NATIONAL RADIO ASTRONOMY OBSERVATORY
POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE ARBOVALE 456-2011

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PAGE 1 OF
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PROJECT: LFST

SUBJECT:

The Wind Area of Members and Space Frames

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The following was derived in order to get quick estimates for wind forces to be expected for space frames, before making the actual design. (Most of it will be nothing new, I guess, but still I would like to distribute these results in a form most easy for application.) In many cases one has to know the wind forces before sizing the members.

1. Single Member

If the unsupported length ℓ of a member is defined by the structure, and once a choice of ℓ/r is made, then r, the radius of gyration needed, is given. Questions: How can we make D, the diameter of the member, as small as possible for minimizing the wind forces? What value of D do we expect? In wich way should a long member be split up, and into how many chords?

First, I select a few shapes (Manual of Steel Construction) and derive the ratio r/D, for the smallest radius of gyration and the largest diameter:

	size inch		weight lb/ft	max.D inch	r_1 inch	r_2 inch	r/D
	14 x	16	426	25.0	7.26	4.34	0.174
	8	8	31	11.3	3.47	2.01	.178
	18.4	16.6	150	20.2	5.27	3.73	.185
	10.5	13.0	56	13.0	2.88	2.96	.221
	8.2	11.5	48	11.5	2.14	2,71	.186
	6.8	8.0	21.5	8.0	1.87	1.89	.234
	24	24	914	34.0	9.01	6.00	.176
	17	19	428	25.4	7.53	4.49	.177
	24	24.1	912	34.0	9.16	6.10	.179
	18	19.3	473	26.4	7.50	4.53	.172
	8	8	113.8	16.0	2.42	3.55	.151
	8	6	78.2	12.0	2.51	2.50	.208
	5	3.5	39.6	7.0	1.55	1.54	.220
						average =	.189

Table 1

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From these examples we see that the ratio r/D is best (largest) if the two radii of gyration are about equal, and that the best values are achieved with a T-shape. From the examples, we have

$$\frac{r}{D} = \left\{\begin{array}{ll} .151 & \text{worst} \\ \underline{.189} & \text{average} \\ .234 & \text{best} \end{array}\right\} \text{ for shapes.} \qquad (1)$$

Second, we consider pipes of outer diameter D and various wall thickness τ, and with help of formula (6) given later on we derive:

τ/D	r/D		
0	0.353	no wall	
1/40 1/20	.345 .336	thin wall	average .334
1/10	.320	medium wall	
1/5	.292	thick wall	
1/2	.250	solid rod	

Table 2

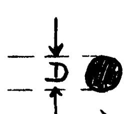
From these examples we have

$$\frac{r}{D} = \left\{\begin{array}{ll} .250 & \text{worst} \\ \underline{.334} & \text{average} \\ .345 & \text{best} \end{array}\right\} \text{ for pipes.} \qquad (2)$$

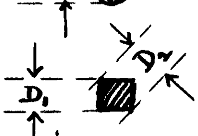
We see that even the worst pipe still is better than the best shape. For same radius of gyration, the average shape picks up more wind force than the average pipe by a factor:

$$\text{wind factor (shape/pipe)} = \frac{.334}{.189} = 1.77 . \qquad (3)$$

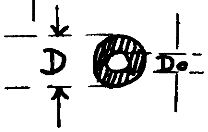
Third, we give some formulae for pipes and rods:



$$r/D = 1/4 = 0.250 \qquad (4)$$



$$\left\{\begin{array}{l} r/D_1 = 0.289 \\ r/D_2 = 0.204 \end{array}\right\} \text{average direction: } 0.233 \qquad (5)$$



$$r = \frac{1}{4}\sqrt{D^2 + D_o^2} \qquad (6)$$

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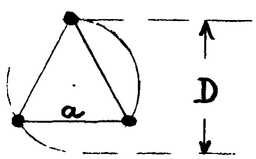
Our first two question can now be answered. Whenever wind forces matter, we should use <u>pipes</u>, because shapes would increase the wind forces by almost 80%. If we choose, for example, $\ell/r = 100$, then a single member of length ℓ will have

