

PROJECT: **LFST**  
 SUBJECT:

## Structures for Homology Calculations

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This is not a finished report, but only a collection of sketches, notes and results, containing all structures suggested and tried with the homology program. A final report may be written at a later time, after having calculated more examples.

### Notations

#### 1. Structure

p = number of structural points (pin joints), including s and r;  
 s = number of surface points where homology is demanded;  
 r = number of holding points where deformations are restricted;  
 m = number of structural members (bars);  
 D = diameter of surface;  
 f = focal length;  
 L = total length of structure, perpendicular to surface;  
 Q = cross sections (bar areas);  
 w = simulated weight of surface on each single surface point.

#### 2. Results

i = number of iterations performed (index 0 means "first guess" from input data);  
 df = change of focal length, looking at zenith  
 dφ = change of axial direction, looking at horizon } = homology parameters;  
 ΔH = rms deviation of all surface points from best-fit paraboloid of revolution;  
 ΔP = rms distance of all surface points from design (actual deformation);  
 ΔX = distance between structural surface center and apex of best-fit paraboloid;  
 ΔA = <sup>maximum</sup> relative change of bar areas;  
 $F_v = \Delta H_{v-1} / \Delta H_v$  = improvement factor for single iterations step, v=1... i;  
 F = geometrical mean of  $F_v$  for all steps performed on this structure;  
 K = ΔP/ΔH = homology factor;  
 G = **1.02** ΔP (300ft/D)<sup>2</sup> = (ΔP of this structure)/(ΔP of octahedral antenna).

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REPORT NO. 13  
CONTRACT NO. \_\_\_\_\_  
PAGE 2 OF \_\_\_\_\_  
DATE \_\_\_\_\_

PROJECT:

SUBJECT:

## Change of Calculating Method

### 1. Non-selective Treatment of Members

The first examples ran nicely, without any trouble. But then the method had to be changed. In (67) of Report 4, the rectangular matrix of homology conditions was split up into a square part and a remaining rectangular part, and the square part was inverted. This is a selective procedure, which treats the members contained in the square part in a different way from those contained in the remaining part. This procedure works, as long as all members of the square part can be defined by the homology conditions, after all other members have been chosen. But the square matrix is singular (and cannot be inverted) if one or more of its members have nothing to do with homology, which, for example, is the case in Structure 2a for 12 members (like 1-2, 1-8, 18-19) which just give stability but have no direct influence on homology.

As a method which gives all members exactly the same treatment, the method of Lagrangean multipliers was chosen, replacing (67) to (79) of Report 4. The new version will be distributed shortly. Up to now, we had no trouble with it.

### 2. Weight Factors $\omega$ , and Calculating Accuracy

With Lagrangean multipliers, it seemed advisable to include the homology parameters  $h_k$  in the minimum condition (71), treating all unknowns exactly the same way, the cross section changes  $dQ_Y$  as well as the  $h_k$ . (If we did not include the  $h_k$ , we would need a lot of additional programming.)

But then, the homology parameters must also be given a "weight factor" for the minimum demand, just as each  $dQ_Y$  is weighted with  $Q_Y^{-2}$  in (71). We called this weight factor for the homology parameters  $\omega$ , <sup>and</sup> left it open to be entered with the input data. A large value of  $\omega$  means that a large importance is assigned to small  $h_k$ , while a small value of  $\omega$  makes the values of the  $h_k$  unimportant.

It turned out that small values of  $\omega$  decrease the calculating accuracy considerably, up to the point where the whole method fails completely. But for larger values of  $\omega$  the method worked alright. For the future, we plan to increase the accuracy by introducing one or two iterative improvements for each inversion of a matrix in the program.

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REPORT NO. 13

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PAGE 3 OF \_\_\_\_\_

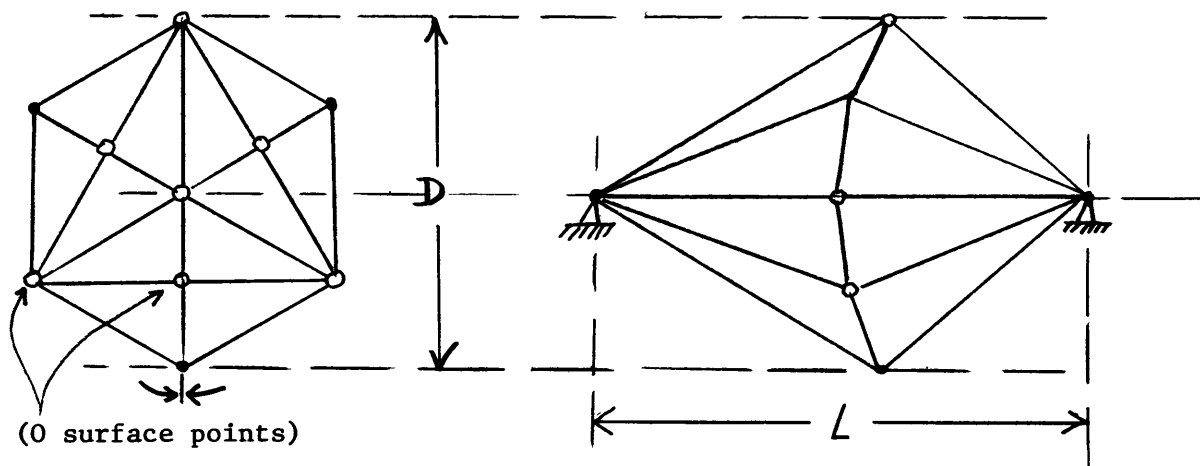
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## Structures 1

### Structure 1a:



$D = 300 \text{ ft}$

$f/D = 0.50$

$p = 12$

$L = 520 \text{ ft}$

$L/D = 1.73$

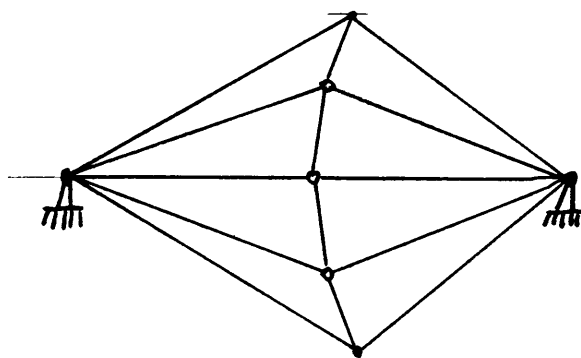
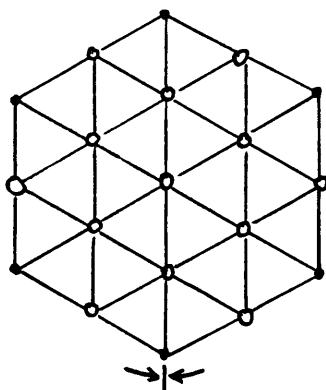
$s = 7$

$f = 150 \text{ ft}$

$r = 3$  (restrictions: 3, 3, 1)

$m = 41$

### Structure 1b:



$D = 260 \text{ ft}$

$f/D = 0.58$

$p = 21$

$L = 390 \text{ ft}$

$L/D = 1.50$

$s = 13$

$f = 150 \text{ ft}$

$r = 3$  (3, 3, 1)

$m = 80$

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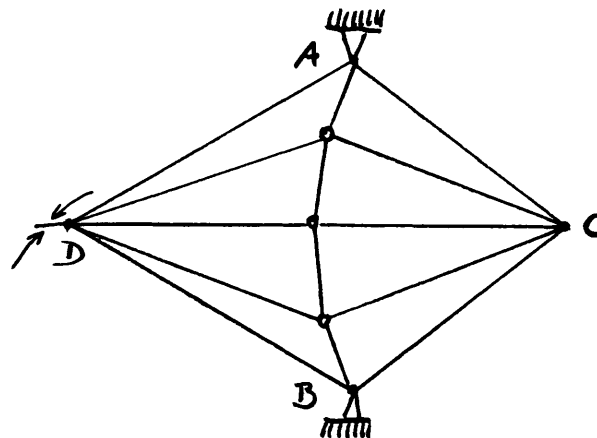
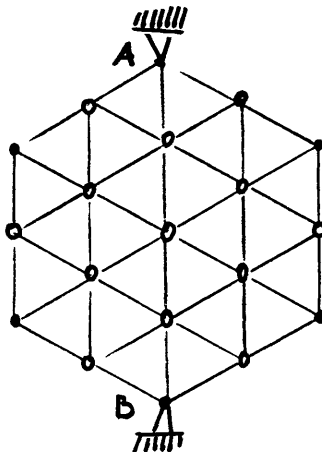
REPORT NO. 13  
CONTRACT NO. \_\_\_\_\_  
PAGE 4 OF \_\_\_\_\_  
DATE August 3, 1966

PROJECT:  
SUBJECT:

Structures 1a and 1b are suggested only for testing the homology program: they are built in such a way that they must, without any doubt, have homology solutions; which means that the program must, if it works, give solutions.

They do not look like a telescope but still have all the essentials: surface points, holding points, additional points. The original suggestion was Structure 1b; but the program, in its first set-up, would have needed too much memory space for it, and thus Structure 1a was taken instead of, with a smaller number of points. Later, the program was changed and then could handle Structure 1b, too.

