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PROJECT: LFST

SUBJECT: Elasticity of long ropes

The effective elasticity, E_s, of a long rope, sagging under its own weight

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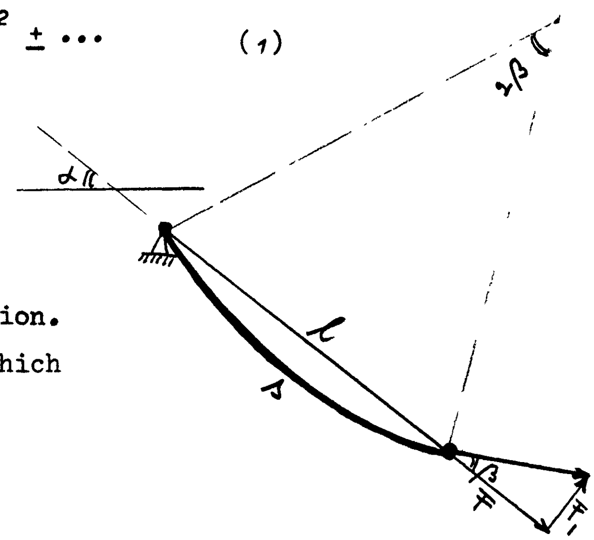
Some structures may use long ropes, for example a guyed tower does. The stiffness of such a structure depends on the modulus of elasticity, E, of the material used, but it also depends on the "sag" of the ropes which decreases the stiffness. We still can treat a long rope in the same way as any solid member, if we define for the rope an "effective modulus of elasticity", $E_s \leq E$, where E is given as the elasticity of the material used, but E_s is a function of the sag. Since I could not find a formula of this type in a few textbooks, and since this question might be important for very large structures, I give the following derivation.

1. We approximate the Catenary of a hanging rope by a circle (assuming a tight rope). Then

$$\left.\begin{aligned} s &= 2r\beta \\ \ell &= 2r \sin\beta \end{aligned}\right\} \quad \frac{s}{\ell} = 1 + \frac{1}{6}\beta^2 \pm \dots \qquad (1)$$



2. In equilibrium, we have

$$F_1 = \frac{1}{2}\rho\ell Q \cos\alpha \qquad (2)$$

with ρ = density of material and Q = cross section. We call ℓ_0 the undeformed length of the rope, which means that

$$\frac{s}{\ell_0} = 1 + \frac{F}{QE} \qquad (3)$$

and we make use of

$$\beta \approx \tan\beta = F_1/F . \qquad (4)$$

With (20, (3) and (4), equation (1) can be written as

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$$\frac{F^3}{QE} = \left(\frac{\ell}{\ell_o} - 1\right) F^2 + \frac{1}{24}\frac{\ell}{\ell_o}(\rho \ell Q \cos\alpha)^2 . \tag{5}$$

3. The effective elasticity we now define as

$$E_s = \frac{\ell}{Q}\frac{dF}{d\ell} . \tag{6}$$

From (5) we find $dF/d\ell$, and after neglecting all terms of higher order we obtain

$$E_s = E\left(1 - \frac{1}{12}\frac{E\rho^2 \ell^2 Q^3}{F^3}\cos^2\alpha\right). \tag{7}$$

4. In order to show more clearly what equation (7) means, we define two quantities:

critical length (material constant) $\ell_c = \dfrac{S^{3/2}}{\rho E^{1/2}}$ (8)

safety factor (free choice) $q = \dfrac{QS}{F} \geq 1$ (9)

where S = maximum allowed stress of material. With these definitions, equation (7) reads finally

$$\underline{\underline{E_s = E\left\{1 - \frac{1}{12} q^3 (\ell/\ell_c)^2 \cos^2\alpha\right\}.}} \tag{10}$$

5. Taking, for example, high-strength Bethlehem ropes, we have $E = 23 \times 10^6$ psi and $S = \text{Yield}/1.85 = 81 \times 10^3$ psi, which gives

$$\ell_c = 430 \text{ m} = 1410 \text{ ft} . \tag{11}$$

6. As an example, we assume $\alpha = 45^\circ$, and we allow $q = 2$ (for taking up wind forces). Equation (10) then becomes

$$E_s = E\left\{1 - (\ell/744\text{m})^2\right\}. \tag{12}$$

If we apply this to the guyed tower in Fig.5 of my Flat-Antenna Report (No.7), we find that the modulus of elasticity goes down by only 4%.

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7. The critical length, ℓ_c, depends strongly on the maximum allowed stress, S, of the material used. It gets very short for normal steel, which means that high-stress steel should be used for long, solid rods, like tensioned diagonals or long guy rods. A few examples are given in the table below.

material	Yield 10^3 psi	E 10^6 psi	ℓ_c meter	ℓ_c feet
prestreched wire rope	150	23	430	1410
solid steel rod	100	30	268	880
	50	30	94	310
	30	30	44	140

The figure below shows the effective elasticity as a function of length, according to formula (10), for which we adopted the values

$\alpha = 45^\circ$ elevation angle above horizontal

$q = 1.5$ safety factor (stress = S/1.5 = Yield/2.77)