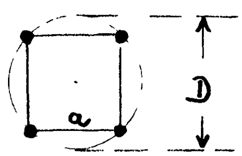
$$\left.\begin{array}{lll} \text{diameter} & D = \dfrac{\ell}{(\ell/r)(r/D)} & = 0.030\,\ell, \\ \text{wind area} & A = D\,\ell & = 0.030\,\ell^2. \end{array}\right\} \text{single pipe, } \ell/r = 100 \qquad (7)$$

2. <u>Long, Built-up Member</u>

As to the third question of how to split up a long member, we first illustrate two possibilities, with 3 and 4 chords placed on a circle of same diameter D:



$$r/D = \tfrac{1}{4}\sqrt{2} = .353; \qquad 3a = 7.35\ r$$



$$r/D = \tfrac{1}{4}\sqrt{2} \doteq .353; \qquad 4a = 8.00\ r \qquad (9)$$

This result is true in general: if we distribute $n \geq 3$ points equally spaced along a circle of diameter D, then the radius of gyration (about <u>any</u> axis) is always the <u>same</u> and is given by (9), independent of n.

But the wind force on the chords increases in proportion to n (at least if the diameter of the chords is defined by ℓ/r; and in prportion to $\sqrt{n}$ if defined by the area needed for strong lateral forces). Also, the amount of material needed for struts and diagonals increases slightly with n. This means:

Take 3 chords, in equilateral triangle. (10)

Choice (10) minimizes the wind force as well as the amount of material needed. It seems that the usual choice of $n = 4$ mainly has the advantage of easy connections with rectangular shapes, which cancels in case of pipes.

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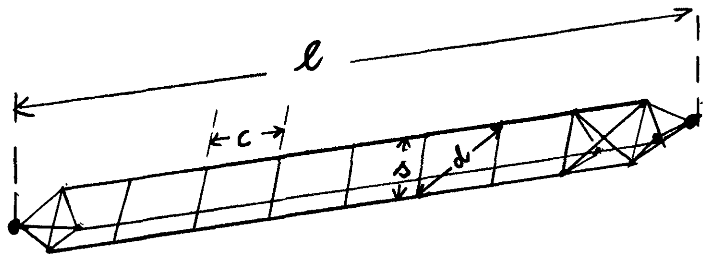

As example, we choose a long member of length ℓ, split into 3 chords. For long, built-up members, one should not take large ℓ/r ratios, and we choose $\ell/r = 30$; this then gives $s = (\sqrt{6}/30)\ell = .0817\ell$. We choose 10 segments, thus $c = \ell/10$. The length of a diagonal, then, is $d = .129\ell$.

For the chords, we take pipes, and choose $\ell/r = 100$, which gives them a diameter of $D_c = 0.03\ell/10$; their total length is $L_c = 3\ell$, and their wind area thus $A_c = 0.009\ell^2$. We have 27 struts, and their total wind area would be $0.0054\ell^2$ if we take again pipes and $\ell/r = 100$; but the wind force for the whole member is largest when the wind is perpendicular to its length, and in this case the struts reduce their wind area to 2/3 because of the projection, which yields $A_s = 0.0036\ell^2$. We have 54 diagonals which should be solid rods in tension, and we assume that their diameter is about 1/4 the diameter of a strut; including the projection, we arrive at $A_d = 0.0048\ell^2$. In summary, we obtain for the whole built-up member of length ℓ:

$$A = 0.018\ell^2. \qquad \left\{ \begin{array}{l} \text{long member, buit-up from pipes} \\ \ell/r = \begin{cases} 30 & \text{for whole member} \\ 100 & \text{for single pieces} \end{cases} \end{array} \right. \tag{11}$$

By comparing formula (11) with (8), we find:

$$\left.\begin{array}{l}\text{Reduction in wind force by} \\ \text{splitting a member, 3 chords}\end{array}\right\} \text{ factor } 0.60 \,. \tag{12}$$