## Structure 1c:



This structure is essentially the same as Structure 1b, only held at different points. Since the program ran so nicely and smoothly on Structures 1a and 1b, and the results were so extremely good, almost suspiciously good, R. Jennings suggested to make a counter-test with a structure which cannot have homology solutions. The easiest way was to take the same structure, but to hold it in such a way that it **must have a strong astigmatism if looking at zenith which cannot be counteracted by just changing the bar areas**. At least, that's what we thought. But, after introduction of  $\omega$ , the program was cleverer than its creators and still gave good convergence. The reason is that the bars meeting at the holding points A and B, actually, just give a suspension to points C and D similar to the suspension of Structure 4. And since points A and B are not defined as surface points, the rest can behave homologically. And does so.

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REPORT NO. 13  
CONTRACT NO. \_\_\_\_\_  
PAGE 5 OF \_\_\_\_\_  
DATE \_\_\_\_\_

PROJECT:

SUBJECT:

## Results of Structure 1

Run	$\omega$	Q inch <sup>2</sup>	w tons	i	df <sub>i</sub> inch	dφ <sub>i</sub> arcmin	ΔH <sub>0</sub> 10 <sup>-3</sup> inch	ΔH <sub>i</sub> 10 <sup>-6</sup> inch	ΔP <sub>i</sub> inch	ΔX <sub>i</sub> inch	ΔA	F	K <sub>0</sub>	K <sub>i</sub> 10 <sup>3</sup>	G
1a, 1	1000	85	20	2	.110	.58	8.6	8.4	.64	.88	.36	32	74	76	1.3
1a, 2	100	85	20	3	.113	3.2	8.6	2.2	.65	3.4	.33	16	76	295	1.3
1a, 3	10	85	20	2	.116	5.7	8.6	3.3	.65	6.2	.29	51	76	197	1.3
1a, 4	1	85	20	3	.116	5.8	8.6	18.0	.65	6.3	.28	8	76	36	1.3
1a, 5	.1	85	20	} first iteration steps diverge											
1a, 6	.01	85	20												
1b	1	65	7	3	.067	4.3	.93	50	.27	4.6	.09	3	290	5	.5
1c	10	65	7	2	1.13	2.0	2.7	2.1	.50	1.9	.15	11	185	240	1.0

## Conclusions

1. The results prove that the mathematical method works, and that physical homology solutions exist for Structure 1. The final accuracies ( $\Delta H_i \approx 10^{-5}$  inch), of course, are only of academic interest; they just show, within the calculating accuracy of the machine, that exact solutions do exist.
2. The speed of convergence is better than expected; the single iteration step gives on the average an improvement by a factor  $F = 20$ , the best factor was 290. For all examples with  $\omega \geq 10$ , the average is  $F = 27$ , and the smallest factor is 6.3. No step diverged until the calculating accuracy was reached.
3. The range of convergence was checked with Structure 1a (with the old program, before the introduction of  $\omega$ ). In one run, all initial cross sections were taken equal (as in all examples of the table above), and in another run, they were randomly changed by  $\pm 20\%$ . The second run converged almost as well as the first one. This means that we do not need a very good first guess to start with.
4. Small values of the homology parameters can be obtained by choosing large  $\omega$ .
5. Structure 1b, with equal cross sections, is almost a perfect solution to start with. But then it was chosen especially for making homology as easy as possible.

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REPORT NO. 13  
CONTRACT NO. \_\_\_\_\_  
PAGE 6 OF \_\_\_\_\_  
DATE August 3, 1966

PROJECT:  
SUBJECT:

## Structure 2

Structure 2, again, is not a telescope, but comes closer to it than Structure 1. Its purpose is to let us study what is essential for homology and what is not. We begin with a sufficiently complicated structure (many degrees of freedom for fulfilling homology), and reduce it stepwise to more simple ones, until we reach one that cannot give exact physical solutions. The principal idea of Structure 2 is an octahedron, held with a suspension in such a way that the basic square of the octahedron can never deform out of its plane. The lower part of the octahedron is considered as the lower part of a single 3-dimensional cell, and the upper part of this cell yields 9 surface points.

Structure 2a isolates, deformation-wise, the center of the cell from its sides. The sides of the cell consist of four 2-dimensional pressure-stable cells as suggested and calculated in my first antenna paper (June 1965, Fig. 9); the center is a 3-dimensional version of the same cell. In this way I made sure that exact solutions must exist.

Structure 2b omits points 15, 16, 17, 18 and connects point 19 directly with points 2, 4, 6, 8. The structure then has  $p = 17$  points, and  $m = 47$  members. It still might have exact solutions, although the deformations of center and sides now are coupled instead of isolated.

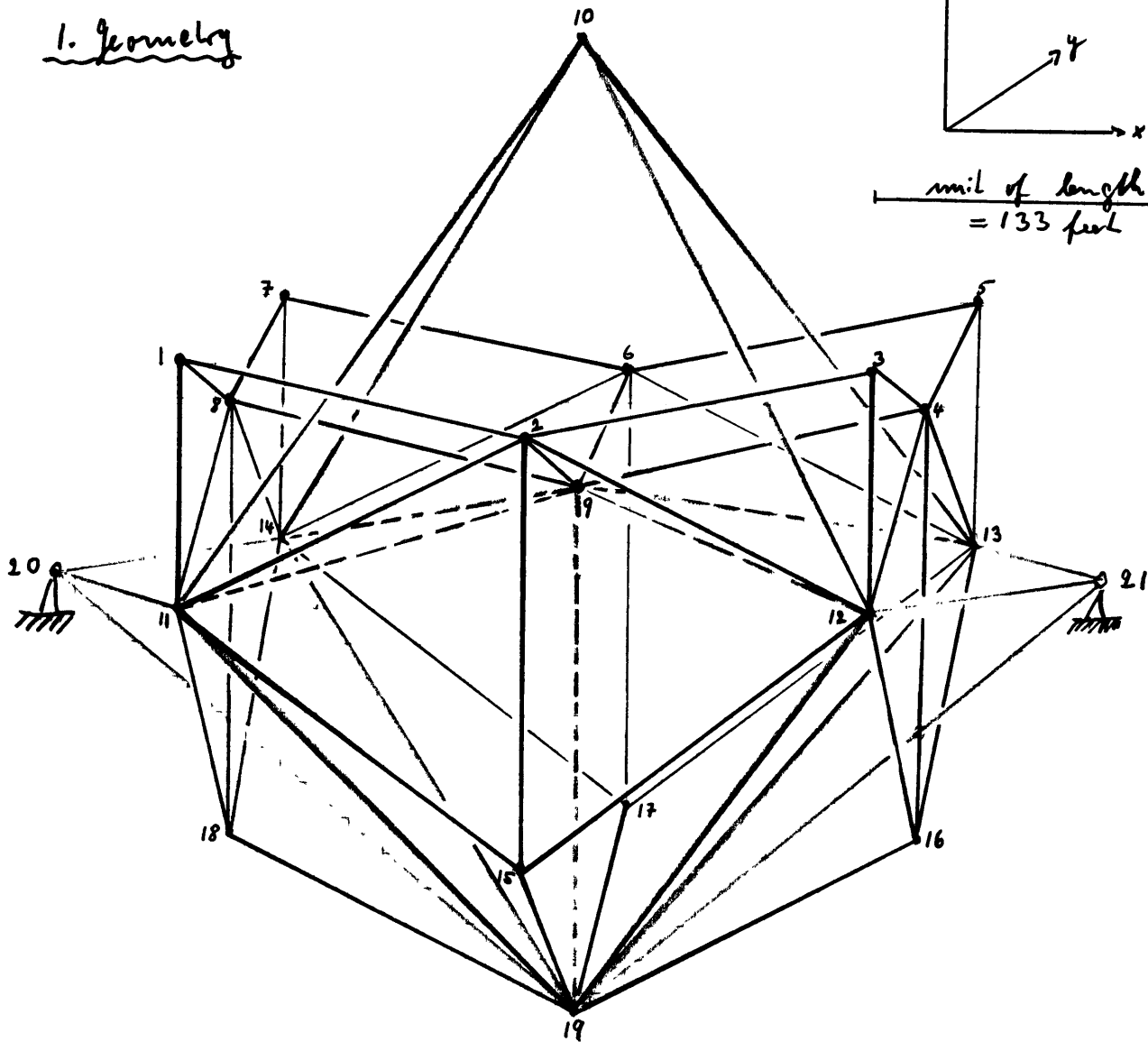
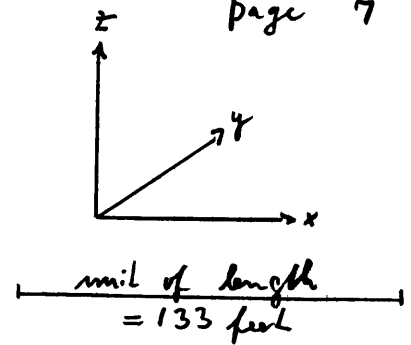
Structure 2c puts the surface right into the basic plane of the octahedron. It has  $p = 13$  and  $m = 35$ ; I doubt that it has exact solutions.

