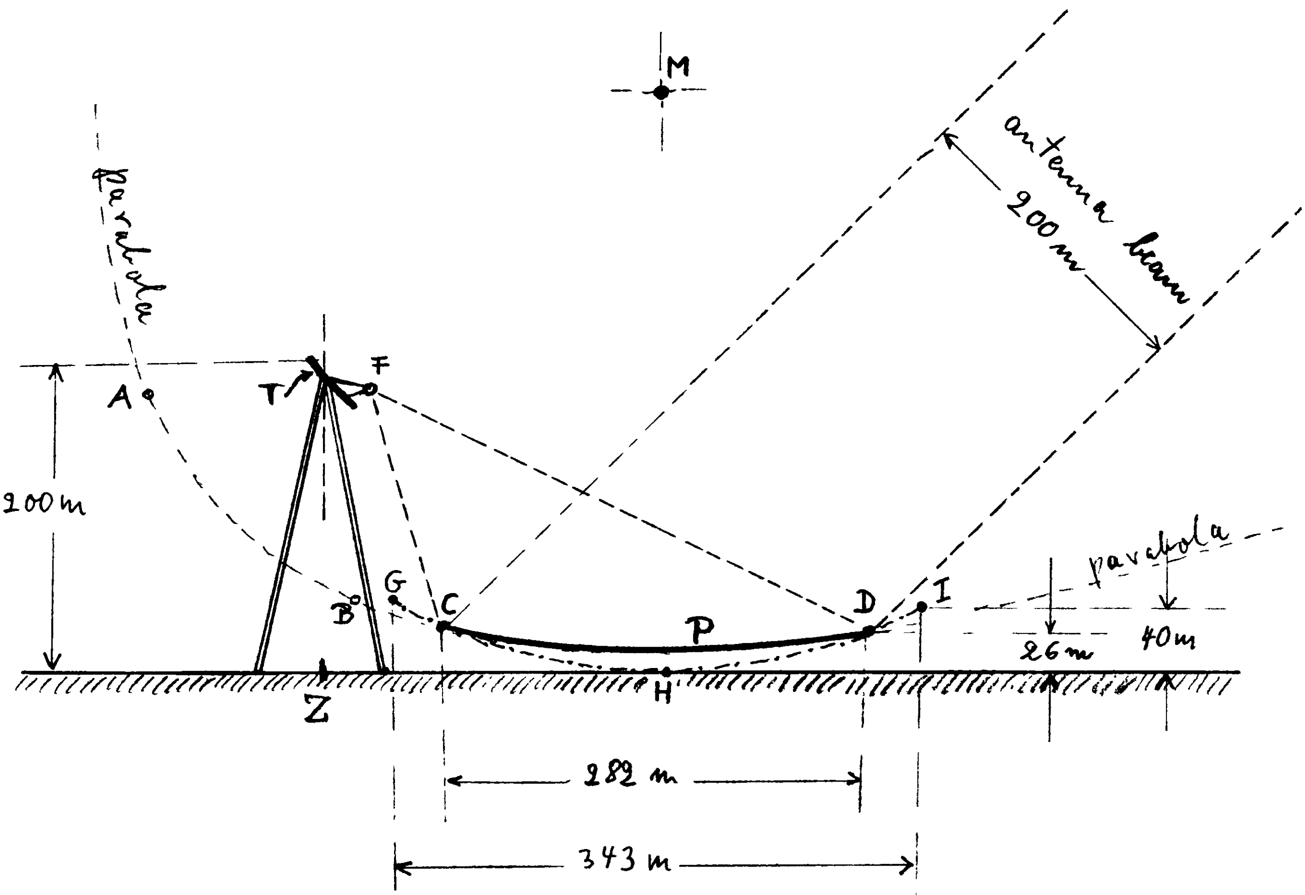
