

# NATIONAL RADIO ASTRONOMY OBSERVATORY

POST OFFICE BOX 2  
GREEN BANK, WEST VIRGINIA 24944  
TELEPHONE ARBOVALE 456-2011

REPORT NO. 6  
CONTRACT NO. \_\_\_\_\_  
PAGE 1 OF 2  
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PROJECT: LFST

SUBJECT: Homology Method

## Approximation to the Inverse of a Matrix =====

S. von Hoerner

In my Repr<sup>t</sup> No. 4 (Calculating Method ...), I used an approximation method for the inverse of a slightly changed matrix, when the inverse of the unchanged matrix is given. Call  $A$  the matrix and  $A^{-1}$  its (known) inverse; add a matrix  $a$  to it which has elements much smaller than those of  $A$ , and call

$$B = A + a. \quad (1)$$

The approximation used in Report 4, then, is

$$B^{-1} \approx A^{-1} - A^{-1}a A^{-1}. \quad (2)$$

R. Jennings suggested to investigate this approximation somewhat closer. Thus, in this present report I show that equation (2) is the first term of an infinite series; I give a condition for the convergence of this series, and another condition for the "usefulness" of equation (2).

Statement: (with  $E$  = unit matrix)

$$B^{-1} = A^{-1} \left\{ E + \sum_{v=1}^{\infty} (-aA^{-1})^v \right\} \quad \text{if the series converges.} \quad (3)$$

Proof:

Multiply (3) from right side with  $A+a$

$$E = E + A^{-1}a + A^{-1} \sum_{v=1}^{\infty} (-aA^{-1})^v A + A^{-1} \sum_{v=1}^{\infty} (-aA^{-1})^v a \quad (4)$$

The term  $A^{-1}a$  cancels the first term of the first series, which then starts with  $v=2$ . We then rewrite this series, using  $( )^v = ( )^{v-1} ( )$ , and  $A^{-1}A=E$ , and obtain

$$E = E - A^{-1} \sum_{v=1}^{\infty} (-aA^{-1})^v + A^{-1} \sum_{v=1}^{\infty} (-aA^{-1})^v. \quad (5)$$

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Both series in (5) cancel each other, and we are left with an identity,  $E=E$ , which proves that statement (3) is correct.

## Convergence:

I have not found a necessary condition for the convergence of (3). But if we define a matrix  $\alpha$  by

$$\alpha = aA^{-1} \quad (6)$$

and call its largest element

$$\alpha_m = \max | \alpha_{ij} | \quad \text{for all } \begin{cases} i = 1 \dots n \\ j = 1 \dots n \end{cases} \quad (7)$$

then the following condition obviously is sufficient for convergence:

$$\alpha_m < 1/n . \quad (8)$$

## The Error of (2):

Equation (2) which is used in Report 4 contains only the first term of the series of equation (3). We call this approximation

$$B_0^{-1} = A^{-1} - A^{-1}a A^{-1}, \quad (9)$$

and its error

$$\Delta B^{-1} = B^{-1} - B_0^{-1}. \quad (10)$$

As the relative error,  $\mathcal{E}$ , we will define  $\Delta B^{-1}$  relative to  $B^{-1}$ , which means  $\Delta B^{-1}$  multiplied with  $B$ :

$$\mathcal{E} = B \Delta B^{-1} = B (B^{-1} - A^{-1} + A^{-1}a A^{-1}). \quad (11)$$

A different way of defining the same quantity  $\mathcal{E}$  is, as can be shown,

$$\mathcal{E} = E - B B_0^{-1}. \quad (12)$$

From (11) we obtain

$$\mathcal{E} = (aA^{-1})^2 = \alpha^2 \quad (13)$$

In order to make (2) a good approximation, it is sufficient to demand

$$\alpha_m \ll 1/n . \quad (14)$$