# Structure #2a

Report 13  
page 7

①

## 1. Geometry



p = 21 points

n = 9 surface points  
s = 3 support points

	restricted:
19	y
20	x y z
21	y z

paraboloid

$$z = 0.3 + \frac{x^2 + y^2}{5}$$

$$f = 1.25$$

m = 59 members

12	surface	—————→
8	octahedron sides	—————
5	" " diagonals	-----
24	in sides of square	—————
4	bracing sides down	—————
6	support	—————
59		

Aperture

square 2 x 2  
= circle r = 1.13

$$\frac{f}{D} = 0.553$$

# Structure #2a

3. Cross Sections ( $\text{area} = 65 \text{ inch}^2$ ) for "first guess."

point:	2. Coordinates			3. Cross Sections (meters) for first group:																					
	x	y	z	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
1	-1	-1	.7		1.0						1.0			1.0											
2	0	-1	.5			1.0						1.0		3.3	3.3			3.7							
3	1	-1	.7				1.0								1.0										
4	1	0	.5					1.0				1.0			3.3	3.3			3.7						
5	1	1	.7						1.0							1.0									
6	0	1	.5							1.0		1.0				3.3	3.3			3.7					
7	-1	1	.7								1.0						1.0								
8	-1	0	.5									1.0					3.3					3.7			
9	0	0	.3											3.3	6.0	6.0	6.0	6.0				6.0			
10	0	0	1.55											7.0	7.0	7.0	7.0								
11	-1	-1	0															5.0			5.0	7.0	9.0		
12	1	-1	0															5.0	5.0			7.0		9.0	
13	1	1	0																5.0	5.0		7.0		9.0	
14	-1	1	0																	5.0	5.0	7.0	9.0		
15	0	-1	-.75																			1.0			
16	1	0	-.75																			1.0			
17	0	1	-.75																			1.0			
18	-1	0	-.75																			1.0			
19	0	0	-1.25																				9.0	9.0	
20	-1.5	0	0																						
21	1.5	0	0																						

4. additional weight for surface points:

points	w
1, 3, 5, 7	2
2, 4, 6, 8	4
9	8

} \* weight of member 1-2

5. Material constants

$\rho, E, S'$  as for steel,  
and same for all members.

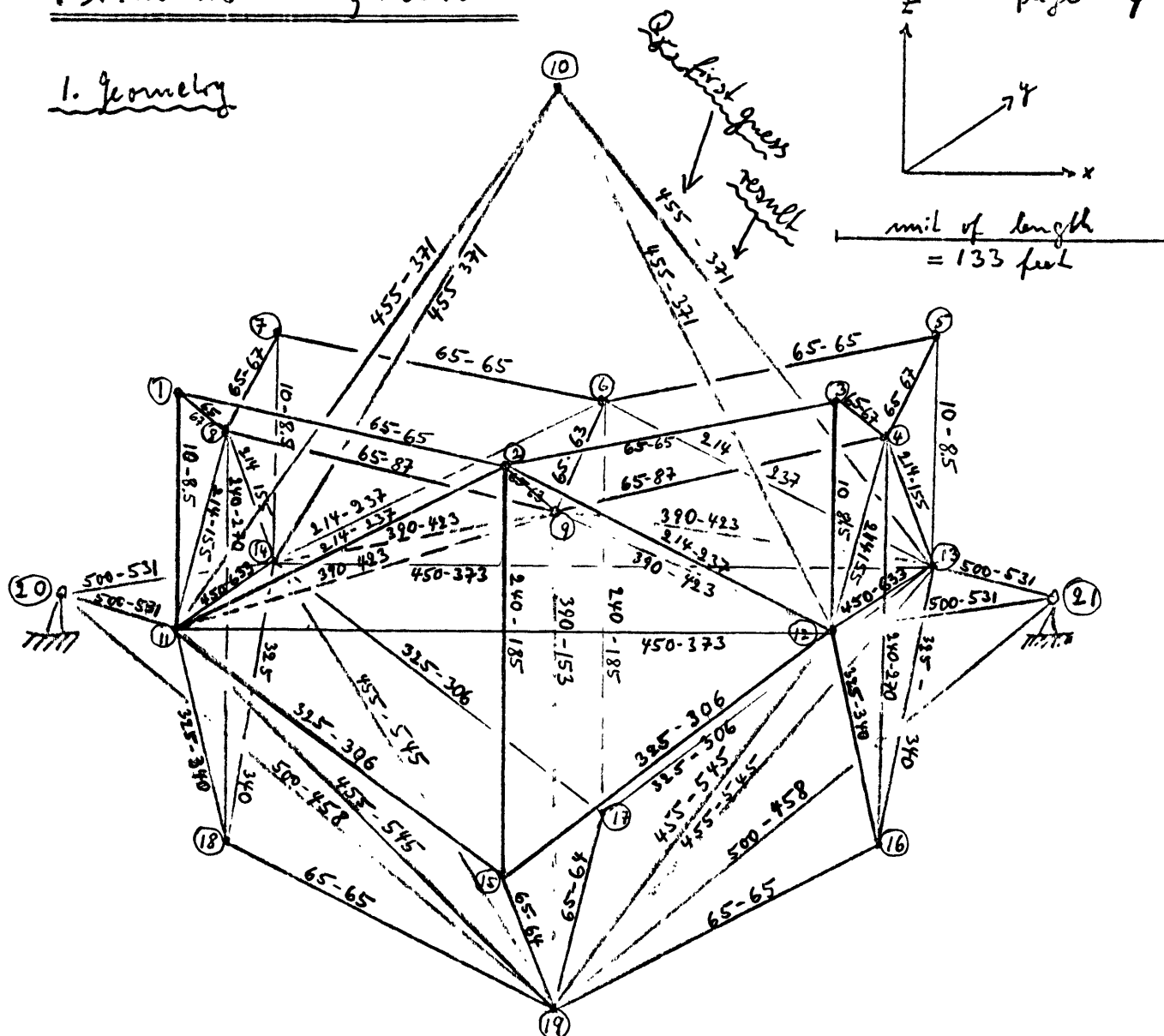


# Structure #2a; Results

Report 13  
page 9

(1)

## 1. Geometry



p = 21 points

n = 9 surface points

s = 3 support points

	restricted:
19	y
20	x y z
21	y z

paraboloid

$$z = 0.3 + \frac{x^2 + y^2}{5}$$

$$f = 1.25$$

m = 63 members

- 12 surface
- 8 octahedron sides
- 5 " " diagonals
- 24 in sides of square
- 4 tracing sides down
- 6 support

59  
4 square sides  
63

Aperture

square  $2 \times 2$

= circle  $r = 1.13$

$$\frac{f}{D} = 0.553$$

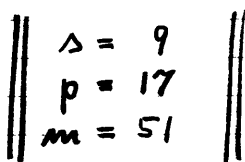
# Ochahedron, 2a; groups of members

example	number	$Q_0$ first guess	$Q_8$ result	$Q_8/Q_0$ factor
1-2	4	65	65	1.00
1-8	4	65	67	1.03
2-9	2	65	63	.97
8-9	2	65	87	1.34
1-11	4	10	8.5	.85
2-11	4	214	237	1.11
11-15	4	325	306	.94
2-15	2	240	185	.77
4-12	4	214	155	.72
12-16	4	325	340	1.05
4-16	2	240	270	1.12
18-19	2	65	65	1.00
15-19	2	65	64	.99
10-11	4	455	371	.82
11-19	4	455	545	1.20
11-12	2	450	373	.83
12-13	2	450	633	max: <span style="border: 1px solid black; padding: 2px;">1.41</span>
9-19	1	390	153	min: <span style="border: 1px solid black; padding: 2px;">.39</span>
9-11	4	390	423	1.08
11-20	4	500	531	1.06
19-20	2	500	458	.92
<u>63</u>				

63 members  
21 groups } factor 3

## 2 symmetries:

- 1) left-right; symmetry in x
- 2) front-back; symmetry in y



	points	$Q_0$	$Q_5$	$\frac{Q_5}{Q_0}$	$\Delta A$
factors $Q$ :	max 12-13	450	1579	3.51	<u><u>2.51</u></u>
	min 8-11	215	115	.53	.47

$$\Delta A = \left| \frac{Q_s}{Q_0} - 1 \right|$$

# Structure 2c

coordinates				Cross sections												
point	x	y	z	1	2	3	4	5	6	7	8	9	10	11	12	13
1	-1	-1	-.4		1.0						1.0		1.0	1.0	2.0	
2	0	-1	-.2			1.0	.7				.7	.7		.7		
3	+1	-1	-.4				1.0						1.0	1.0		2.0
4	+1	0	-.2					1.0	.7			.7		.7		
5	+1	+1	-.4						1.0				1.0	1.0		2.0
6	0	+1	-.2							1.0	.7	.7		.7		
7	-1	+1	-.4								1.0		1.0	1.0	2.0	
8	-1	0	-.2									.7		.7		
9	0	0	0											.7		
10	0	0	-1.25													
11	0	0	+1.45												3.0	3.0
12	-1.5	0	-.4													
13	+1.5	0	-.4													

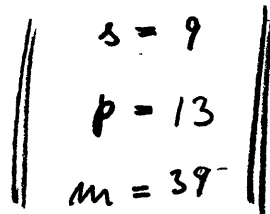
$$\text{parabola } z = -\frac{x^2 + y^2}{4f} \quad ; \quad f = 1.25 \quad \left. \vphantom{\begin{matrix} z = -\frac{x^2 + y^2}{4f} \\ f = 1.25 \end{matrix}} \right\} f = 166 \text{ ft}$$

unit of length = 133 feet

unit of cross section = 50 inch<sup>2</sup>

surface weight: same as #2a

with results



		points	$Q_o$	$Q_4$	$g\%$	$\Delta A$
factors Q	max	2-11	35	70	2.00	<u><u>1.00</u></u>
	min	1-12	100	40	.40	.60

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REPORT NO. 13  
CONTRACT NO. \_\_\_\_\_  
PAGE 14 OF \_\_\_\_\_  
DATE \_\_\_\_\_

PROJECT:  
SUBJECT:

## Results of Structure 2

Run	$\omega$	Q inch <sup>2</sup>	w tons	i	df <sub>i</sub> inch	dφ <sub>i</sub> arcmin	ΔH <sub>0</sub> 10 <sup>-3</sup> inch	ΔH <sub>1</sub> 10 <sup>-5</sup> inch	ΔP <sub>1</sub> inch	ΔX <sub>1</sub> inch	ΔA	F	K <sub>0</sub>	K <sub>1</sub> 10 <sup>3</sup>	G
2a	200	10-500	30	5	4.36	2.3	133	1.8	2.2	6.3	.61	6	17	120	4.4
2b	200	10-500	30	5	2.83	9.9	157	45.9	1.9	17.8	2.51	3	12	4	3.7
2c	200	35-150	30	4	1.43	1.5	331	1.6	1.1	1.0	1.00	12	3	70	2.2

### 1. Changes of Structure, for getting Convergence

In the original form, the iterations diverged (Pages 7 and 8, Structure 2a). We then introduced three changes:

a) Cross sections of members 1-11, 3-12, 5-13, 7-14 made only 10 inch<sup>2</sup> instead of 65.