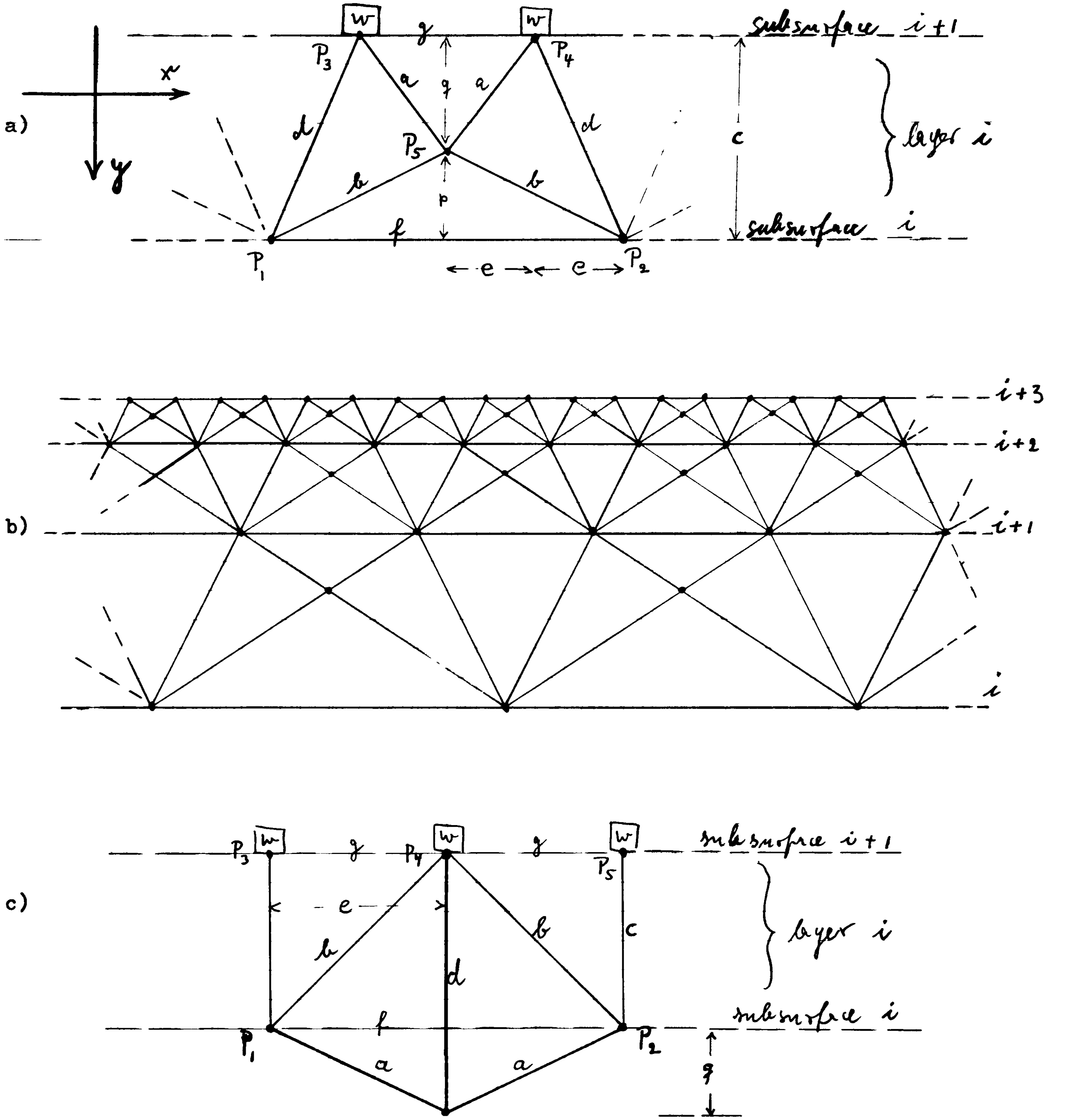