Finally, we call $A_t = s\ell$ the total area of the member for wind direction perpendicular to it, and $V = (\sqrt{3}/4)s^2\ell$ its volumn. The wind area as given by (11) then can be written as

$$A = 0.220\, A_t \,, \tag{13}$$

or

$$A = 0.886\, V^{2/3}. \tag{14}$$

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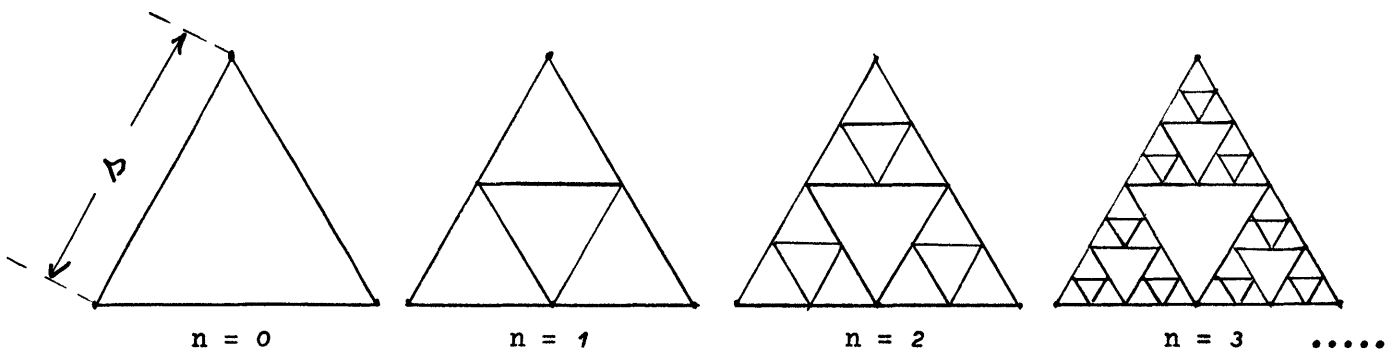
3. Space Frames

We now consider space frames which are not elongated, as a built-up member is, but which are more of a "round" shape. They can be treated in general, because fortunately the wind force does not depend on the number n of subsystems of secondary members, if they are arranged in an economical way, for example:



If the above drawing is extended into the third dimension, forming a tetrahedron, then the introduction of every additional subsystem ($n \rightarrow n+1$) doubles the total length of all members, but it halves all diameters, and the wind area thus stays constant.

We take again pipes with $\ell/r = 100$ and $r/D = .334$, and if diagonals are needed, we use rods with 1/4 the diameter of a compression member. We define, in accordance with (13) and (14), two numerical factors, q_a and q_v, as

$$A = q_a A_t = q_v V^{2/3} \qquad (15)$$

Omitting all details, and with s = length of one side, we arrive at the following values:

Form	A	q_a	q_v
Tetrahedron	$0.141\ s^2$	0.336	0.571
Octahedron	$.332\ s^2$	.321	.530
Cube	$.545\ s^2$	.372	.526
	average =	.34	.54

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In order to check these estimates, we use O. Heine's recent design of the support structure for the FTT. The actual wind area he calculates for this structure is $\underline{A = 4380\ ft^2}$. From his drawings we find a total area of $A_t = 16000\ ft^2$; we multiply by 0.34 from Table 3 and obtain an estimated wind area of $A = 5400\ ft^2$, which is 23% too high. Or, using the volumn of his structure, $V = 720\,000\ ft^3$, we get $V^{2/3} = 8020\ ft^2$; we multiply by 0.54 from Table 3 and obtain the estimated wind area as $A = 4300\ ft^2$, which is 2% low. We see that our primitive estimates are good enough and thus could be quite useful. This comparison is fair, because O.Heine actually used pipes, with $\ell/r = 75$ for orthogonal members and $\ell/r = 130$ for diagonal ones, which in the average comes close to the value $\ell/r = 100$ we used for our estimates.

4. <u>Summary</u>

a. If wind forces matter we should use <u>pipes</u> throughout, since rectangular shapes increase the wind area by almost 80%. If we choose $\ell/r = 100$, the wind area of a single member then is

$$A = 0.030\,\ell^2.$$

b. Space frames of total area A_t and volumn V, which are neither too flat nor elongated, have a wind area of

$$A = 0.34\ A_t$$

or

$$A = 0.54\ V^{2/3},$$

independent of the number of subsystems of secondary members, if they are arranged economically.

c. The only way of reducing the wind area below these values, is splitting up the main members (instead of using secondary members connecting them). The best way is to use <u>3 chords</u> in an equilateral triangle. This reduces the wind area by 40%.

d. Similarly, also towers should have <u>3 legs</u> or chords only, and if guyed, 3 guy ropes. The stiffness does not depend on this number, but the wind force does.