We simply had forgotten to apply equation (49) of the antenna-design paper.

b) Introduction of the four sides of the basic square, 11-12, 12-13, 13-14, 14-11 on page 9 and 11 for Structures 2a and 2b, and 1-3, 3-5, 5-7, 7-1 on page 13 for Structure 2c. These sides are redundant, but increase the stiffness.

If Structure 2 represents a telescope, the sides are alright for 2a and 2b. But in Structure 2c the sides are in front of the surface and cast shadow. We will try again without sides, using various first guesses; maybe we find a solution.

c) Lowering the support points (which is actually drawn only in page 11, but is used in all three structures), for decreasing the sag in zenith position.

After these changes, all three structures converged nicely. But there was no time for investigating which of these changes are essential.

PROJECT:  
SUBJECT:

## 2. Resulting Cross Sections

On page 9, 11, 13 the cross sections actually used as first guess (left) and the resulting cross sections after reaching homology (right) are written at each member.

- a) Symmetry. We see that we have exact symmetry in x and y (as it should be). On page 10 all members are grouped together which behave identically. There are 21 groups as compared to 63 members. If the program were rewritten, taking care of these symmetries, the number of unknowns thus would be reduced by a factor 3.
- b) Amount of Change. Structure 1 was almost a perfect homology solution at the first guess. This is different for Structure 2, where 2b for example gave initial deviations from homology of  $\Delta H = .157$  inch, which would limit the shortest wavelength to  $\lambda_0 = 6.4$  cm. Here, the program has much more "work to do", which is shown by the large changes in some of the cross sections, up to a factor of 3.6
- That so large a change can be reached, shows that the range of convergence is large. On the other side, it is not unlimited either, as was shown by the divergence of the first tries, where some members were wrong by a factor of 7.7.

## 3. Application

Structures 2 were not really meant to be telescopes (page 6). But if Structure 2c could be made working without the side members, then a good telescope could be obtained by just adding some more surface points, connecting each surface point to its neighbors and down to point 11, just as points 2, 4, 6, 8 are already held now. But, as of now, the best one is Structure 2b; with nine surface points, a 300-ft telescope could observe down to  $\lambda = 2$  cm wavelength. But 2b looks too clumsy and heavy.

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REPORT NO. 13  
CONTRACT NO. \_\_\_\_\_  
PAGE 16 OF \_\_\_\_\_  
DATE July 28, 1966

PROJECT:

SUBJECT:

## Structure 3 =====

This is not a telescope; it is only the mirror part, to be put into the round opening of the "floating sphere" telescope as designed by O. Heine. The ring at the opening is assumed to keep its shape; how to get a non-deforming ring without extra costs is described in my Report 12.

Structure 3a simulates a single, thin shell, in order to investigate whether a shell has homology solutions or at least can approximate one closely enough. If not so, a Structure 3b should be tried as suggested in Report 12, consisting of a shell with a special suspension.

## Results

Run	$\omega$	Q inch <sup>2</sup>	w tons	i	df <sub>i</sub> inch	dp <sub>i</sub> arcmin	$\Delta H_0$ 10 <sup>-3</sup> inch	$\Delta H_1$ 10 <sup>-6</sup> inch	$\Delta P_1$ inch	$\Delta X_1$ inch	$\Delta A$	F	K <sub>0</sub>	K <sub>1</sub> 10 <sup>3</sup>	G
3a	200	80	30	4	2.2	.72	86	6.8	.78	1.2	.39	11	9	114	6.3

The single, thin shell gave a good solution on first try. The result, however, is not a shell which could be replaced by a membrane, it must be framework, since it demands heavier rings and lighter radial members. Page 17 shows the resulting cross sections only for the center part, all remaining changes were below 10%. At the first guess, all cross sections were 80 inch<sup>2</sup>.

This good convergence to a homology solution, together with Report 12 (Mirror for the Floating Sphere, July 22, 1966), show that the floating sphere could relatively easy be supplied with a homologous mirror of any accuracy wanted. With Structure 3a, with 19 surface points, observation can go to  $\lambda = 2$  cm.



3 a:  
Structure  
(floating-sphere mirror)  
July 21, 1966; S.v.H.

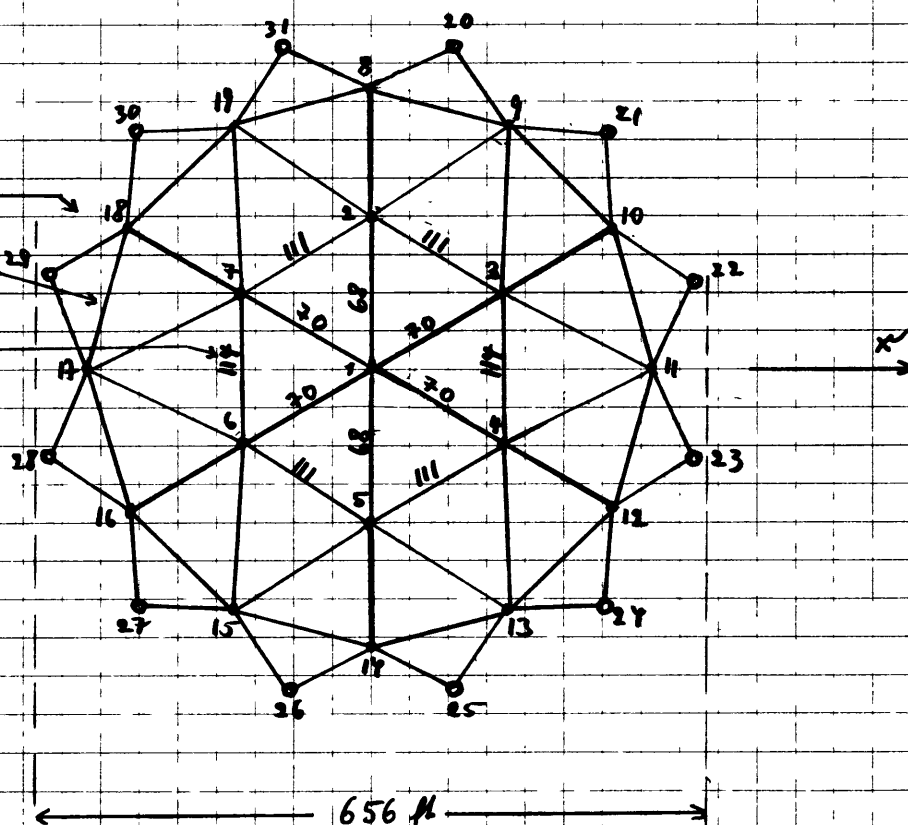
o 12 holding points  
o 19 mirror points  
31

3 circles:

$r_1 = 328 \text{ ft}$

$r_2 = 274 \text{ ft}$

$r_3 = 152 \text{ ft}$



Parabola:  
$$z = -\frac{x^2 + y^2}{4f}$$

$f = 328 \text{ ft}$   
 $D = 656 \text{ ft}$   
 $\frac{f}{D} = 0.5$

all  $Q = 80 \text{ inch}^2$   
all  $w = 30 \text{ tons}$

$p = 31 \text{ points}$   
 $m = 66 \text{ members}$

This structure simulates a simple shell, suspended in a non-deforming ring.  
I think it will not have an exact homology solution, but maybe it is already  
a good approach.

# NATIONAL RADIO ASTRONOMY OBSERVATORY

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REPORT NO. 13  
CONTRACT NO. \_\_\_\_\_  
PAGE 18 OF \_\_\_\_\_  
DATE August 3, 1966

PROJECT:  
SUBJECT:

## Structure 4

This is the first structure actually meant to be a telescope (it was suggested in Report 1, of October 20, 1965). It has 21 surface points, the number of all points is 34, and the number of members is 128. For comparison: with 34 points, the minimum number of members is 96 for a stable structure, the maximum number of members is 561, and 128 is pretty close to the minimum.

Structure 4 consists of a suspension (held at points 33 and 34), holding an octahedron (points 23, 24, 25, 26, 31, 32); this yields five basic homologous points (23, 24, 25, 26, 22). From these five basic points, we reach the surface with a single layer, yielding 21 homologous surface points (1 ... 21).

The layer consists of two types of cells which alternate around the center, and which have several points and members in common. We have four cells of each type. Both cells are of the pressure-stable kind; for example, if point 23 is pressed toward point 26, then point 30 moves down, but all surface points keep constant height (if the cross sections have certain values); again, if 22 is pressed toward 26, then 30 and 29 move down, but 1, 5 and 21 keep constant height.