Figure 5. Fixed-elevation transit telescope with large surface flat on the ground. The parabolic mirror CPD moves on wheels in a flat cylindrical trough GHI by 10° around M. The trough moves on horizontal circular tracks by 360° around Z. In stow position, the rim of the mirror is only 26 m above ground.



<u>Figure 6</u>. Two types of cells, where layer i+1 is parallel to layer i for any elevation.

a) Most simple type, general solution.

b) Particular solution (c = 2e = 3q) of same type, three layers.

c) Cell which keeps height constant if P_1P_2 is compressed.

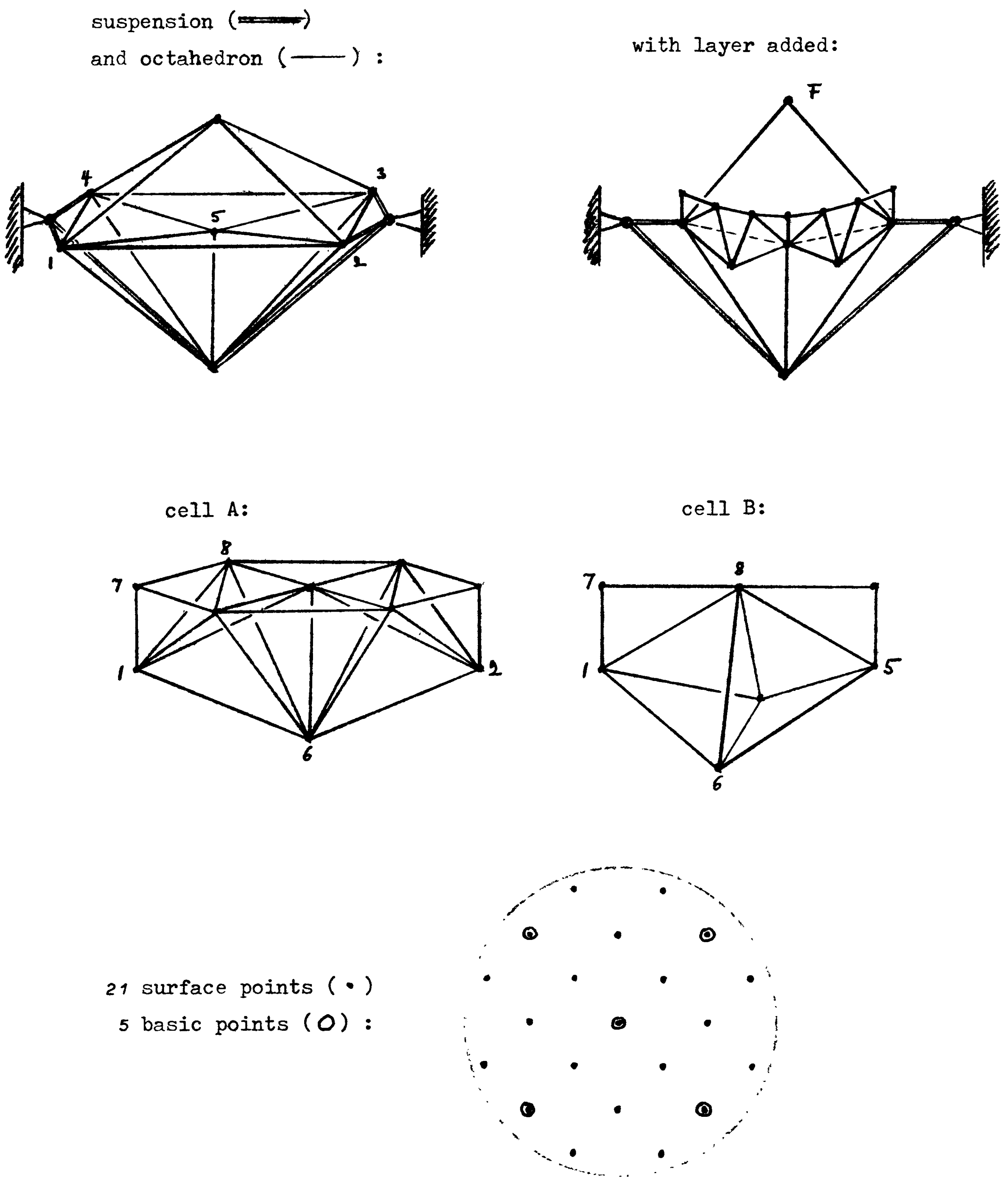


Fig. 7. Antenna structure for homology deformations;
with suspension, octahedron, one layer, and 21 surface points.
(Identical numbers mean identical points in the structure)