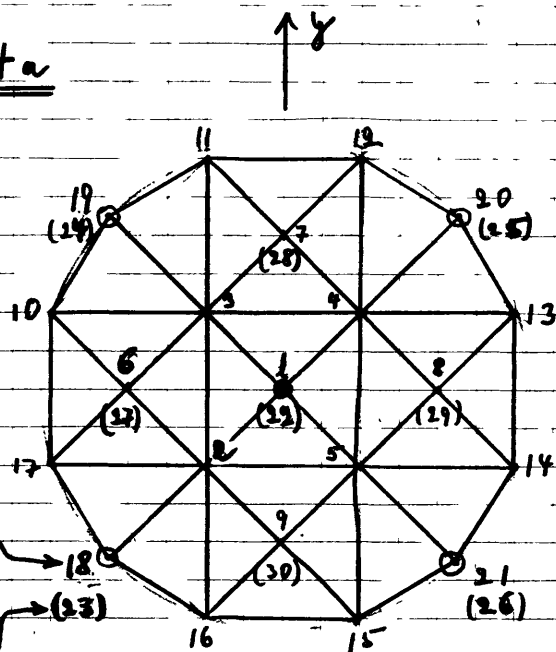
A number of 21 surface points is most probably all we need for the near future. The antenna diameter D then is divided into four parts, which means that a thick (non-homologous) structure in between neighboring points deforms only 1/16 of the deformation of a conventional, non-homologous telescope of diameter D. With other words, having 21 homologous surface points, we can pass the gravitational limit by a factor of 16 in wavelength or a factor of 4 in diameter. Some examples are given below.

D (feet)	$\lambda$ (cm)	
	conventional	with 21 hom. surf. points
600	30	1.9
450	17	1.1
300	7.5	.5
210	3.7	.23
140	1.6	.10
85	.6	.04

Structure 4a

1) Surface:

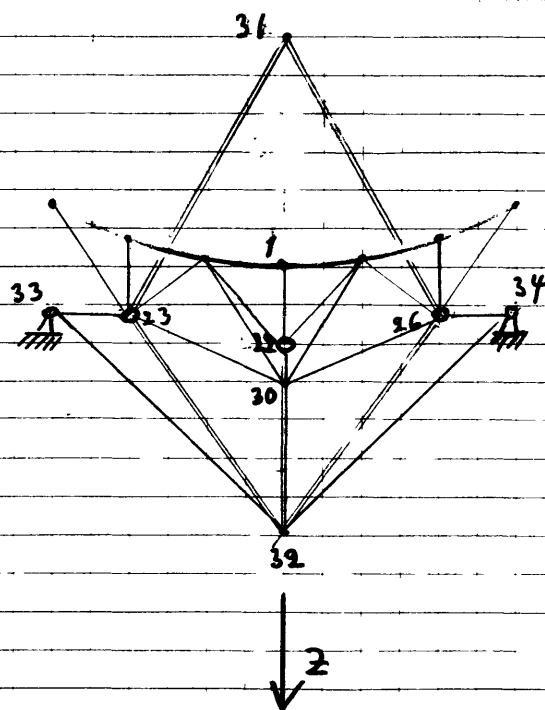
upper point,  
in surface  
lower point  
in structure



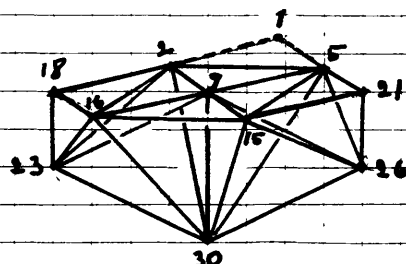
$$\begin{aligned} s &= 21 \\ p &= 34 \\ m &= 128 \end{aligned}$$

holding points	restricted deformation
32	$\gamma$
33	$x \gamma z$
34	$x \gamma z$

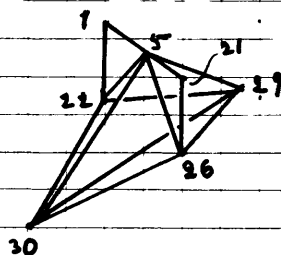
2) Side view



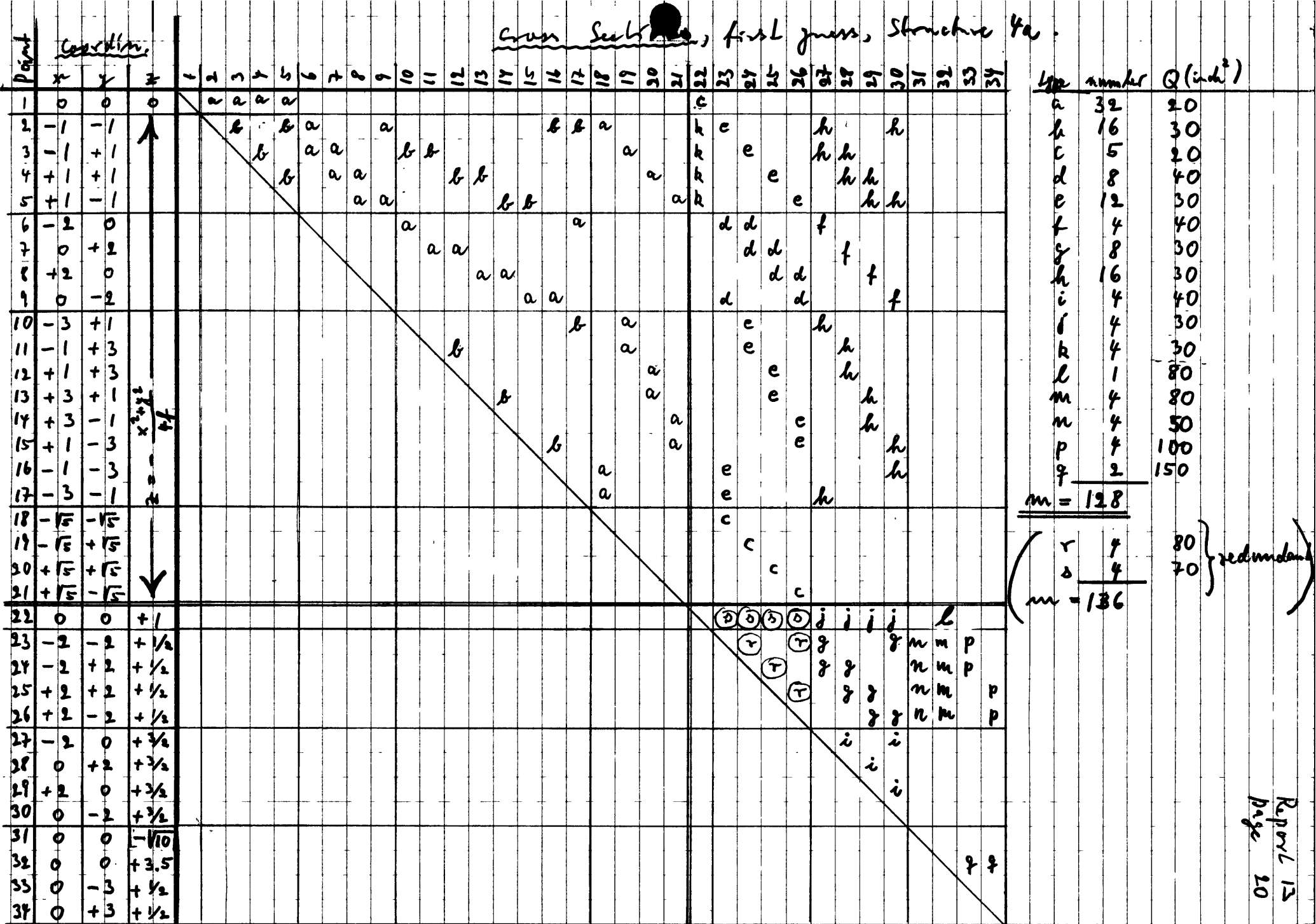
3) Cells:



Cell 1



Cell 2



Example:  $D = 300 \text{ ft}$  : unit of length =  $47.4 \text{ ft} = 569 \text{ inch}$  //  $m$  points:  $W = 15 \text{ tons}$   
 $f/D = 0.5$  :  $f = 150 \text{ ft} = 1800 \text{ inch}$

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REPORT NO. 13  
CONTRACT NO. \_\_\_\_\_  
PAGE 21 OF \_\_\_\_\_  
DATE \_\_\_\_\_

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PROJECT:

SUBJECT:

## Memory Limit for Present Program

When Structure 4 was tried, the computer said: "NOT ENOUGH MEMORY". This means completely rewriting the program in such a way that actually only one quadrant of the structure is calculated (assuming structures which are symmetrical in x and y). Since this would give a very long delay, we first tried whether we could find a useful telescope structure with the present program: a structure with less than 30 points.

Structure 2c: We tried to get rid of the side bars in front of the surface (see page 14 and 15), but did not succeed. Several variations of 2c with side bars converged nicely, but without sidebars we always obtained negative bar areas. Thus, we gave up on this one.

Structure 2d: Since 2b had given convergence, we tried to increase the number of surface points from 9 to 20, with a total of 29 points, and 95 members. This failed and gave negative areas; but we think with a few more additional points and members it would have worked. Because of the memory limit, we did not follow this line.

## Structure 2e

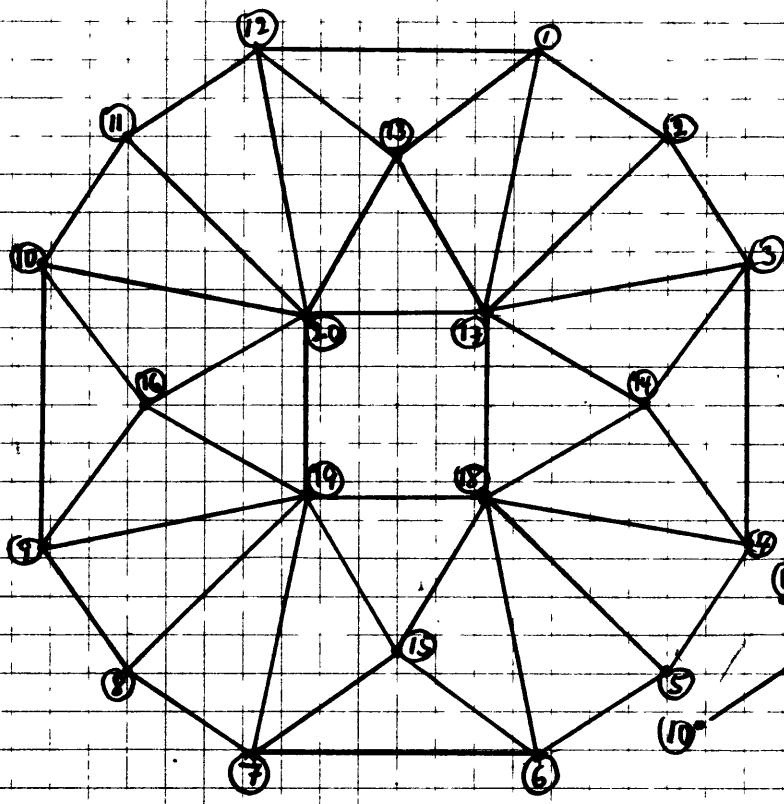
Since 20 surface points for <sup>a</sup>total of 29 points seemed not possible, we went back a step to only 13 surface points. Structure 2e is the logical continuation of 2c with side bars, adding one more layer in a most simple way.

Most probably, 2e is about all we can get out of the present program.

# Structure 2d

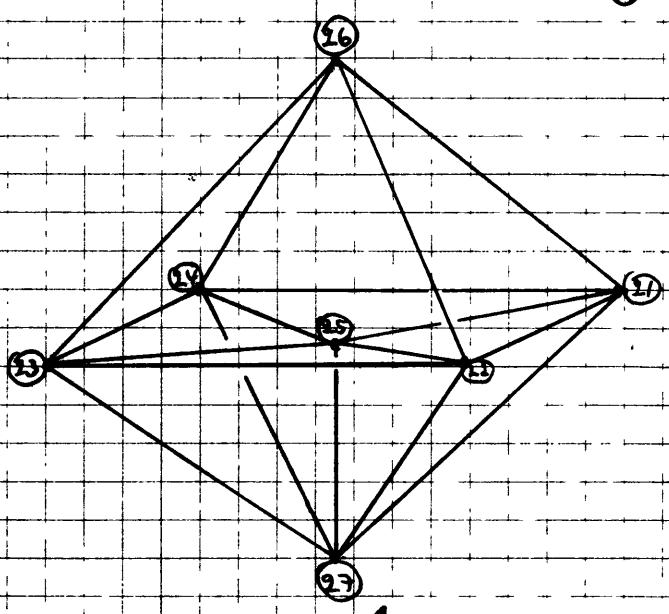
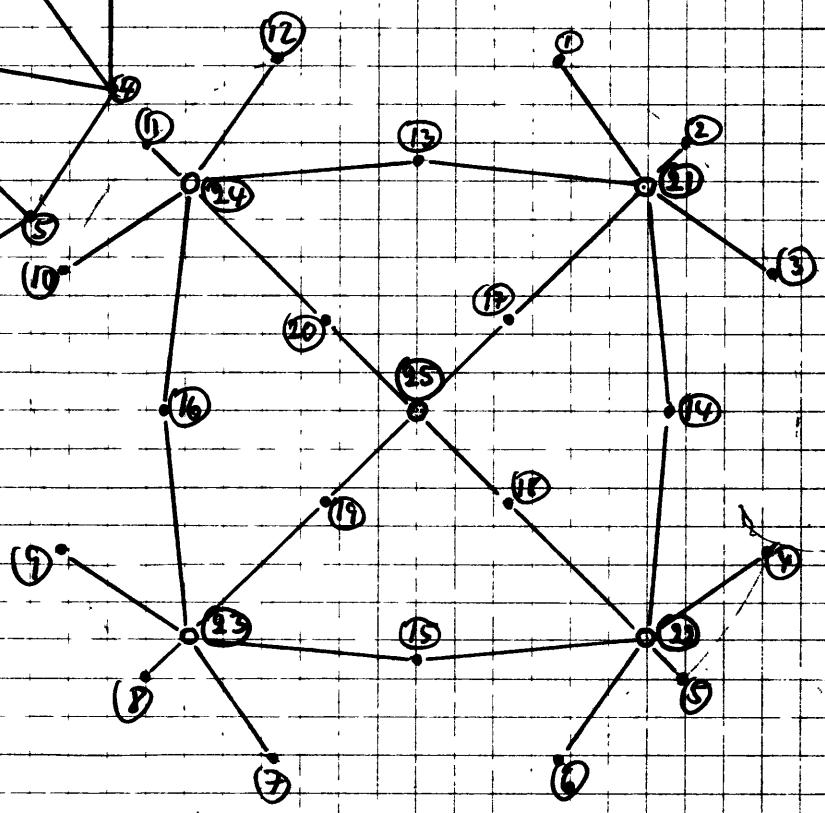
(page 22)

$$\begin{array}{l|l} \Delta = 20 & D = 300 \text{ ft} \\ p = 29 & f = 150 \text{ ft} \\ m = 95 & \end{array}$$



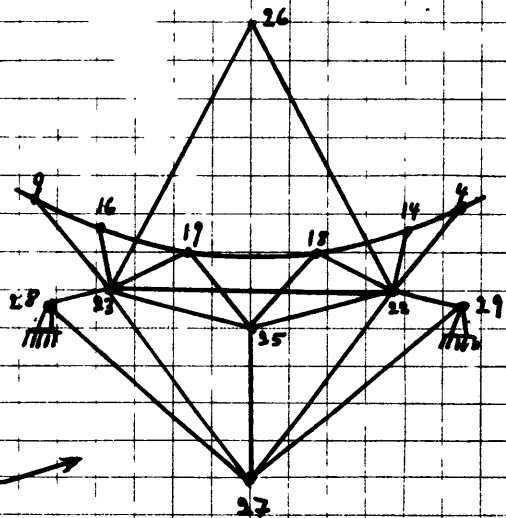
44 members in surface

28 members between surface and basic square

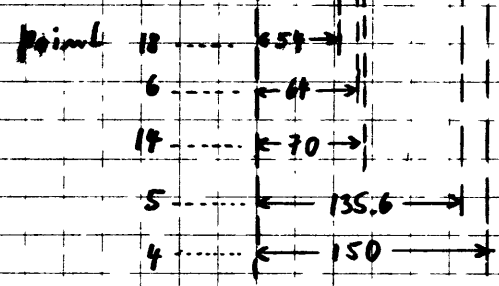
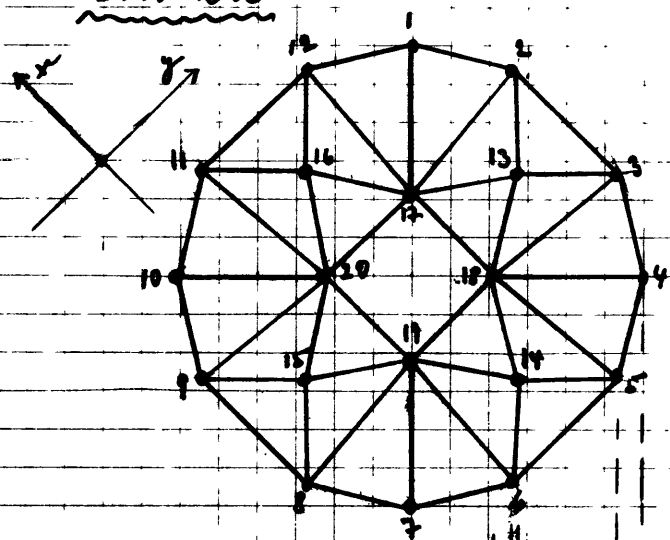


octahedron

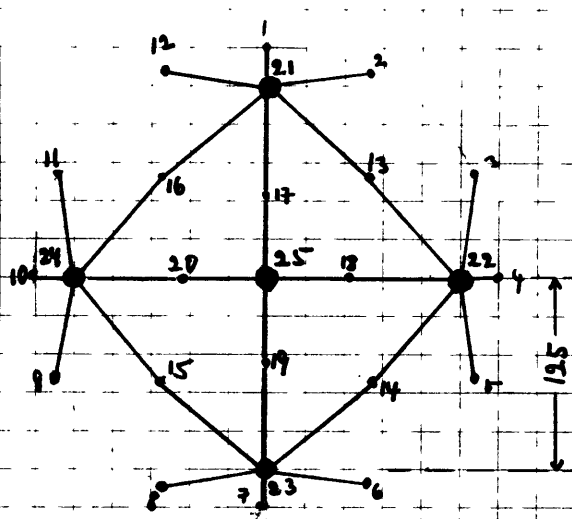
side view



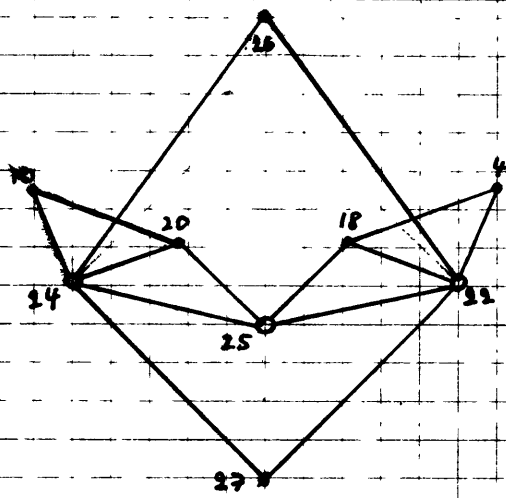
Str. 2d



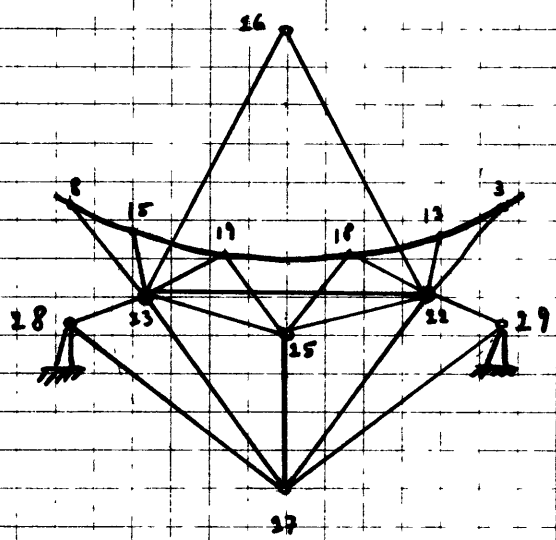
a) Members in surface  
(= 44)



b) Members between surface and basic square  
(= 28)



c) Members in plane  
4-10-27



d) Side view

$$\Delta = 20$$

$$p = 29$$

$$m = 95$$

$$D = 300 \text{ ft}$$

$$f = 150 \text{ ft}$$

If actual surface cantilevers 17 ft,  
then effective  $D = 328 \text{ ft} = 100 \text{ m}$

# Structure 2 d

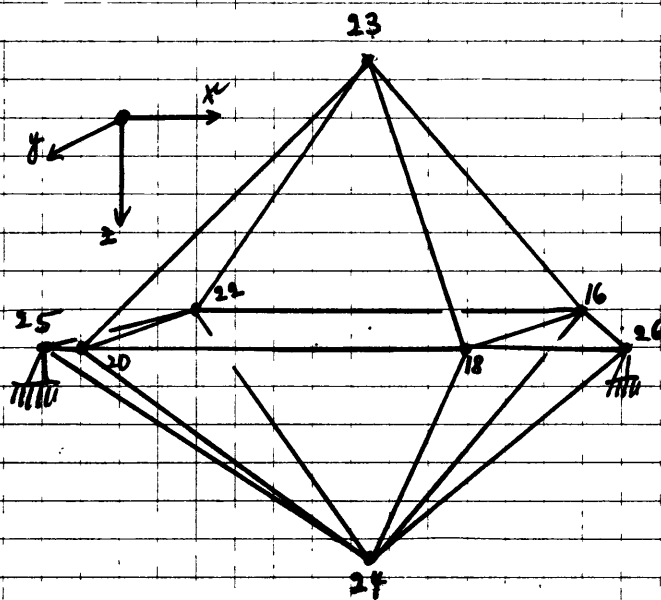
page 24

No.	points	Q
1	1 - 2	50
2	1 - 12	50
3	1 - 13	50
4	17	50
5	21	80
6	2 - 3	50
7	17	50
8	21	15
9	3 - 4	50
10	14	50
1	17	50
2	21	80
3	4 - 5	50
4	14	50
5	18	50
6	22	80
7	5 - 6	50
8	18	50
9	22	15
20	6 - 7	50
1	15	50
2	18	50
3	22	80
4	7 - 8	50
5	15	50
6	19	50
7	23	80
8	8 - 9	50
9	19	50
30	23	15
1	9 - 10	50
2	16	50
3	19	50
4	23	80
5	10 - 11	50
6	16	50
7	20	50
8	24	80
9	11 - 12	50
40	20	50
1	24	15
2	12 - 13	50
3	20	50
4	24	80
5	13 - 17	50
6	20	50
7	24	80
48	24	80

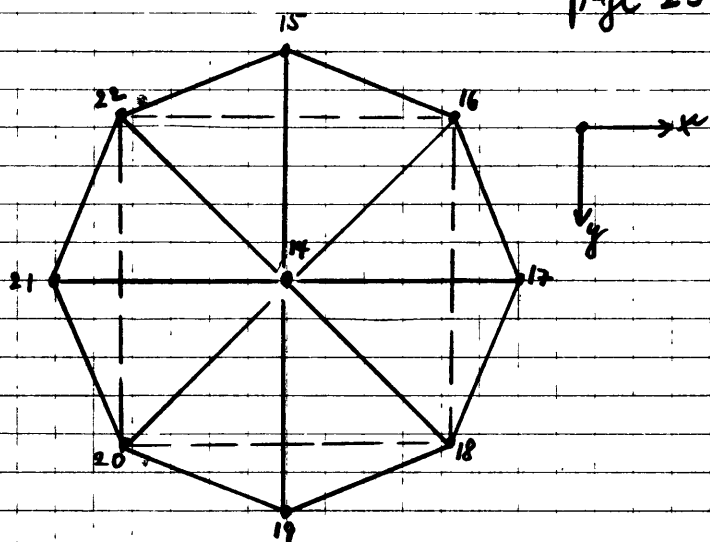
No.	points	Q
49	14 - 17	50
50	18	50
1	21	80
2	22	80
3	15 - 18	50
4	19	50
5	22	80
6	23	80
7	16 - 19	50
8	20	50
9	23	80
60	24	80
1	17 - 18	50
2	20	50
3	21	80
4	25	80
5	18 - 19	50
6	22	80
7	25	80
8	19 - 20	50
9	23	80
70	25	80
1	20 - 24	80
2	25	80
3	21 - 22	200
4	24	200
5	25	180
6	26	150
7	27	250
8	29	350
9	22 - 23	200
80	25	180
1	26	150
2	27	250
3	29	350
4	23 - 24	200
5	25	180
6	26	150
7	27	250
8	28	350
9	24 - 25	180
90	26	150
1	27	250
2	28	350
3	25 - 27	100
4	27 - 28	600
95	29	600



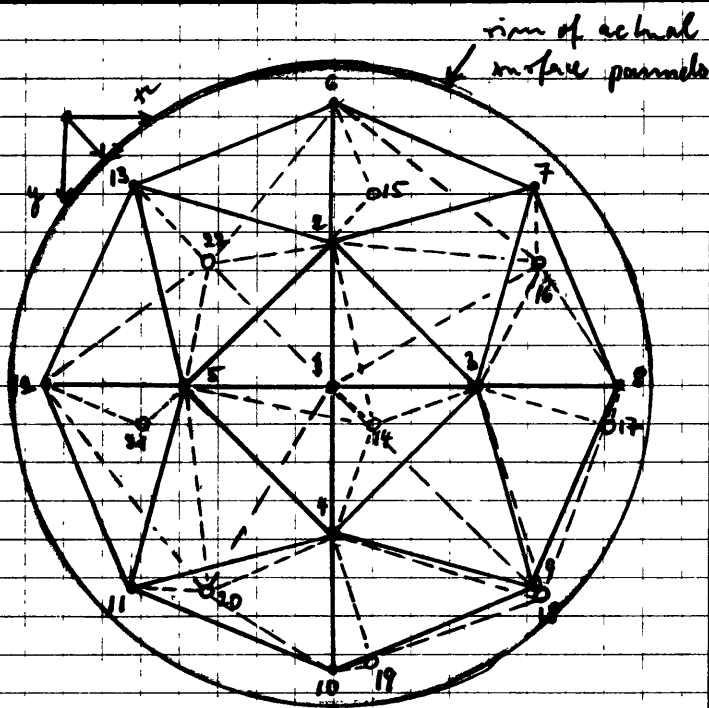
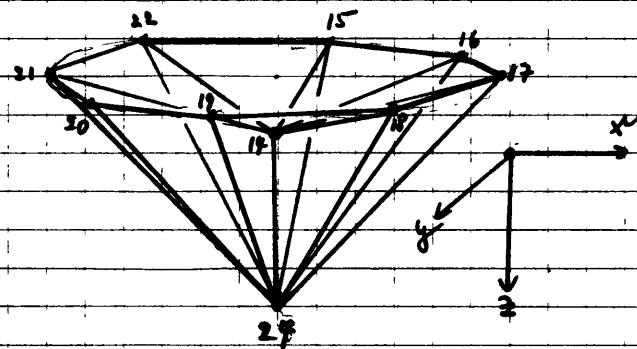
Structure 2c, 4



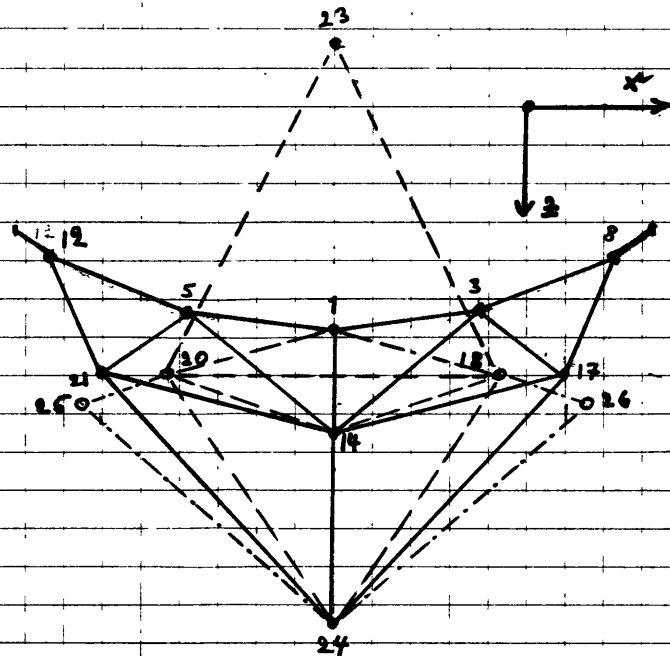
1) Octahedron and suspension  
(12) (6)



2) basic octagon and layer 1



3) Surface and layer 2  
(28) (37)



4) Side view of plane 12-8-24,  
octahedron and suspension

$$\begin{aligned} s &= 13 \\ p &= 26 \\ m &= 184 \end{aligned}$$

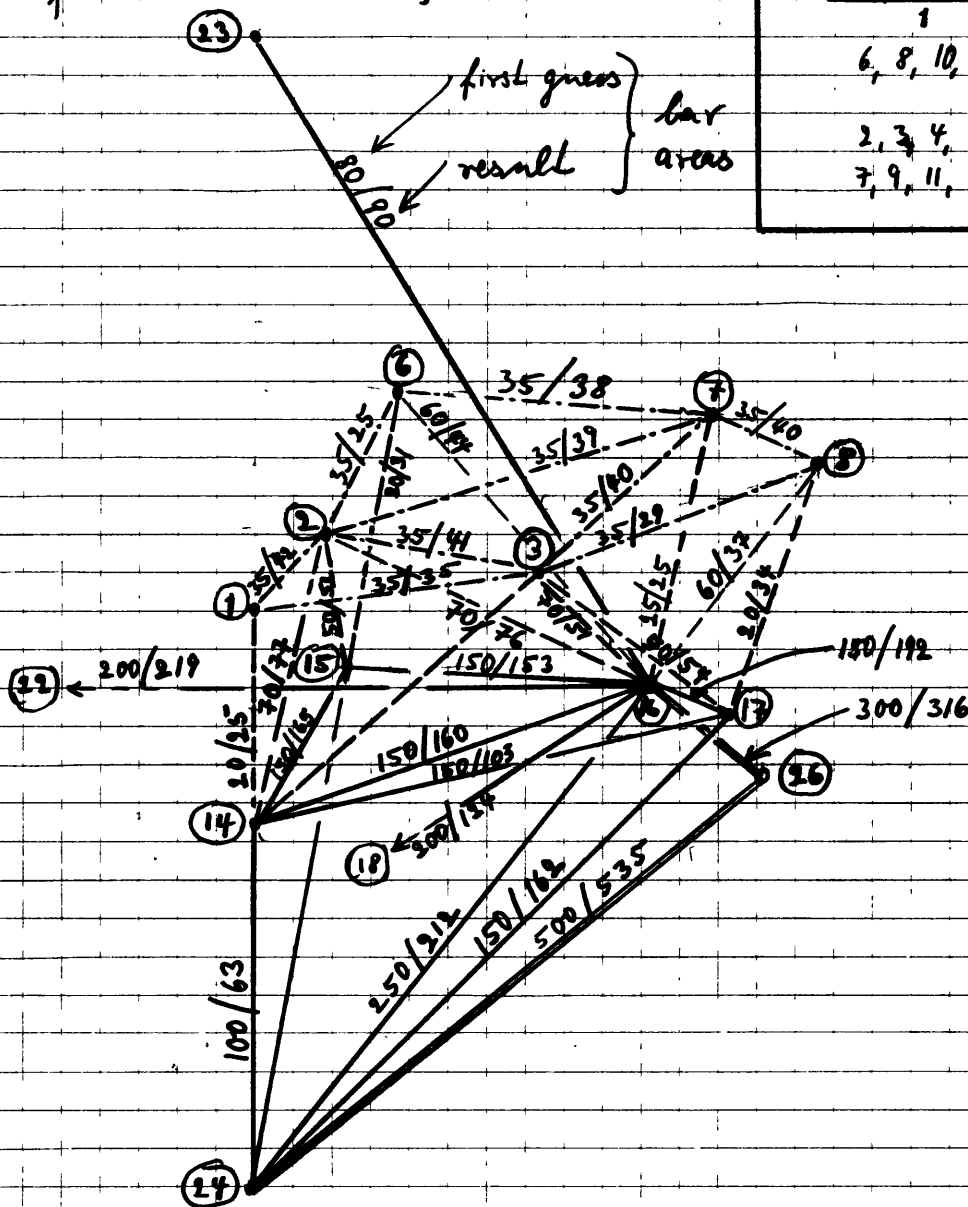
$$\begin{aligned} D &= 300 \mu \text{ (actual surface)} \\ f &= 136 \mu \\ H/D &= .453 \end{aligned}$$

# Structure 2e, Results

(one quadrant shown)

## Surface Loads:

points	$w_0$
1	
6, 8, 10, 12	18 hrs = 40,000 lb
2, 3, 4, 5	
7, 9, 11, 13	26 hrs = 58,000 lb



## Structure 2e

one quadrant, side view

- surface
- layer 2
- octahedron and octagon
- ===== suspension

# 2e, Coordinates (in feet)

page 27

point	x	y	z
1	0	0	0
2	0	-68	↑
3	68	0	
4	0	68	
5	-68	0	
6	0	-136	
7	96.3	-96.3	↑ x+y z
8	136	0	
9	96.3	96.3	
10	0	136	↑ x z
11	-96.3	96.3	
12	-136	0	
13	-96.3	-96.3	↓
14	0	0	
15	0	-110	↑
16	78	-78	
17	110	0	
18	78	78	
19	0	110	22
20	-78	78	↓
21	-110	0	
22	-78	-78	
23	0	0	-136
24	0	0	136
25	-120	0	35
26	120	0	35

# Structure 2 e

page 28

member	prints	Q <sub>0</sub>
1	1 - 2	35
2	3	35
3	4	35
4	5	35
5	14	20 x
6	2 - 3	35
7	5	35
8	6	35
9	7	35
10	13	35
1	14	70
2	15	50
3	16	70
4	22	70
5	3 - 4	35
6	7	35
7	8	35
8	9	35
9	14	70
20	16	70
1	17	50
2	18	70
3	4 - 5	35
4	9	35
5	10	35
6	11	35
7	14	70
8	18	70
9	19	50
30	20	70
1	5 - 11	35
2	12	35
3	13	35
4	14	70
5	20	70
6	21	50
7	22	70
8	6 - 7	35
9	13	35
40	15	20 x
1	16	60
2	22	60
3	7 - 8	35
4	16	35 x
5	8 - 9	35
6	16	60
7	17	20 x
8	18	60
9	9 - 10	35
50	18	25 x

member	prints	Q <sub>0</sub>
51	10 - 11	35
2	18	60
3	19	20 x
4	20	60
5	11 - 12	35
6	20	25 x
7	12 - 13	35
8	20	60
9	21	20 x
60	22	60
1	13 - 22	25 x
2	14 - 15	150
3	16	150
4	17	150
5	18	150
6	19	150
7	20	150
8	21	150
9	22	150
70	24	100°
1	15 - 16	150
2	22	150
3	24	150
4	16 - 17	150
5	18	200
6	22	200
7	23	80
8	24	250
9	26	300
80	17 - 18	150
1	24	150
2	18 - 19	150
3	20	200
4	23	80
5	24	250
6	26	300
7	19 - 20	150
8	24	150
9	20 - 21	150
90	22	200
1	23	80
2	24	250
3	25	300
4	21 - 22	150
5	24	150
6	22 - 23	80
7	24	250
8	25	300
9	24 - 25	500
100	26	500

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REPORT NO. 13

CONTRACT NO. \_\_\_\_\_

PAGE 29 OF \_\_\_\_\_

DATE \_\_\_\_\_

PROJECT:

SUBJECT:

Results of Structure 2e

Rum	$\omega$	Q	w	i	$df_i$	$d\gamma_i$	$\Delta H_o$	$\Delta H_i$	$\Delta P_i$	$\Delta A$	F	$K_o$	$K_i$	G
		inch <sup>2</sup>	tons		inch	arcmin	10 <sup>-3</sup> inch	10 <sup>-5</sup> inch	inch				10 <sup>3</sup>	
2e	1000	25-500	18-26	3	2.1	1.0	170	20	.52	.70	9.5	3.3	2.6	1.0

Structure 2e converged nicely on first try. Nothing seems critical about it; the resulting changes in the cross sections are not too large, the maximum increase is a factor 1.70 (member 8-17, page 26) and the maximum decrease a factor 1.62 (member 8-16); this means the resulting final cross sections are neither too large nor too small.

With 13 homologous surface points, a 300-foot telescope could observe down to about 8 mm wavelength, and a 450-foot telescope to 1.5 cm. The largest telescope for 1 mm wavelength would be about 90 feet large. Which means that Structure 2e can be offered as a good solution.

The question of its total weight cannot be answered until Section IX of Report 4 is programmed, which now is the next thing to be done. But meanwhile some very rough estimates, for a diameter of  $D = 400$  feet, gave the following results. If defined by survival, the total weight would be about 3000 tons, and with this weight one could observe down to about 3 cm wavelength for winds up to 17 mph, which means 3/4 of all time. If standing in a radome, the weight might be about 300 - 400 tons. The thermal limit is 3 cm wavelength for temperature differences in the structure up to 5°C, which is the case in full sunshine with best protective paint. For comparison: the weight, with the (arbitrary) cross sections as given on page 26, is about 2300 tons. All numbers given are for normal steel.